Title: Tayler instability in Protoneutron stars

Speakers: Valentin Skoutnev

Collection/Series: Magnetic Fields Around Compact Objects Workshop

Subject: Strong Gravity

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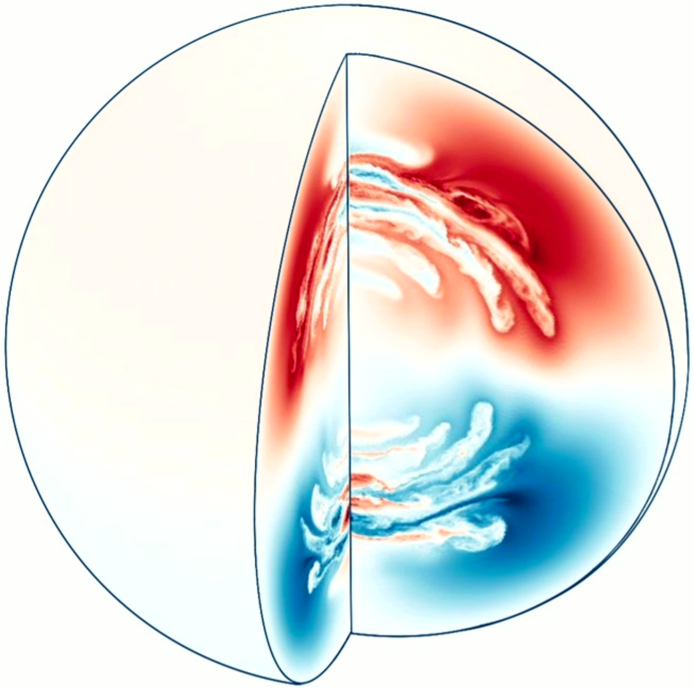
Abstract:

Amplification of magnetic fields by differential rotation and feedback by magnetic instabilities is one of the main mechanisms for magnetizing a protoneutron star. I will discuss a recent revision of the Tayler instability of strong toroidal fields and its implications for the stably stratified interior of protoneutron stars. If time permits, I will briefly highlight new simulations quantifying the efficiency of the chiral dynamo instability.



Tayler instability in protoneutron stars

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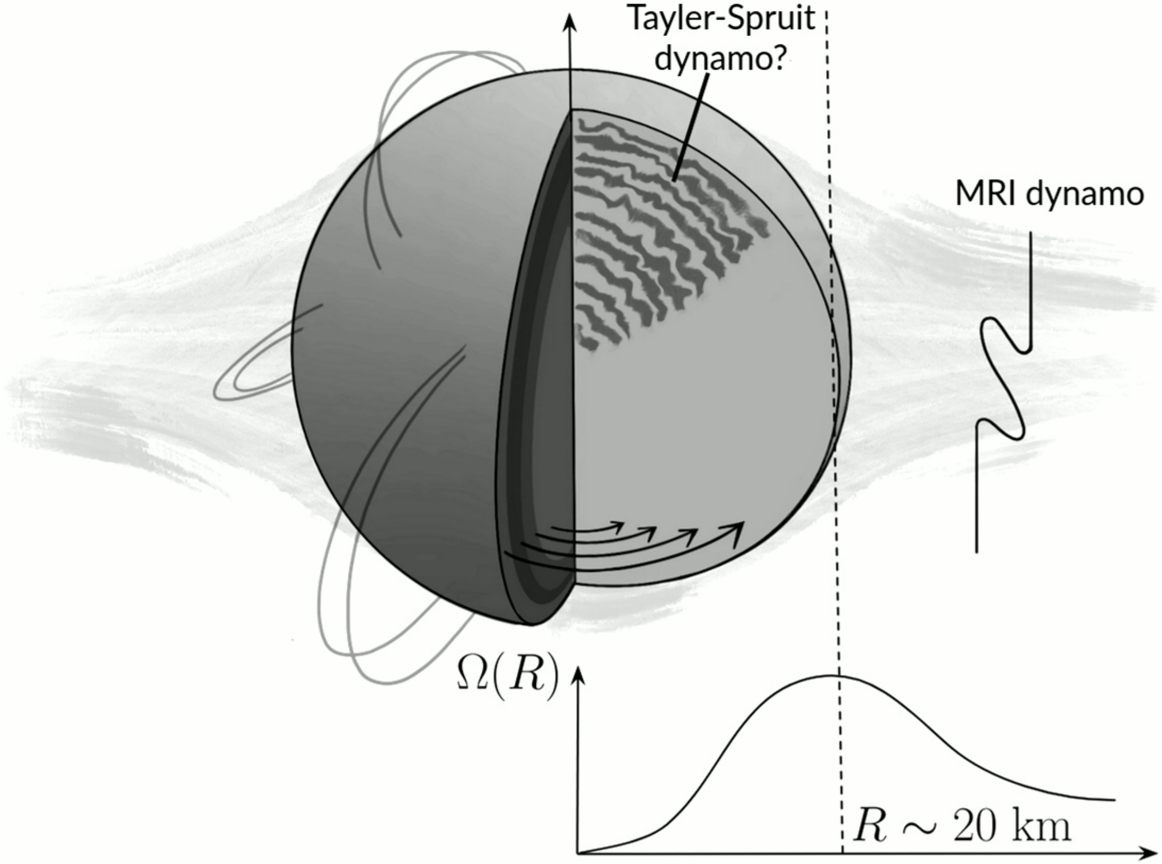
MPA

Skoutnev, Beloborodov, 2024a. Tayler instability revisited. *ApJ*.
Skoutnev, Beloborodov, 2024b. Zones of Tayler instability in stars. *ApJ (in review)*.
Skoutnev, Beloborodov, Nattila. *in prep*.

In collaboration with Andrei Beloborodov

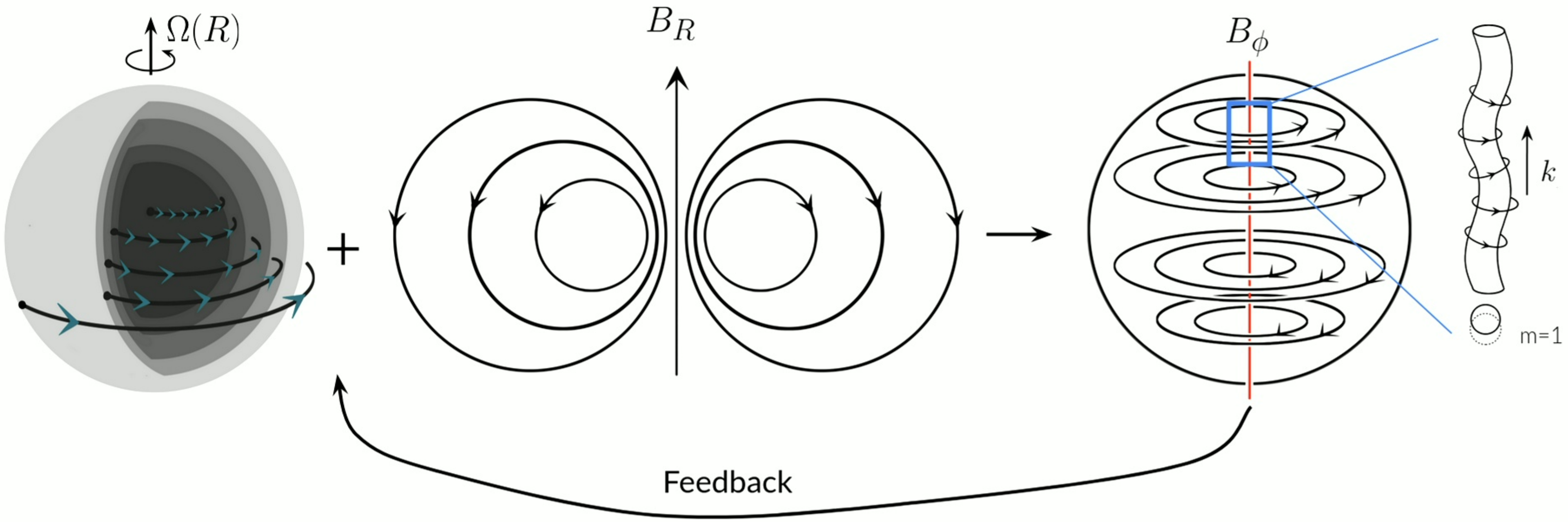
Dynamos in a PNS/merger remnant



$$t_{\mathrm{KH}} \sim \frac{GM^2}{RL_\nu} \sim 10\,\mathrm{s}$$

Differential rotation & magnetic field

Tayler instability of toroidal fields



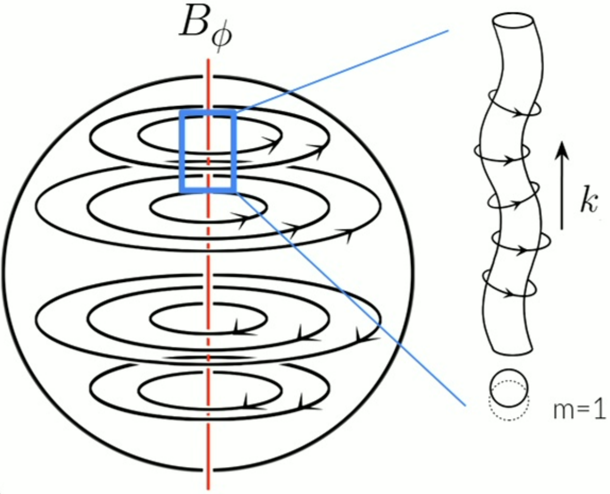
SPRUIT 2002

Tayler Instability Revisited

Complete characterization of all unstable roots in “Tayler instability revisited”
SKOUTNEV & BELOBORODOV 2024a. ApJ.

Previous analysis:

- Based on marginal stability
- Missed the most unstable wavenumbers, their stability criteria, and some roots.



$$D(\omega) = \left[\overbrace{\omega_\nu\omega_\eta}^{\text{Inertia}} - \overbrace{m_\star^2\omega_{\rm A}^2}^{\text{Alfvén}} - \overbrace{\left(\frac{k_\theta^2 N^2}{k^2}\right)\left(\frac{\omega_\eta}{\omega_\kappa}\right)}^{\text{Buoyancy}}\right]\left[\overbrace{\omega_\nu\omega_\eta}^{\text{Inertia}} - \overbrace{m^2\omega_{\rm A}^2}^{\text{Alfvén}}\right] - \left[\overbrace{2\Omega\frac{k_z}{k}\omega_\eta}^{\text{Coriolis}} + \overbrace{2m\omega_{\rm A}^2\frac{k_z}{k}}^{\text{Hoop Stress}}\right]^2 = 0,$$

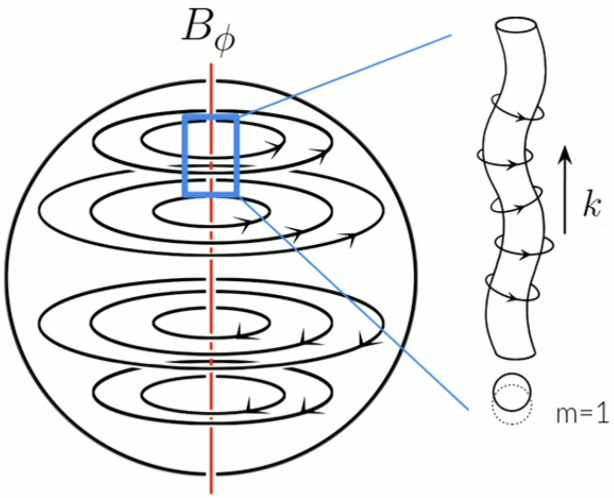
$$\omega_s \equiv \omega + isk^2,\ s \in \{\nu, \eta, \kappa\}$$

Tayler Instability Revisited

Four ingredients:

- Stable stratification
- Rotation
- Toroidal magnetic field
- Diffusivities: ν, η, κ

$$N > \Omega > \omega_{\rm A} \equiv \frac{B_\phi}{\sqrt{4\pi\rho R^2}}$$



Stable stratification (no rotation)

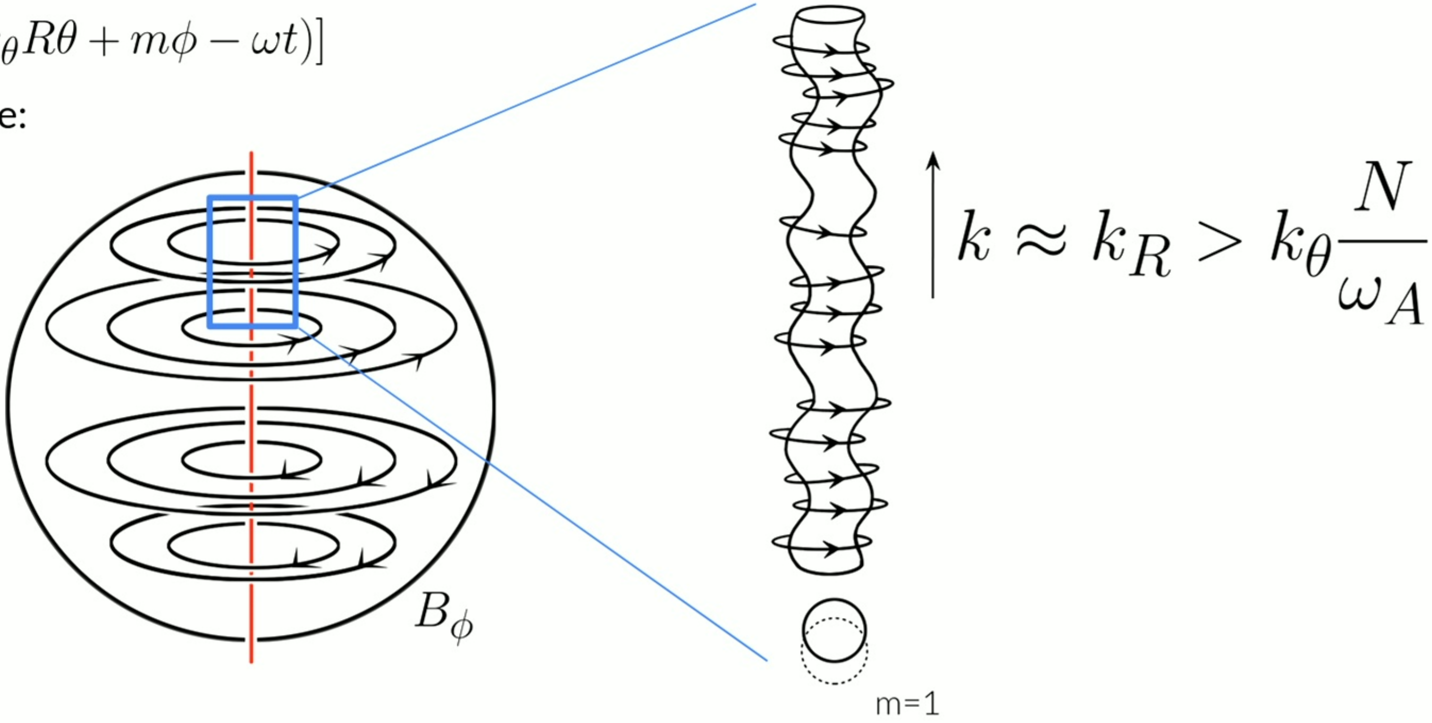
TAYLER 1973

- Unstable modes have **short** radial length scales

$$\delta \boldsymbol{B} \propto \exp[i(k_R R + k_\theta R\theta + m\phi - \omega t)]$$

- Frequency and growth rate:

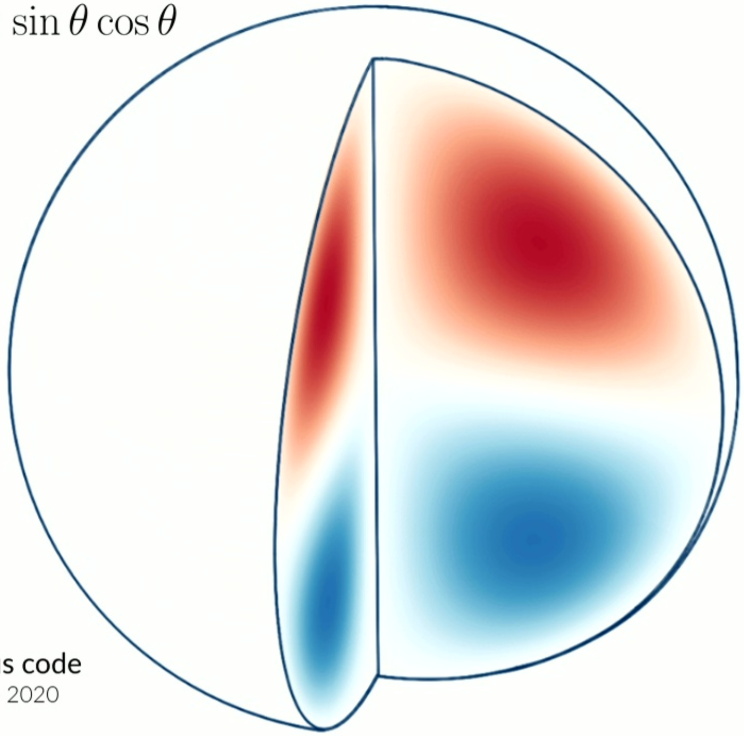
$$\omega_r = 0$$
$$\gamma = \omega_\text{A}$$



DNS of TI with no rotation

SKOUTNEV, BELOBORODOV, NATTILA, 2025. *in prep.*

$B_\phi \propto \sin\theta\cos\theta$



Dedalus code

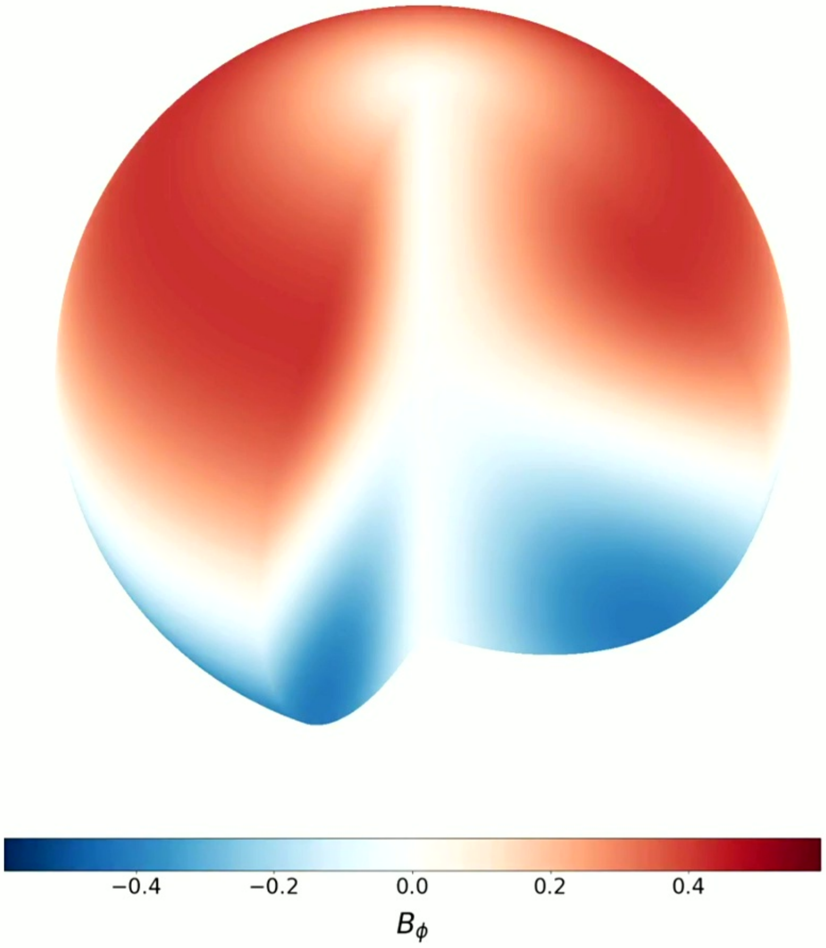
BURNS+ 2020

$$\frac{N}{\omega_\mathrm{A}} = 8$$

(expect ~ 8 wavelengths)

$$\frac{\omega_\mathrm{A}}{\eta/R^2} = 10^5$$

$$\nu = \eta = \kappa$$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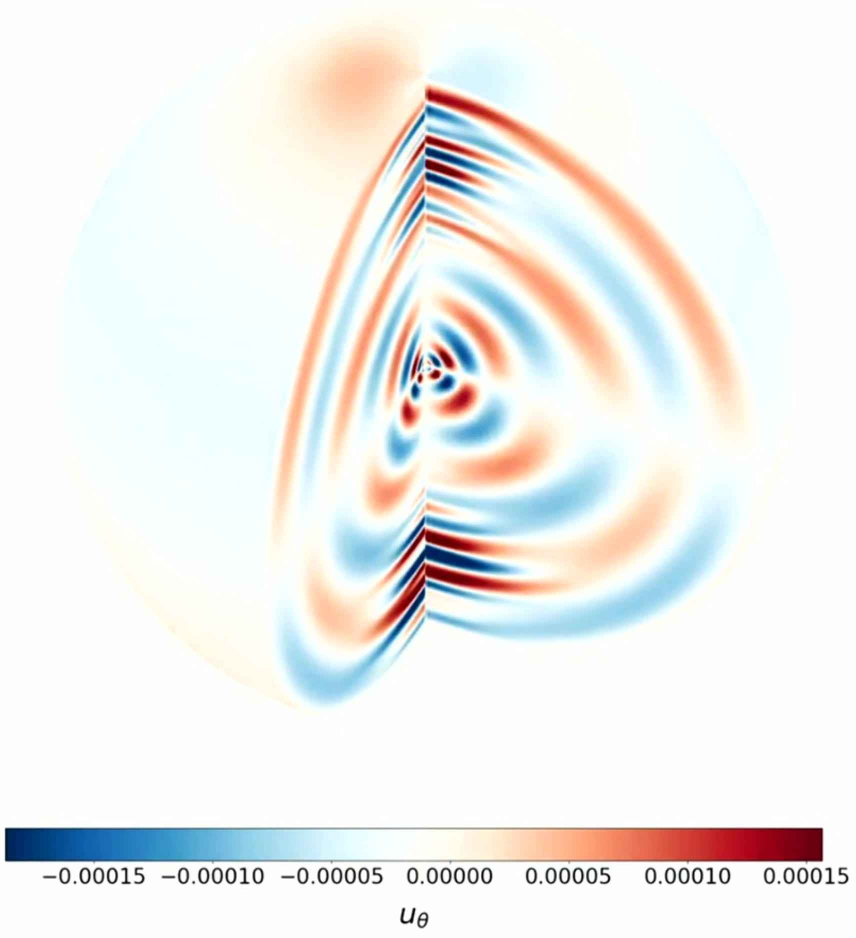
0.0
0.2
0.4
B_ϕ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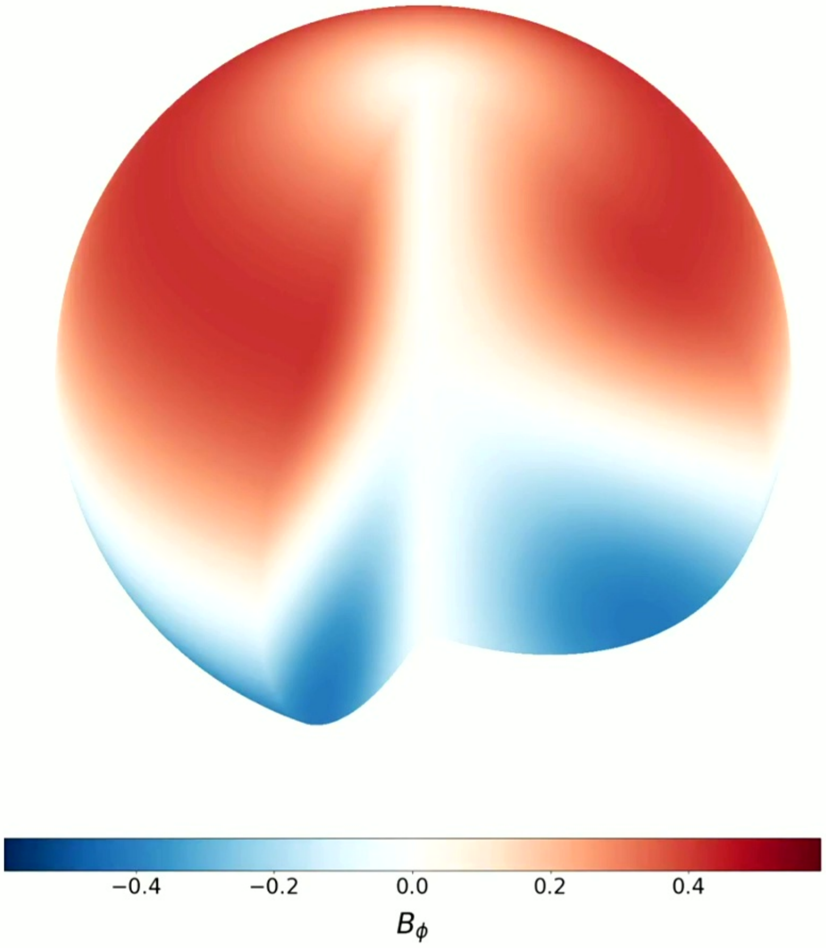
−0.00005
0.00000
0.00005
0.00010
0.00015
u_θ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0.0
0.2
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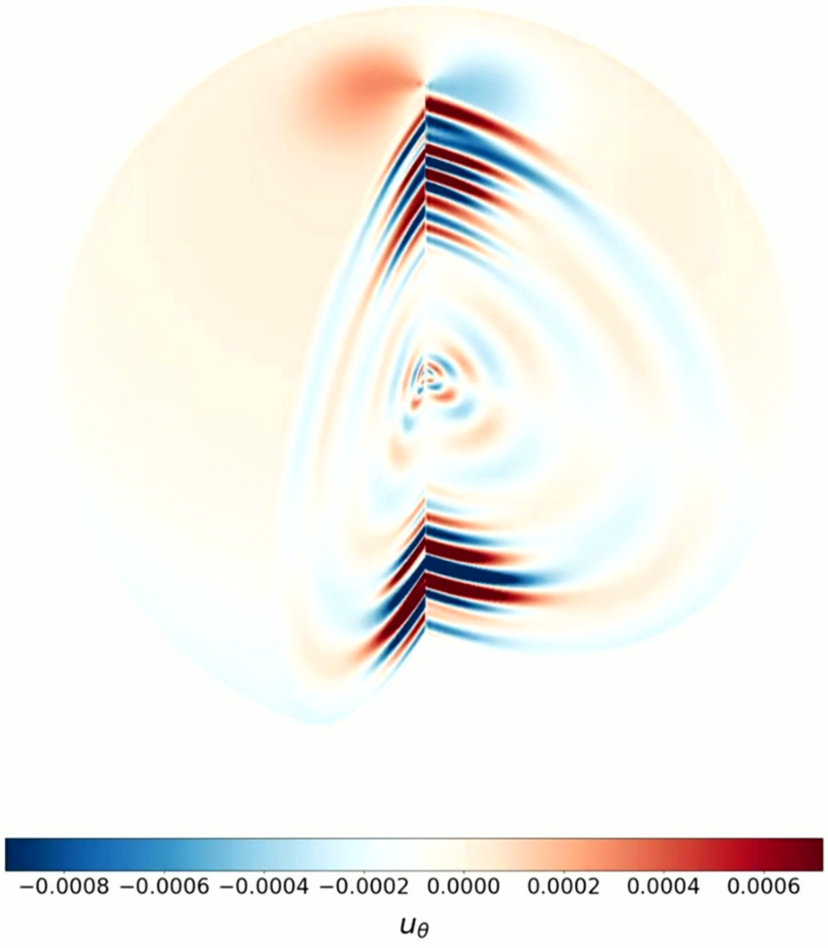
B_ϕ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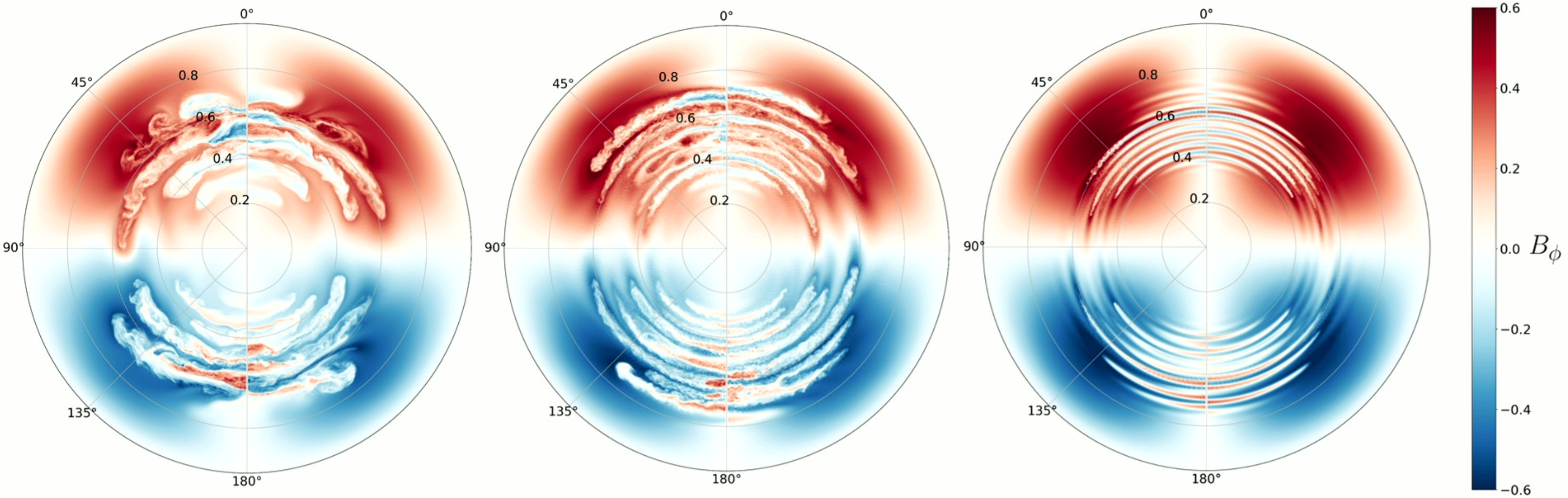
−0.0006
−0.0004
−0.0002
0.0000
0.0002
0.0004
0.0006
u_θ

Increasing Stratification



$$\frac{N}{\omega_{\rm A}} = 4$$

$$\frac{N}{\omega_{\rm A}} = 8$$

$$\frac{N}{\omega_{\rm A}} = 32$$

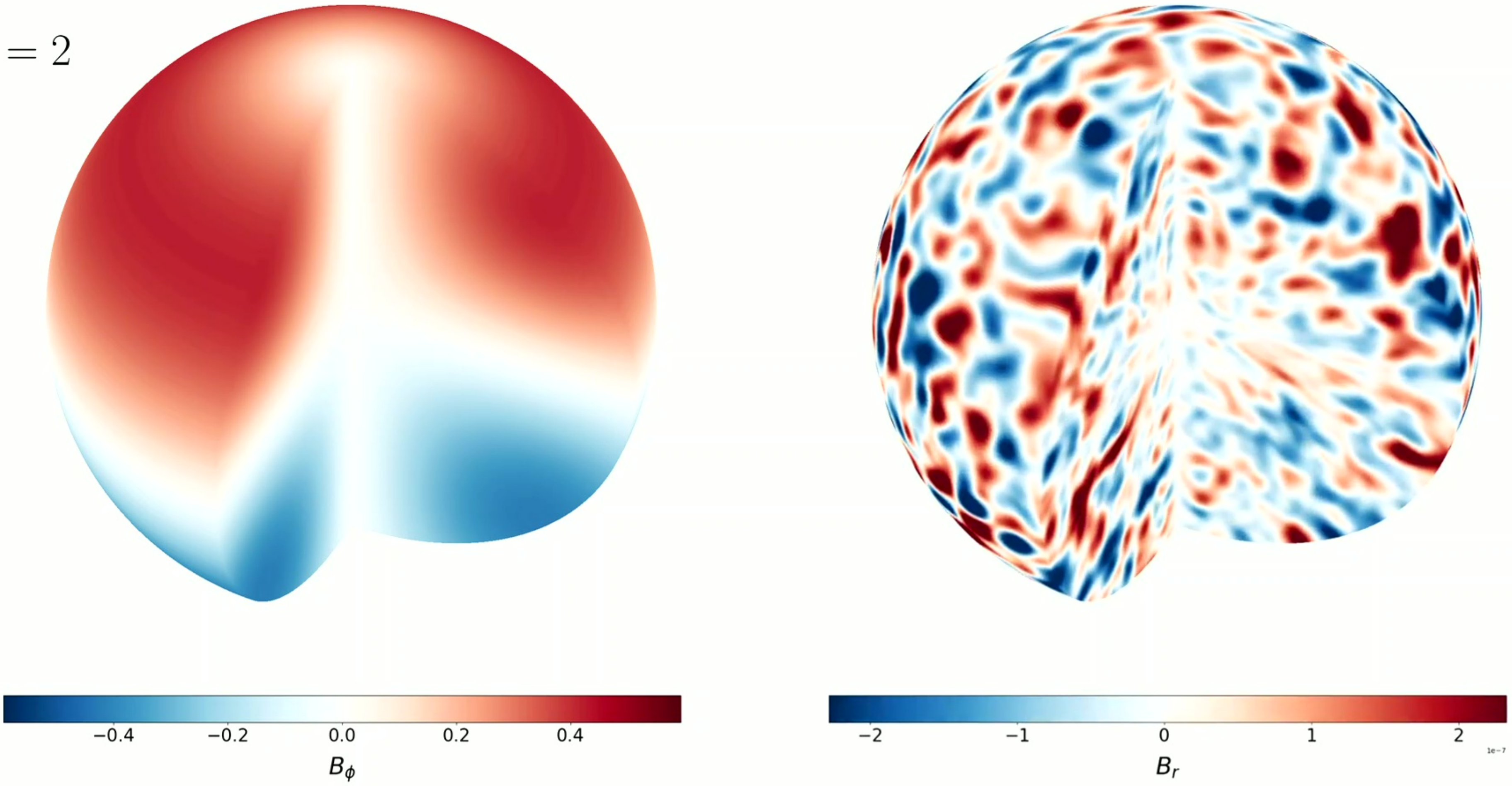
Rotation

SKOUTNEV & BELOBORODOV 2024a

- No instability in ideal MHD
- A rapidly rotating, magnetized fluid supports two classes of waves

Magnetostrophic waves	**Inertial waves**
Coriolis force ~ Lorentz force	Coriolis force ~ Inertia
Low frequency $\omega_r \sim \frac{\omega_A^2}{2\Omega}$	High frequency $\omega_r \sim 2\Omega$

$$\frac{\Omega}{\omega_A} = 2$$



Rotation

SKOUTNEV & BELOBORODOV 2024a

- Diffusivities break rotational constraint when: $t_{\mathrm{diffusion}}(k) \sim \frac{1}{2\Omega}$

$$k_{\kappa_{\mathrm{th}}} = \left(\frac{k_\theta^2 N_{\mathrm{th}}^2}{2\Omega\kappa_{\mathrm{th}}}\right)^{\frac{1}{4}} \qquad k_\nu = \left(\frac{2\Omega}{\nu}\right)^{\frac{1}{2}} \qquad k_\eta = \left(\frac{2\Omega}{\eta}\right)^{\frac{1}{2}}$$

Thermal diffusion

Viscosity

Magnetic diffusion

Magnetostrophic waves

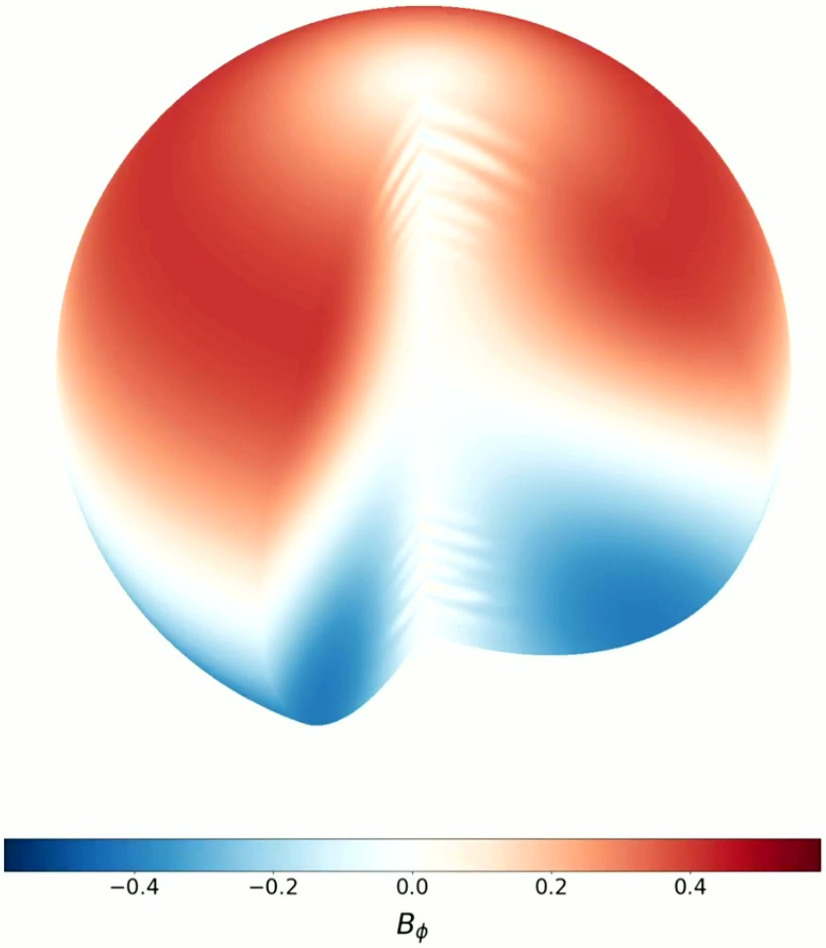
Low frequency: $\omega_r \sim \frac{\omega_A^2}{2\Omega}$

Growth rate: $\gamma \sim \frac{\omega_{\mathrm{A}}^2}{4\Omega}$

Inertial waves

High frequency: $\omega_r \sim 2\Omega$

Growth rate: $\gamma \sim \frac{\omega_{\mathrm{A}}^2}{4\Omega}$


−0.4
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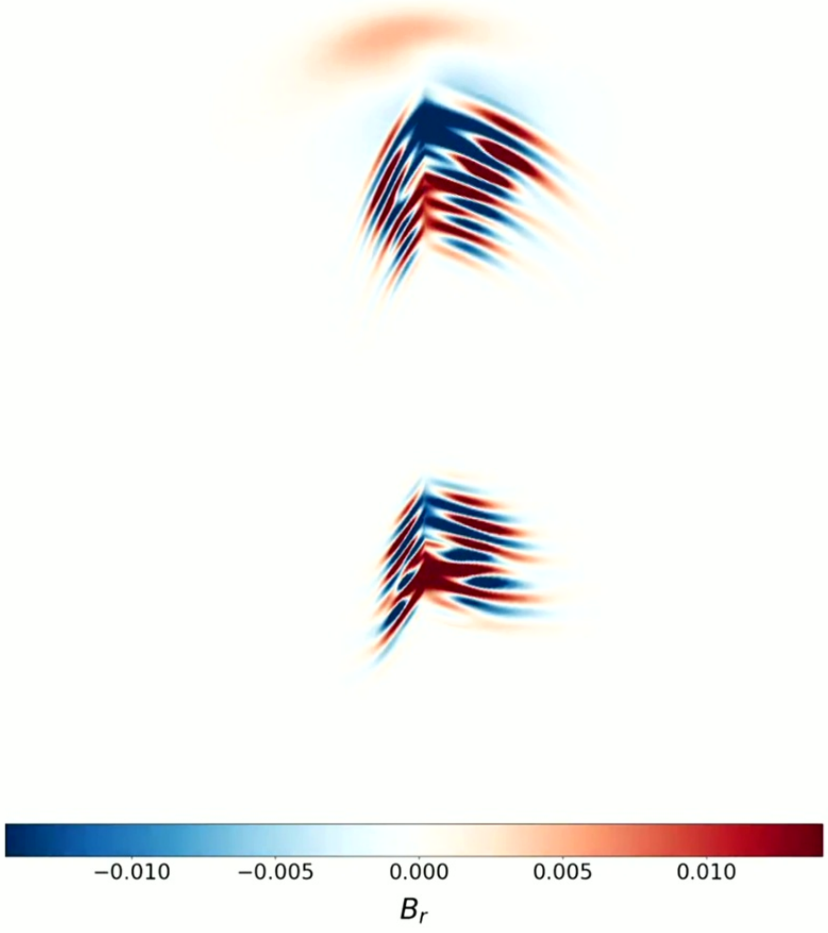
0.0
0.2
0.4
B_ϕ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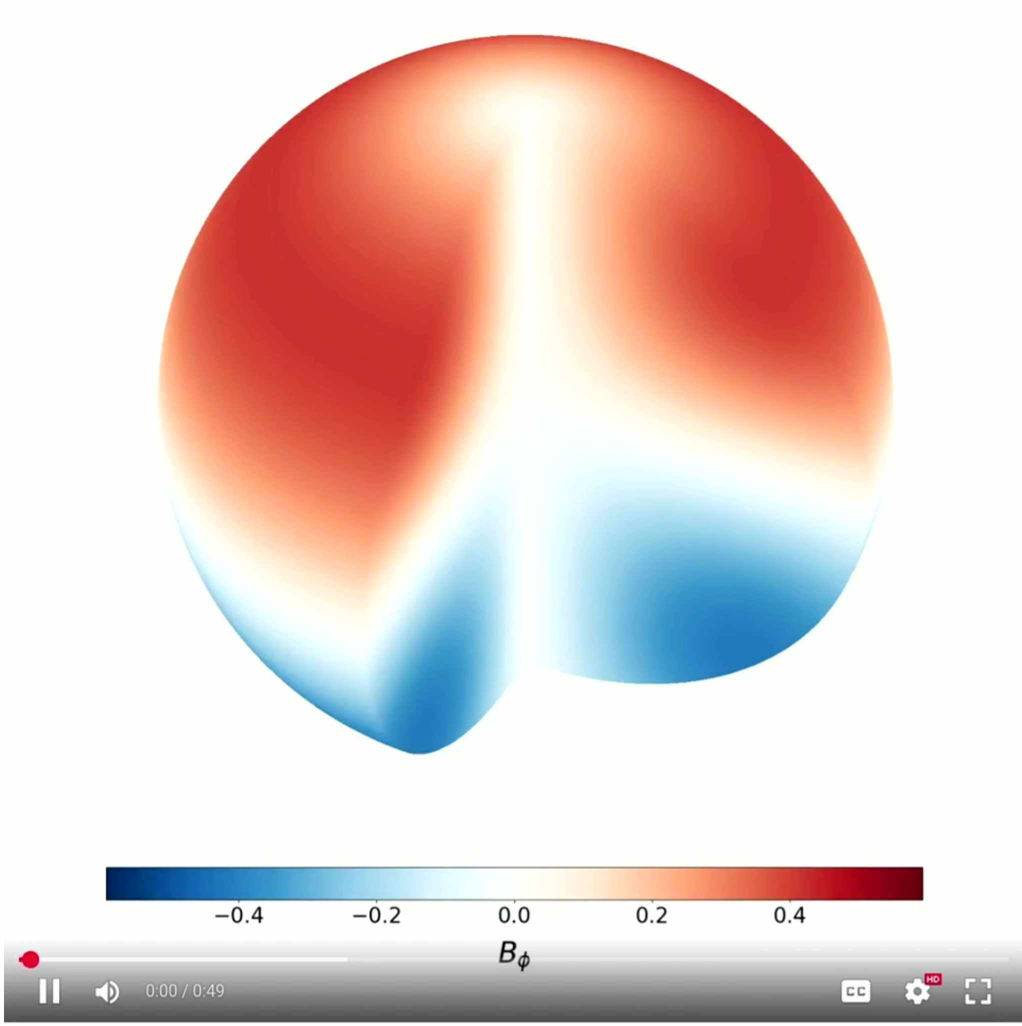
0.010
B_r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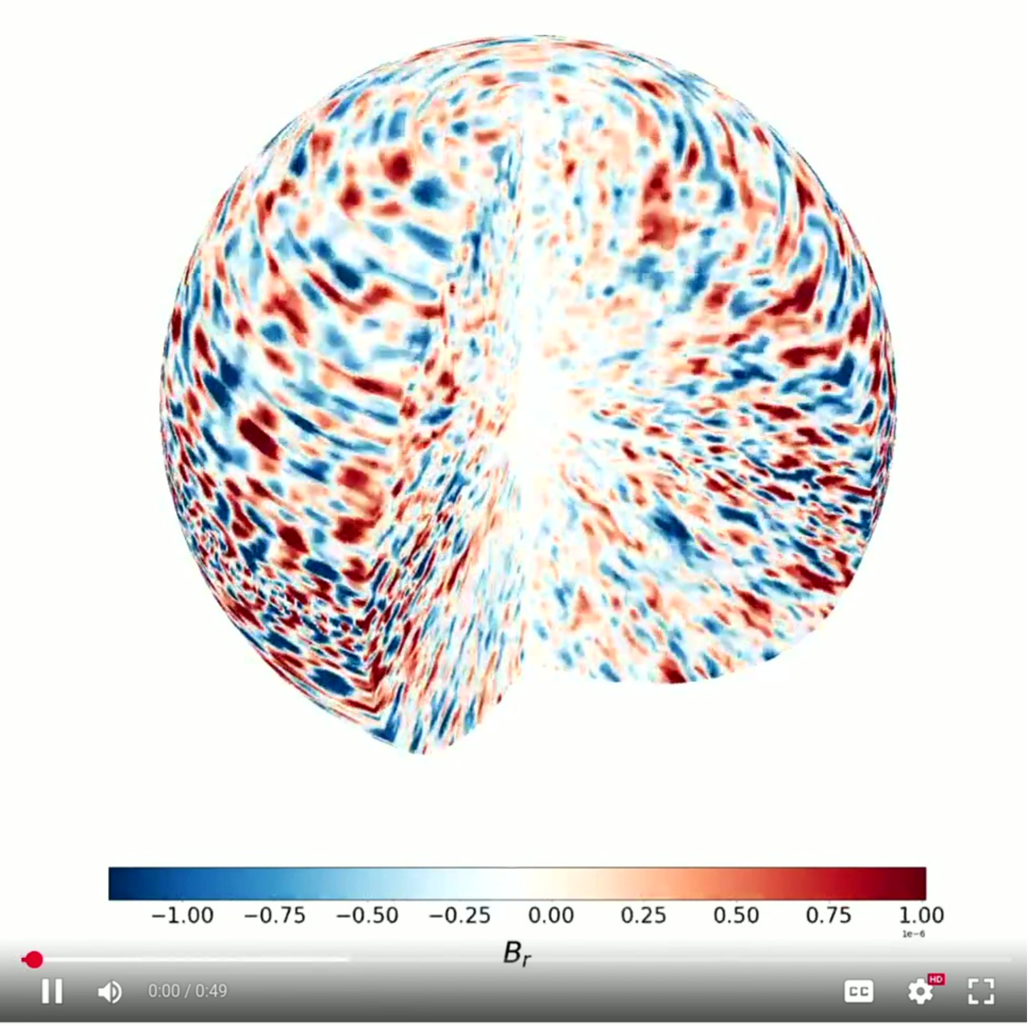
−0.2
0.0
0.2
0.4
B_ϕ
0:00 / 0:49


−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
1e−6
B_r
0:00 / 0:49

Tayler instability in a protoneutron star

Skoutnev & Beloborodov (2024a). *ApJ*.
Skoutnev & Beloborodov (2025). *in prep*.

- With the revisited stability analysis, we now know the wavelengths of the instability

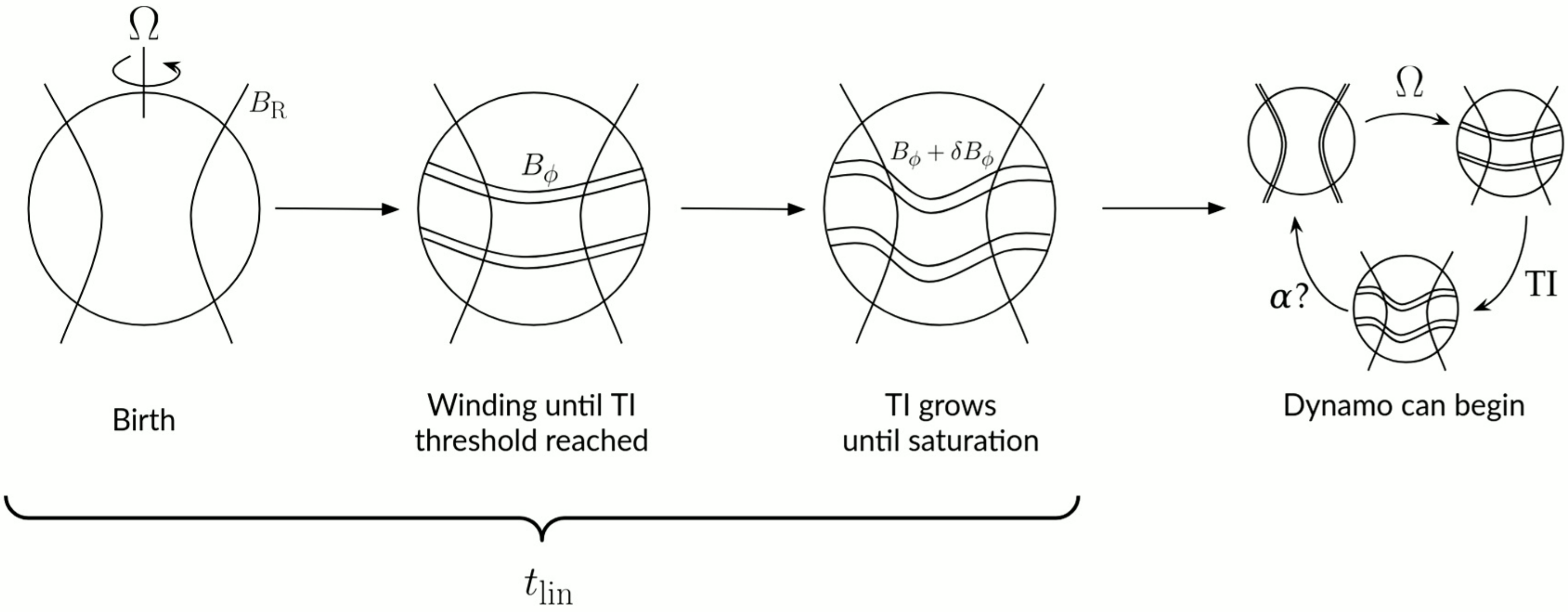
Diffusive-buoyancy enabled mode: $\lambda_\kappa \sim 50\ \mathrm{m}$

Viscosity enabled mode: $\lambda_\nu \sim\ 5\mathrm{m}$

- Growth rate is slow. On magnetostrophic timescales:

$$\frac{1}{\gamma} \sim \omega_{\mathrm{A}}^{-1}\left(\frac{\Omega}{\omega_{\mathrm{A}}}\right) \sim 1\ \mathrm{s}\ \frac{\Omega_3 \rho_{14} R_6^2}{B_{\phi,15}^2}$$

Can Tayler instability contribute to the PNS/merger dynamo?



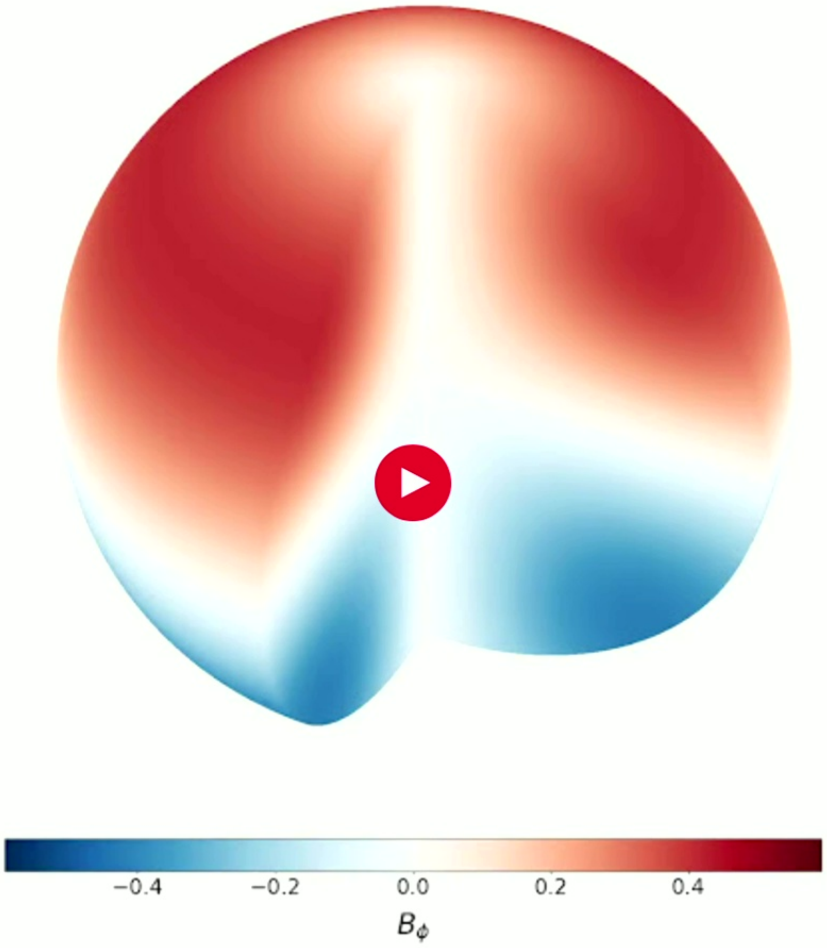
Birth

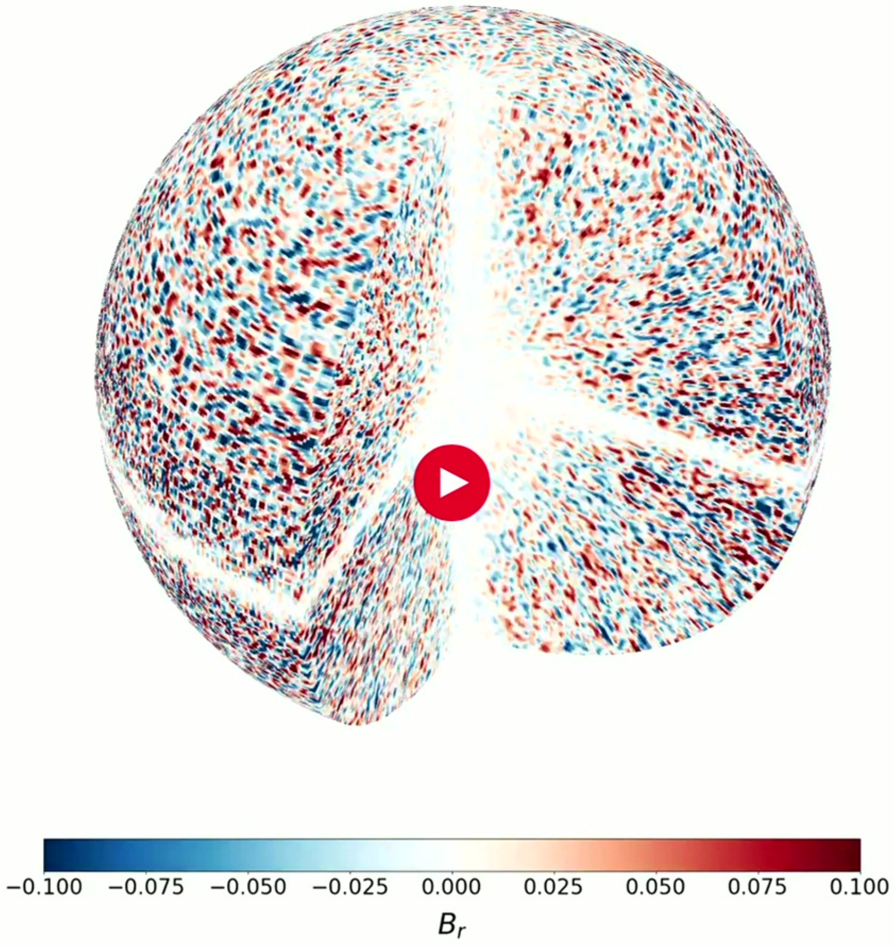
Winding until TI threshold reached

TI grows until saturation

Dynamo can begin

Linear phase must fit in Kelvin-Helmholtz time $\mathrm{t}_\mathrm{KH} \approx 10\mathrm{s}$


−0.4
−0.2
0.0
0.2
0.4
B_ϕ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0.025
0.050
0.075
0.100
B_r