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Predictability is Typical in Gravitational Collapse

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Conventional wisdowm

Typical high energy/temperature states in quantum gravity are black holes.

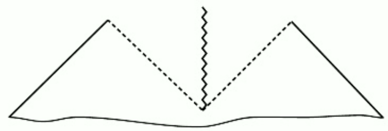
This lore underlies much of modern understanding of quantum black holes, including:

1. Black hole thermodynamics (BH exteriors as thermal ensembles);
2. Black hole microstate counting; e.g. Strominger-Vafa in the contexto of string theory
3. Black hole scrambling calculations starting with Sekino-Susskind
4. Maximal chaos of near-equilibrium black holes. in AdS: Maldacena-Shenker-Stanford; recently Chen-Lin-Shenker

However...

The status of black hole typicality, often assumed implicitly, is conjectural at best:

- There are theorems that guarantee the existence of singularities e.g. Penrose, '69 but...
- There are *no* theorems that guarantee event horizon formation; very littlie is known about how prevalent event horizons are in dynamical states.



Event horizon formation is only established under assumption of *global hyperbolicity* or at least strong asymptotic predictability. But...

Global hyperbolicity isn't true in many states

There are examples of generic, regular gravitational initial data sets evolving to form singularities without horizons:

1. Gregory-Laflamme, "pinch-off" instability in $D \geq 5$; Lehner, Pretorius; Figueras, Kunesch, Tunyasuvunakool; Figueras, Kunesch, Lehner, and Tunyasuvunakool...

Image credit: P. Figueras

2. Charged solutions in AdS_4 Horowitz, Santos; Crisford, Horowitz, Santos
3. Scalar matter in AdS_4 Folkestad and possibly (speculatively) in AF_4. Santos, Eperon
4. The evaporating black hole.

The Options

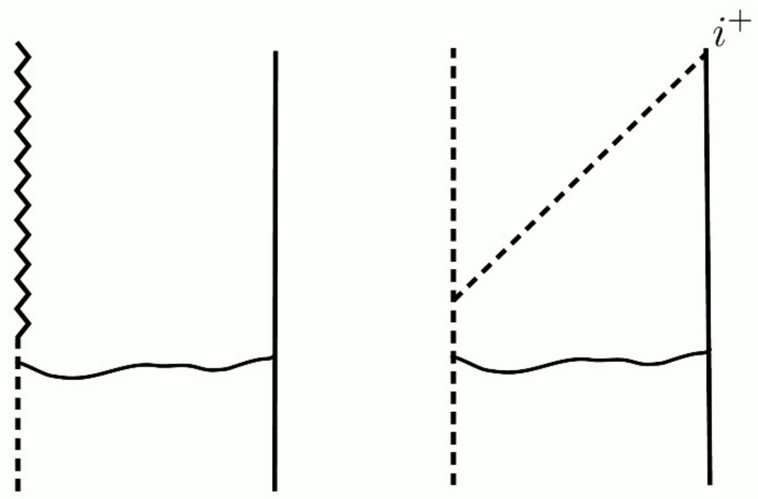
The existence of generic (open sets of) initial data that evolve to form naked singularities presents two options:

1. Horizonless gravitational collapse is typical in quantum gravity high energy states;
2. Horizonless gravitational collapse is typical in classical GR, but is *atypical* in quantum gravity.

Option 1 requires a “conspiracy”: naked singularities must exactly mimic BH typical state behavior.

A Strange 'Conspiracy'

But the two systems are not only dramatically different – typicality of black holes has been specifically linked to properties of the event horizon at adiabaticity. Naked singularities are (a) horizonless, and (b) highly dynamical.



Option 2: matter that supports naked singularities has no UV completion?

Evidence in favor:

- Many of the known examples of generic naked singularities violate the weak gravity conjecture;
- It is possible to prove the AdS Penrose Inequality from consistency of the AdS/CFT dictionary NE, Horowitz '19: this inequality is an upper bound on the area of compact minimal surfaces in terms of the asymptotic mass. Its validity is necessary (but not sufficient) to rule out naked singularities.

Option 2: matter that supports naked singularities has no UV completion?'

Problems:

- Evaporating black holes are expected to end in a brief naked singularity – which should occur in typical states.
- Pinch-off/Gregory-Laflamme type instability is in pure $D \geq 5$ gravity as a consequence of the evolution of ordinary black holes. Surely that is also typical...
- Certain timelike "singularities" admit a resolution in classical string theory, without access to nonperturbative QG. If they admit a resolution, surely they can also exist...

Implications of Option 2

Consistency of Option 2 requires two classes of naked singularities:

1. Ones that are “large” and/or whose resolution requires nonperturbative QG.
2. Ones that are “small” or can possibly be resolved without access to nonperturbative QG;

The problems are (a) the apparent typicality of the first category, and (b) distringuishing between the two categories.

Horizonless, Nonperturbative Singularities

Want to argue that horizonless singularities are atypical unless they can be in the second category: small or perturbatively resolvable.

A Difficulty

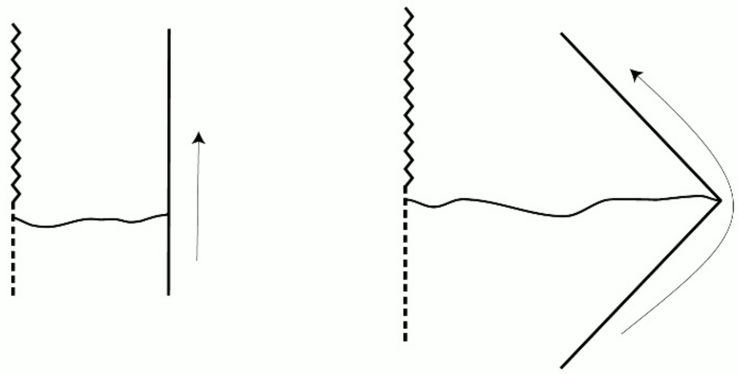
The definition of a singularity – the existence of incomplete, inextendible geodesics – is classical.

Global hyperbolicity is violated in both types of singularities because it is insensitive to how "small" or "large" (or nonperturbative) a singularity is.

Want: a *semiclassical* generalization of global hyperbolicity

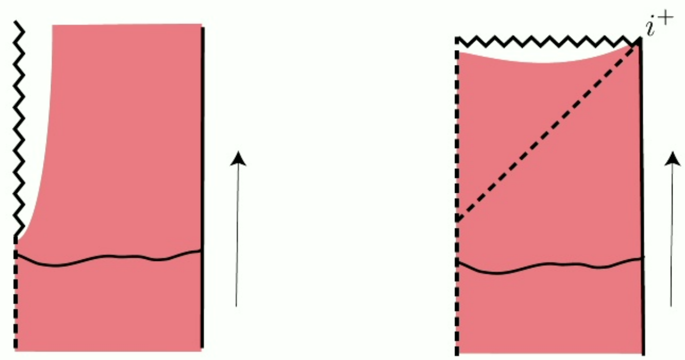
We want to notion of global hyperbolicity that can distinguish between the two cases; then we would like to prove that this notion is valid in typical states.

Step 1: assume there is a notion of time evolution in the fundamental description (e.g. asymptotic time).



Towards Semiclassical global hyperbolicity

Consider a "semiclassical" experimentalist, living in the low curvature region.



The experimentalist makes measurements on the system – e.g. by doing scattering experiments. To maintain semiclassicality, the experimentalist is limited to a number of experiments that is very small compared with e^M, where $M = \log \dim \mathcal{H}_{QG} \sim (G\hbar)^{-1}$.

The experimentalist then processes the data obtained in the measurements for a short time (short compared with times that scale exponentially in $1/G\hbar$).

Towards Semiclassical global hyperbolicity

The experimentalist then morphs into a theorist, whose goal in life is to produce a prediction for the time evolution operator U of the system that the experimentalist was observing.

If the theorist produces a guess O_{guess} that successfully reproduces the expectation values of all operators under the *actual* time evolution:

$$\langle \mathcal{O} \rangle_{O_{\text{guess}}(\psi)} \approx \langle \mathcal{O} \rangle_{U(\psi)}$$

We define such time evolution U to be *effectively predictable*. Predictivity can persist *after the semiclassical theory breaks down*.

Semiclassical global hyperbolicity = effective predictability

We define a semiclassical generalization of global hyperbolicity:

Effective predictability

The time evolution U of a system of size $\log|\mathcal{H}|$ is *effectively predictable* if there exists a quantum algorithm that runs in a time no longer than $\text{poly}(\log|\mathcal{H}|)$, which queries the system no more than $\text{poly}(\log|\mathcal{H}|)$ times, that can guess an operator O_{guess} such that for all operators $\mathcal{O}$ and all states ψ (in the code subspace):

$$\langle\mathcal{O}\rangle_{U(\psi)} = \langle\mathcal{O}\rangle_{O_{guess}(\psi)} + \frac{1}{\text{poly}(\log|\mathcal{H}|)}$$

Otherwise, U is *effectively unpredictable*.

Goal of This Talk

Goal

Show that:

1. All globally hyperbolic spacetimes are effectively predictable;
2. The evaporating black hole is effectively predictable;
3. By a natural definition of "smallness', small singularities are predictable;
4. Unpredictable timelike singularities are atypical.

Main Technical Tool: The Pauli Basis

We work in semiclassical gravity with a UV cutoff, so that

$$\mathrm{d}im\mathcal{H}_{SC} = e^{M}$$

and M is large.
The time evolution in the cutoff theory, which includes boundary conditions inherited from QG at any timelike singularities, is denoted C:

$$C : \mathcal{B}(H^M) \to \mathcal{B}(H^M)$$

Allowing for the possibility that $\mathcal{H}_{SC}$ may change over time, H^M denotes a large Hilbert space that contains both $\mathcal{H}_{SC}$ and its image under C.

Main Technical Tool: The Pauli Basis

Predictability in the Pauli Basis Huang, Chen, Preskill; Caro

If C is supported on $\mathcal{O}(\text{poly}(M))$ of the elements of the Pauli basis, it is *effectively predictable*.

A local operator propagating in the lightcone takes $\mathcal{O}(1)$ Paulis to $\mathcal{O}(1)$ Paulis.

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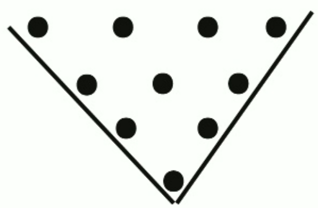
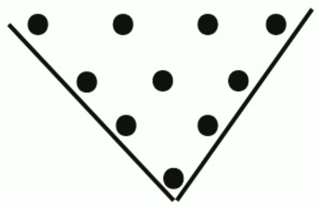
If C is supported on $\mathcal{O}(\text{poly}(M))$ of the elements of the Pauli basis, it is *effectively predictable*.

A local operator propagating in the lightcone takes $\mathcal{O}(1)$ Paulis to $\mathcal{O}(1)$ Paulis. If you compose it with itself $\text{poly}(M)$ times it will take $\mathcal{O}(1)$ Paulis to $\mathcal{O}(\text{poly}(M))$ Paulis: remaining predictable.



Globally Hyperbolic Spacetimes are Effectively Predictable



It will only cease to be effectively predictable at exponentially late times, which require nonperturbative quantum gravity.

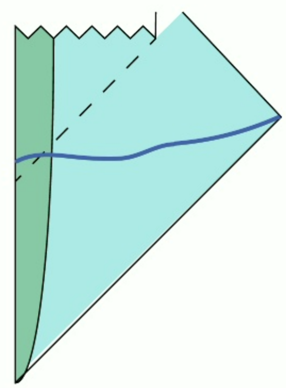
So we find that our generalization of global hyperbolicity to the semiclassical regime does include standard global hyperbolicity.

Black Hole Evaporation in the EFT Perspective

Recall: we are trying to ascertain the extent to which a semiclassical observer can predict the evolution of semiclassical spacetime.

We are *not* (in this talk) trying to figure out the quantum gravity evolution of the system.

In the evaporating black hole, the entire evolution up to times of $\mathcal{O}(t_{\text{evap}})$ is the domain of dependence of a single time slice:



Near Evaporation

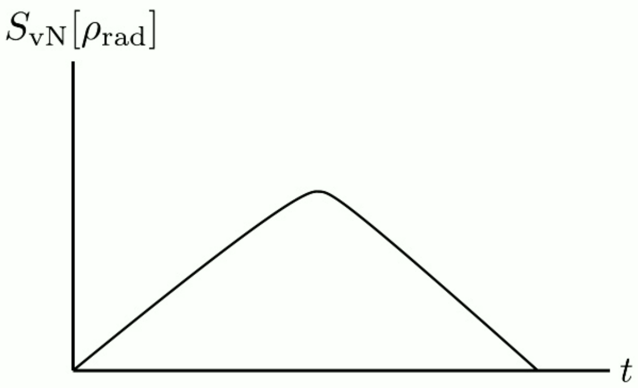
- Curvatures diverge in $1/\ell_P$ at times of order $t_{\text{evap}} - \mathcal{O}(1)$.
- At these times, the area of the black hole is $\mathcal{O}(G_N)$.
- By now, in full nonperturbative quantum gravity, the black hole has an entropy

$$S_{BH} = \frac{\text{Area}[BH]}{4G} \sim \mathcal{O}(1)$$

- So number of bits remaining in the black hole is $e^{\mathcal{O}(1)} \sim \mathcal{O}(1)$.

Can a poly(M) complexity quantum algorithm always guess an operator that acts on $\mathcal{O}(1)$ qubits, no matter how complex that operator is?

Evolution Through the Evaporation Point is Effectively Predictable



While the evaporating black hole isn't globally hyperbolic, the singularity is "small" in the sense of having a very limited amount of information – only $\mathcal{O}(1)$ bits. Such singularities should be included in any semiclassical generalization of global hyperbolicity – and they are for us!

More Generally...

More generally, we would like singularities that are small in duration (by some definition) to be included in effective predictability.

The evaporating black hole suggests the singularities should be small information theoretically.

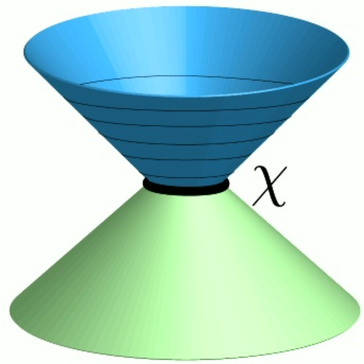
Rough idea: use the Bekenstein/Bousso bound.

Information Bound

The Bousso bound is a conjecture that generalizes the Bekenstein bound:

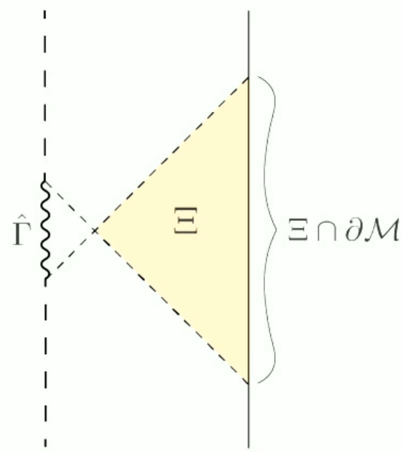
$$\frac{\text{Area}[\chi]}{4G} > S$$

where χ is a spacelike surface that splits a time slice in two and S is the von Neumann entropy across χ on the time slice.



If χ circumscribes the singularity and $[Area](\chi) \sim \mathcal{O}(G)$, we recover the case of the evaporating black hole: there are $\mathcal{O}(1)$ bits (at most) in the singularity.

Schematically



If Area$[\chi] \sim \mathcal{O}(G)$, then assuming the Bousso bound, we call the singularity "small".

All small singularities are effectively predictable.

Overview

All Globally Hyperbolic Spacetimes are Effectively Predictable

The Evaporating Black Hole is Effectively Predictable

Small Singularities in General

Unpredictable timelike singularities are atypical

Conclusion

Large timelike singularities

- Such singularities, if they exist, are *unpredictable*.
- Can we show that they are very atypical?

Large timelike singularities

- Such singularities, if they exist, are *unpredictable*.
- Can we show that they are very atypical?
- Yes, but for this we need some assumptions about the relation between the semiclassical description and the full quantum gravity theory.
- We will also need time slices that intersect the singularity to intersect an asymptotic boundary.
- This means we cannot comment on the fate of the Cauchy horizon of e.g. Reissner-Nordstrom.

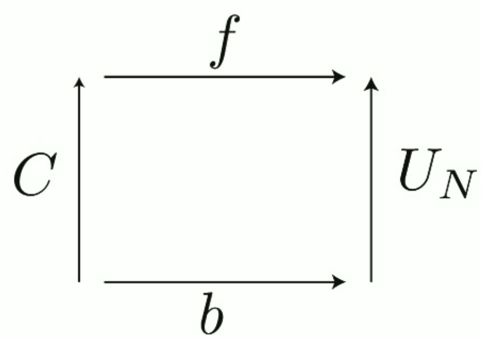
The Workhorse: Equivariance

We assume:

- On any time slice (at infinite $M \sim (G\hbar)^{-1}$), there is a map between the effective theory and the fundamental QG description:

$$\mathcal{H}_{\text{EFT}} \to \mathcal{H}_{\text{QG}}$$

- The full quantum gravity description is unitary;
- The map between effective and fundamental is equivariant:



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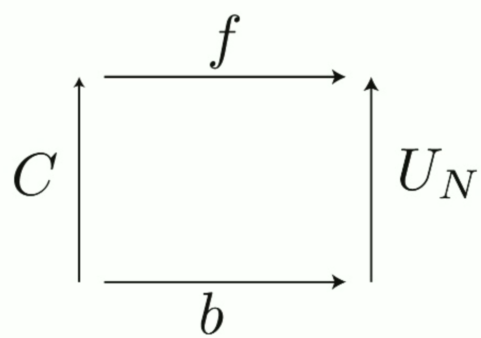
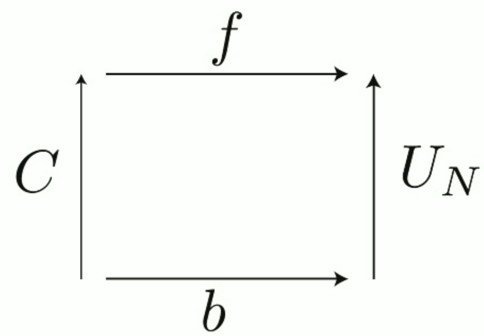


The Setup



To show atypicality, we ask what happens when we perturb the system: if we perturb the initial data by some perturbation, the map b changes by a perturbation:

$$b \to b + \delta b$$

Correspondingly, f changes:

$$f \to f + \delta f$$

On $\mathcal{O}(1)$ times, a singularity would be stable if a generic δb were to correspond to a generic δf.

The Result

We consider δb drawn at random from a Gaussian random distribution with some variance and vanishing mean.
Unitarity of U_N and unpredictability of C^M, together with equivariance, require that:

$$\mathrm{var}[\delta f] = \frac{\mathrm{var}[\delta b]}{e^{M^\alpha}}$$

for some $\alpha > 0$.

That is, a generic perturbation δb with some spread can only preserve both unpredictability and equivariance if it becomes exponentially fine-tuned with time evolution.

We calculate the volume of perturbations δb compared with the volume of the perturbations δf that can maintain equivariance: it is exponentially small.

Unpredictable timelike singularities are atypical.

Effective Predictability

Under effective predictability:

1. All globally hyperbolic spacetimes remain effectively predictable;
2. The evaporating black hole – which is expected to be typical – is effectively predictable.
3. All small singularities, as quantified by the Bousso/Bekenstein bound, are effectively predictable.
4. Unpredictable singularities are exponentially fine-tuned/atypical.

Up Ahead

- Closed universes: can we use an observer's clock for equivariance, to prove unpredictability is atypical?
- What about Cauchy horizons of non-extremal black holes?
- Can we prove semiclassical singularity theorems that would apply to spacetimes that are effectively predictable even if they are not globally hyperbolic?

Thank you!