Title: My journey from quantum coordinates to quantum reference frames

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Abstract:

In this talk I will review my work on relational observables in perturbative quantum gravity and put it in context of quantizing coordinates in gravity. I will then discuss more recent work on quantum reference frames and give some outlook on how these two strands could fit together.

Outline of the talk

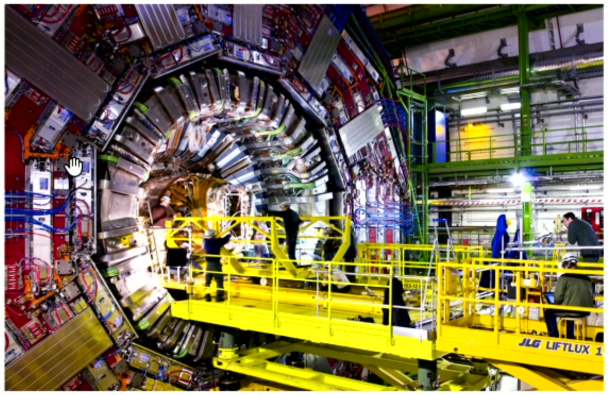
1. QFT
   - Algebraic QFT
   - pAQFT

2. AQFT and QRFs
   - Types of local algebras
   - Measurement theory and QRFs

Matter at small scales and high velocities

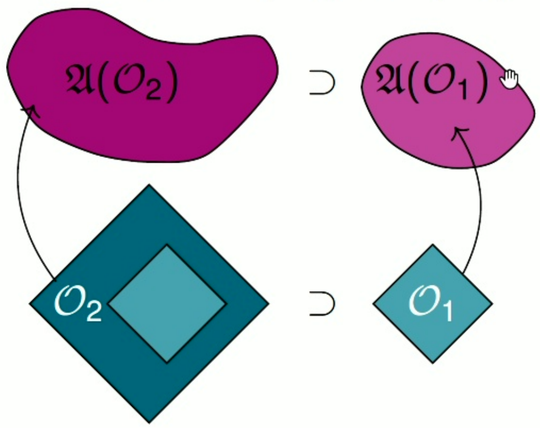
Quantum field theory (QFT) is a framework which allows to combine special relativity (SR) with quantum mechanics (QM).

- Input from SR: causality, structure of Minkowski spacetime, notions of future past and spacelike separation.
- Input from QM: observables as elements of a certain operator algebra, states, expectation values, correlations, entanglement.



Algebraic quantum field theory

- A convenient framework to investigate conceptual problems in QFT is the Algebraic Quantum Field Theory.
- Axiomatic framework of Haag-Kastler: a model is defined by associating to each region $\mathcal{O}$ of Minkowski spacetime $\mathbb{M}$ an algebra $\mathfrak{A}(\mathcal{O})$ of observables that can be measured in $\mathcal{O}$.
- The physical notion of subsystems is realized by the condition of isotony, i.e.: $\mathcal{O}_1 \subset \mathcal{O}_2 \Rightarrow \mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)$. We obtain a net of algebras.



QFT on curved spacetimes

- We mix quantum and GR but consider situations where the quantum gravity effects can be considered as small, so the spacetime is fixed.
- Replace $\mathbb{M}$ with another background M, e.g. Schwarzschild, FLRW, de Sitter, ...
- Assign algebras $\mathfrak{A}(\mathcal{O})$ to regions $\mathcal{O} \subset \mathcal{M}$.

Perturbative algebraic quantum field theory

- Perturbative algebraic quantum field theory (pAQFT) is a mathematically rigorous framework that allows to build interacting QFT models on curved spacetimes using formal power series.
- Main contributions:
  - Free theory obtained by the formal deformation quantization of Poisson (Peierls) bracket: $\star$-product ([Dütsch-Fredenhagen 00, Brunetti-Fredenhagen 00, Brunetti-Dütsch-Fredenhagen 09, . . .]).
  - Interaction introduced in the causal approach to renormalization due to Epstein and Glaser ([Epstein-Glaser 73]),
  - Generalization to gauge theories using homological algebra ([Hollands 08, Fredenhagen-KR 11]).

The (p)AQFT perspective on quantum observables

From classical to quantum

Perturbative algebraic QFT (pAQFT) is a machinery to turn functionals of classical field configurations (classical observables) into quantum observables. The choice of diff invariant objects is made on the classical level.

- The aim of this program is to study some aspects of observables in QG that are accessible to QFT methods and to learn more about the algebraic structure they define.
- The ultimate goal is to break away from the classical picture and have an intrinsically quantum formulation.

Towards full diffeomorphism invariance

- Diffeomorphism invariant observables in perturbative quantum gravity have been introduced in [Brunetti, Fredenhagen, KR 2016].
- Consider gravity coupled to matter fields, collectively denoted by φ. The variables are $\Gamma = (g, \varphi)$, where g is the metric.
- Consider four scalars X^μ_Γ, $\mu = 0, \ldots, 3$ which will parametrize points of spacetime. The fields X^μ_Γ transform under diffeomorphisms χ as

$$X^\mu_{\chi^*\Gamma} = X^\mu_\Gamma \circ \chi \,,$$

- One can think of the choice of X^μ as the choice of observer (or reference frame).
- Fix a background Γ_0 such that the map

$$X_{\Gamma_0} : x \mapsto (X^0_{\Gamma_0}, \ldots, X^3_{\Gamma_0})$$

is injective.

Relational observables

Taking the further step that the reference frame is a quantum system, we may conclude that:

- Take $\Gamma = \Gamma_0 + \gamma$ sufficiently near to Γ_0 and set
$$\alpha_\Gamma = X_\Gamma^{-1} \circ X_{\Gamma_0} \,.$$
- α_Γ transforms under diffeomorphisms as
$$\alpha_{\chi^*\Gamma} = \chi^{-1} \circ \alpha_\Gamma \,.$$
- Take another local field $A_\Gamma(x)$ (e.g. a metric scalar). Then $\mathcal{A}_\Gamma := A_\Gamma \circ \alpha_\Gamma$ is invariant under diffeomorphisms.
- Such observables can be quantized in the framework of pAQFT.
- Under construction: connect these to QRFs in the sense of Loveridge et.al.

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Examples

- On generic backgrounds g_0, without matter, one can use traces of the powers of the Ricci operator:

$$X_g^a := \mathrm{Tr}(\mathbf{R}^a), \qquad a \in \{1, 2, 3, 4\}$$

- More examples: [Bergmann 61, Bergmann-Komar 60].
- When matter fields are present in the model, also these can serve as coordinates, e.g. the dust fields in the Brown-Kuchař model [Brown-Kuchař 95]; 4 scalar fields coupled to the metric.
- For an explicit construction on a cosmological background see my work with R. Brunetti, K. Fredenhagen, T.-P. Hack and N. Pinnamonti: *Cosmological perturbation theory and quantum gravity*, [JHEP 2016].
- See also papers by Fröb et. al. [JCAP 2017, CQG 2018].

Types of von Neumann algebras

We model $\mathfrak{A}(\mathcal{O})$ as von Neumann algebras. These come in various sorts, roughly corresponding to different physical settings:

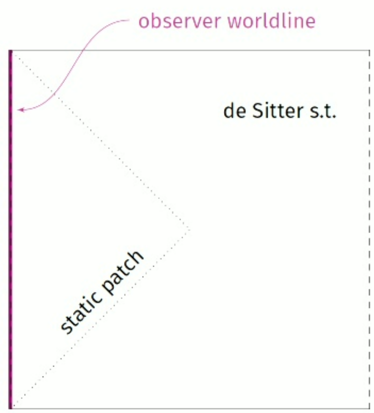
- Type I_n (qubits) $\leftarrow$ finite trace,
- Type I_∞ (quantum mechanics) $\leftarrow$ semifinite trace,
- Type II_1 (quantum stat. mech.) $\leftarrow$ finite trace,
- Type II_∞ (quantum stat. mech.) $\leftarrow$ semifinite trace,
- Type III (quantum field theory) $\leftarrow$ no (semi)finite trace.

Can be distinguished by properties of traces they admit. (Traces are weakly continuous positive linear maps $\tau : \mathcal{M} \to \mathbb{C}$ with $\tau(AB) = \tau(BA)$. Useful for calculating entropy!)

Type change

- Observable algebras $\mathfrak{A}(\mathcal{O})$ in QFT are necessarily of type III, so entropy calculations for local regions diverge! Can we fix this?
- On the other hand, is it enough to just have a net of algebras on its own?
- As we operationally describe measurement in the presence of symmetries, we end up introducing a reference frame (made precise later).
- This can lead to dramatic consequences for the algebraic structure of the theory.
- In [Witten 2022; Kudler-Flam, Leutheusser and Satishchandran 2024] show that in toy models of quantum gravity on a black hole exterior background the type of von Neumann algebras can change!
- CLPW [Chandrasekaran, Longo, Penington and Witten 2023] describe a type reduction phenomenon to type II_1 for time translation invariant observables of a quantum field on a de Sitter static patch in conjunction with some ‘observer’ quantum system.

The CLPW model

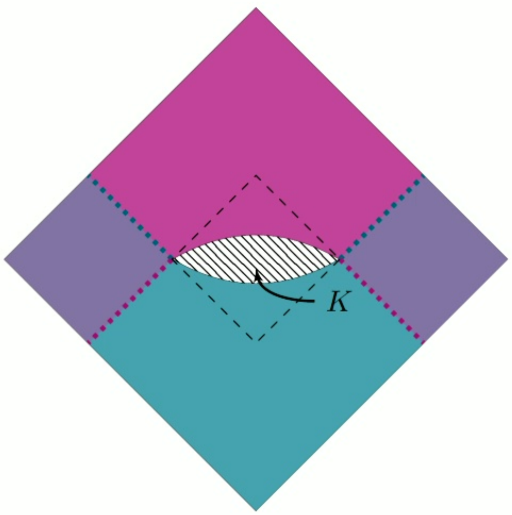


- Consider an 'observer system' (QM clock for the worldline proper time) with Hilbert space $\mathcal{H}_R = L^2(\mathbb{R}^+)$ and Hamiltonian $H_R\psi(E) = E\psi(E)$ (in energy space).
- The static patch is the region where the observer can direct and receive results of experiments.
- Consider a quantum field on the de Sitter background in a Bunch-Davies type state (restricts as a KMS state to the static patch).
- It is described by the von Neumann algebra $\mathcal{M}_S \subset B(\mathcal{H}_S)$ of QFT observables on static patch with unitary rep. of time translation $U_S : \mathbb{R} \to \mathbf{U}(\mathcal{H}_S)$.

Physical observables are the joint observables of the clock and QFT that are invariant i.e. $(\mathcal{M}_S \otimes B(\mathcal{H}_R))^{U_S \otimes \exp(iH_R \cdot)}$. These form a type II_1 algebra, in particular they admit a finite trace.

Sketch of the measurement theory

How to perform a measurement for some QFT $\mathfrak{A}$ on a spacetime M? [Fewster and Verch 2020].

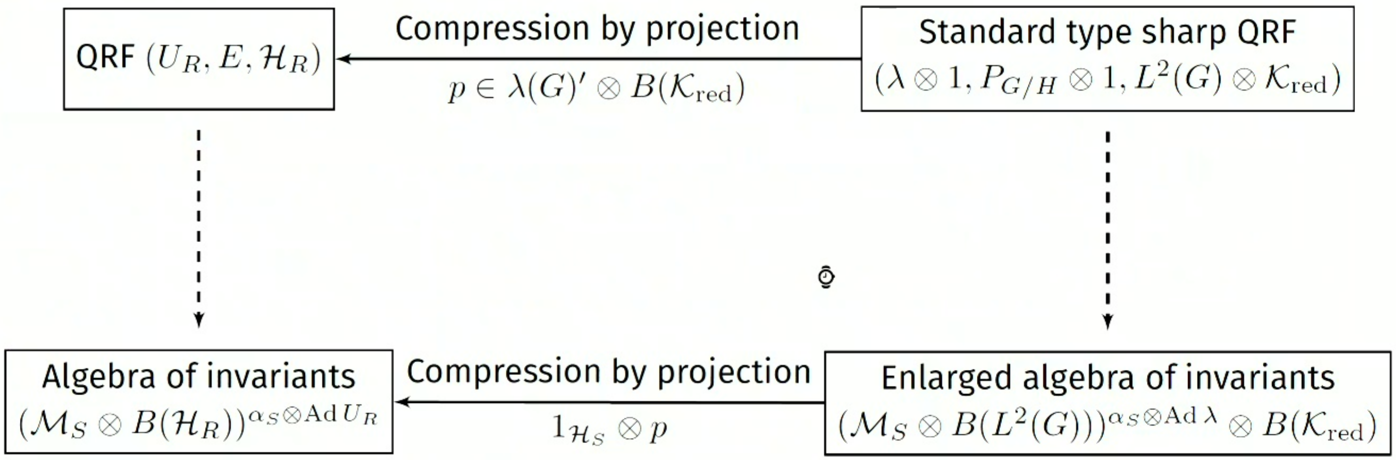


How this fits into our viewpoint

- We demonstrate how this fits into a general QFT-measurement and QRF framework [arXiv:2403.11973].
- Observer system $\rightarrow$ (operational) quantum reference frame (Loveridge et.al.).
- Measurements on de Sitter static patch are performed via relativistic local measurement schemes (in the Fewster-Verch sense) defined 'relative to' QRF.
- Requirements on QRF related to localization and thermal properties of QFT imply measurable observables are 'at most' type II_∞ (admit a semifinite trace).
- Assuming furthermore good thermal properties of QRF (at same temperature as QFT), the algebra is type II_1 i.e. admits a finite trace.

Invariant algebras and the crossed product

Let $(U_R, E, \mathcal{H}_R)$ be a compactly stabilised ('sufficiently good resolution') QRF for 'sufficiently nice' group G. We can always present such QRF as compression of a 'standard' one:



Here $\mathcal{M}_S \subset B(\mathcal{H}_S)$ is the von Neumann algebra of the system with $\alpha_S : G \to \mathrm{Aut}(\mathcal{M}_S)$ a weakly continuous group action; λ denotes the left action of G on $L^2(G)$.

Crossed products and modular theory

- In the setting of covariant QFT's on stationary spacetime, if the QFT admits a faithful normal β-KMS state (thermal state), then the time translation action on the associated von Neumann algebra can be (up to reparametrisation) identified with a modular action.
- For $\sigma : \mathbb{R} \to \mathrm{Aut}(\mathcal{M})$ a modular action, the modular crossed product $\mathcal{M} \rtimes_\sigma \mathbb{R}$ is semifinite.

Conclusions and Outlook

- For local measurement schemes on a background with symmetries, one can use QRFs to define a notion of relative measurement schemes, in terms of relativised induced observables.
- The localisability of the coupling zone forces one to use a particular class of QRFs that give rise to a crossed product parameterisation of the algebra of invariants containing the 'measurable observables'.
- Using the relations between thermal states, modular theory and the crossed product, we formulate conditions for this algebra of invariants to be (semi)finite. Good thermal properties of both the system QFT and the QRF at equal temperature imply finiteness.
- It would be interesting to see how similar analysis could be applied to diffeomorphism invariant relational observables of perturbative QG.

Thank you very much for your attention!