Title: Non-Locality induces Isometry and Factorisation in Holography

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Collection/Series: Emmy Noether Workshop: Quantum Space Time

Subject: Quantum Gravity

Date: March 13, 2025 - 10:00 AM

URL: https://pirsa.org/25030064

Abstract:

Within the AdS/CFT correspondence, two manifestations of the black hole information paradox are given by the nonisometric nature of the bulk-boundary map and by the factorisation puzzle. By considering timeshifted microstates of the eternal black hole, we demonstrate that both these puzzles may be simultaneously resolved by taking into account non-local quantum corrections that correspond to wormholes arising from state averaging. This is achieved by showing, using a resolvent technique, that the resulting Hilbert space for an eternal black hole in Anti-de Sitter space is finite-dimensional with a discrete energy spectrum. The latter gives rise to a transition to a type I von Neumann algebra. Talk based on 2411.09616.

Non-locality induces isometry and factorisation in holography

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Overview

- AdS/CFT correspondence
- Geometry and entanglement
- Wormholes and factorization in AdS/CFT
- Berry phase and its relation to von Neumann algebras
- Non-local quantum corrections

Talk based on

- Berry phase in quantum mechanics and wormholes

Nogueira, Banerjee, Dorband, J.E., Meyer, van den Brink
arXiv:2109.06190, PRD

- Berry phases in AdS_3/CFT_2

Banerjee, Dorband, J.E., Meyer, Weigel arXiv:2202.11717, JHEP

- Berry phases and von Neumann algebras

Banerjee, Dorband, J.E., Weigel arXiv:2306.00055, JHEP

- Non-locality induces isometry and factorisation in holography

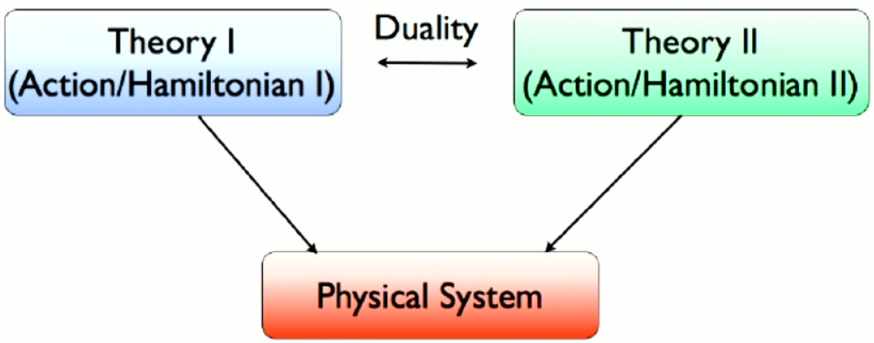
Banerjee, J.E., Karl arXiv:2411.09616

I. Lightning review of the AdS/CFT correspondence

Duality: A physical theory has two equivalent formulations

Same dynamics

One-to-one map between states



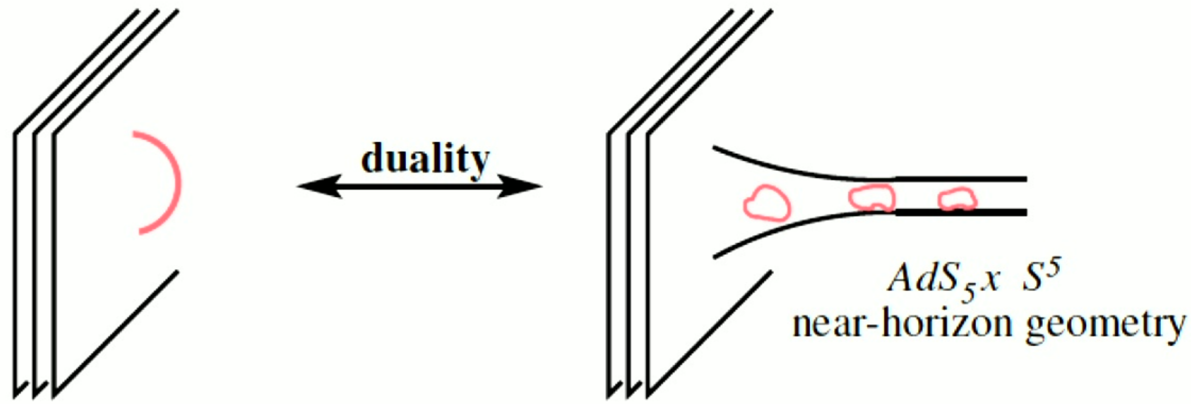
Gauge/Gravity Duality:

Gauge Theory Quantum Field Theory	$\Leftrightarrow$	Gravity theory in higher dimensions

String theory origin of the AdS/CFT correspondence

Maldacena 1997

D3 branes in 10d



$\Downarrow$ Low energy limit

Supersymmetric $SU(N)$ gauge theory in four dimensions $(N \to \infty)$

Supergravity on the space $AdS_5 \times S^5$

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Limits in AdS/CFT

SU(N) Super Yang-Mills theory for any N, any λ ⟺ Type IIB string theory on $AdS_5 \times S^5$

$$\lambda = g_{\mathrm{YM}}{}^2 N$$
$$N \to \infty$$

$$g_s = g_{\mathrm{YM}}{}^2 = \frac{\lambda}{N} \to 0$$

SU(N) Super Yang-Mills theory in large N limit ⟺ Classical string theory on $AdS_5 \times S^5$

$$4\pi\lambda = \frac{L^4}{{\ell_s}^4} \to \infty$$

SU(N) Super Yang-Mills theory for large N, large λ ⟺ Type IIB supergravity on $AdS_5 \times S^5$

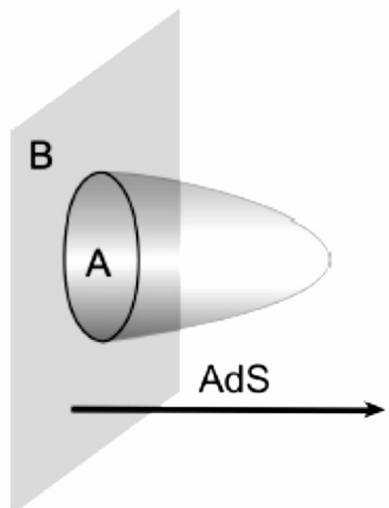
Entanglement entropy in Gauge/Gravity Duality

(Ryu, Takayanagi Phys.Rev.Lett. 96 (2006) 181602)

Leading term in entanglement entropy given by
area of minimal surface in holographic dimension

$$\rho_A = \mathrm{Tr}_B \rho_{\mathrm{tot}}$$

$$S_A = -\mathrm{Tr}_A \rho_A \ln \rho_A$$



II. Black holes and wormholes in AdS/CFT and the factorization puzzle

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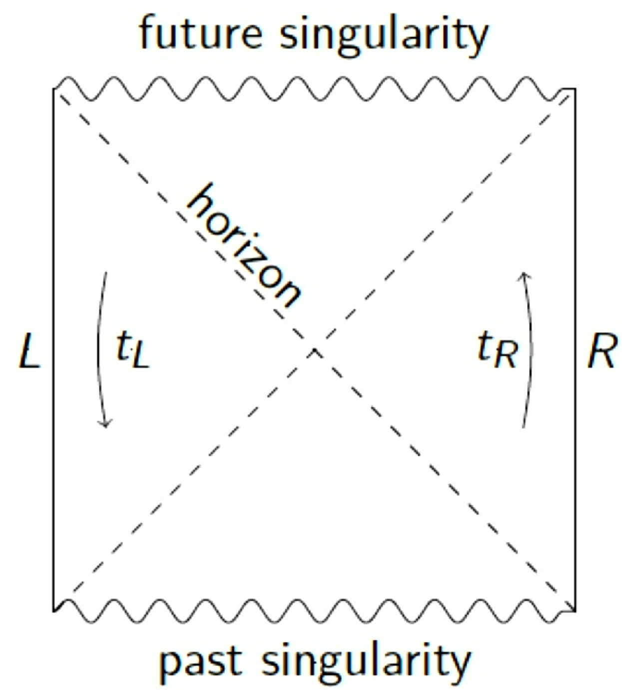
Eternal AdS black hole

- Global coordinates (Kruskal)
- Non-traversable wormhole
- Singularity in time coordinate: Time-like Killing vector switches sign at horizon



Eternal AdS black hole

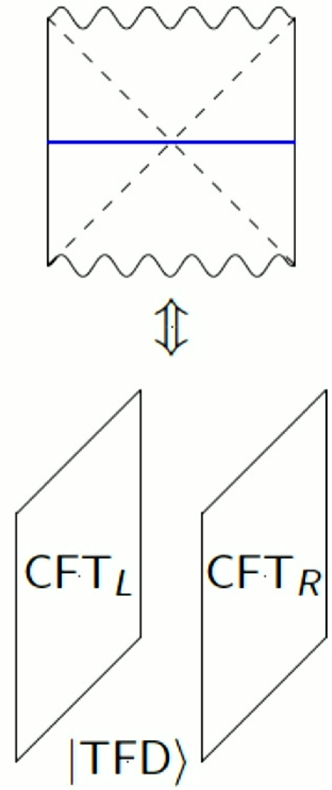
- Eternal black hole in AdS spacetime is dual to two copies of the boundary CFT, entangled in the TFD state

  J. Maldacena, [hep-th/0106112]

- TFD state is the purification of a thermal state of one CFT

$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\frac{\beta}{2} E_n} |n\rangle_L |n\rangle_R^* ,$$

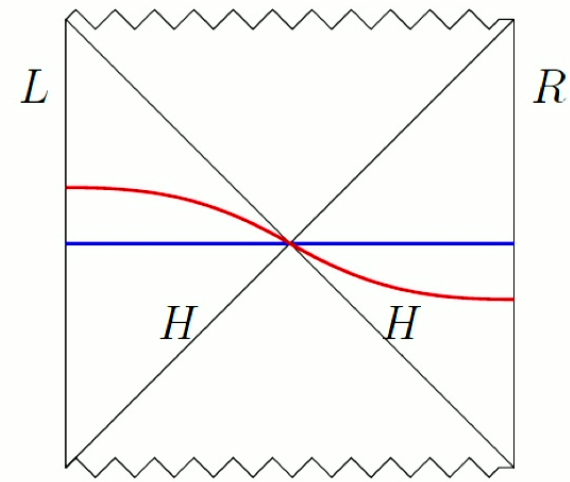
$$\mathrm{tr}_R |\mathrm{TFD}\rangle\langle\mathrm{TFD}| = \frac{1}{Z} e^{-\beta H_L} = \rho_\beta$$



Factorization puzzle

Maldacena+Maoz '13; Harlow '16

- The two CFTs have disjoint Hilbert spaces since there is no interaction between them, $\mathcal{H}_L \otimes \mathcal{H}_R$
- The wormhole Hilbert space does not factorize
- Apparent contradiction?
- Holographic map non-isometric



Wormholes in quantum mechanics

H. **Verlinde 2021**
2003.13117
2105.02129

$$Z(\beta) = \mathrm{tr}(e^{-\beta H})$$

$$Z(D) = \int [dX]\, e^{\int_D \Omega - \oint_{\partial D} H dt} \qquad D =$$

generalized coordinates and momenta X^a, symplectic form $\Omega = \frac{1}{2}\omega_{ab} dX^a \wedge dX^b$

Exact symplectic structure: $\Omega = d\alpha, \quad \int_D \Omega = \oint_{\partial D} \alpha$

$$Z(\beta) = Z(D)$$

If symplectic structure is non-exact:

$$\langle Z(\beta)^n \rangle = Z(\Sigma_n) \qquad \Sigma_n =$$

III. Berry phases and von Neumann algebras

Berry phase

- Maurer-Cartan form: Connection on a group manifold *M* defined for any group element σ
- $A_{\mathrm{MC}} = \sigma^{-1} \mathrm{d}\sigma$
- Berry connection: Ground state expectation value of the Maurer-Cartan form

$$A_{\mathrm{B}}(\lambda) = \mathrm{i} \langle \psi_0 | A_{\mathrm{MC}} | \psi_0 \rangle$$

- Berry curvature: $F_{\mathrm{B}}(\lambda) = \mathrm{i} \langle \psi_0 | \omega_{\mathrm{KK}} | \psi_0 \rangle$
- Berry phase: $\Phi_B = \int F_B$

$$\omega_{\mathrm{KK}} = \mathrm{d} A_{\mathrm{MC}}$$

Kirillov-Kostant symplectic form

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von Neumann algebras

von Neumann 1930

Jefferson; Liu, Leutheusser; Witten; Chandrasekaran, Penington, Witten

Concept of algebraic QFT
for classifying operator algebras w. r. t. entanglement properties

Type I - density matrix and trace (as in quantum mechanics),

admits irreducible representations

Type II - trace prescription, but does not act irreducibly

Type III - no trace prescription (eg. free QFTs)

Two-spin system: Berry phase and type I Von Neumann algebra

- Coupled spins in external magnetic field:
  Electronic Zeeman interaction in hydrogen atom $H = J\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 - 2\mu_B B S_{1z}$

- Ground state $|\psi_0\rangle = -\frac{\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}}{\sqrt{2}}|\uparrow\downarrow\rangle - \frac{\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}}{\sqrt{2}}|\downarrow\uparrow\rangle$ $\tan\alpha = 2\mu_B \frac{B}{J}$

- Projective Hilbert space $\mathbb{C}\mathrm{P}^3$ (= SU(4)/U(3))

Schmidt decomposition $|\psi_0\rangle = \sum_{i=\uparrow,\downarrow} \kappa_i |i, \tilde{i}\rangle$

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$\kappa_\uparrow = \sqrt{\frac{1 - \sin\alpha}{2}}$ $\kappa_\downarrow = \sqrt{\frac{1 + \sin\alpha}{2}}$

Two-spin system: Berry phase and type I Von Neumann algebra

Entanglement entropy	$S_{\mathrm{EE}} = -\sum_{i=\uparrow,\downarrow} \kappa_i^2 \ln \kappa_i^2 = \sin\alpha \ln \frac{1-\sin\alpha}{\cos\alpha} - \ln \frac{\cos\alpha}{2}$		
No entanglement (J=0)	$S_{EE} = 0$	Entanglement orbit	$\mathbb{C}\mathrm{P}^1 \times \mathbb{C}\mathrm{P}^1$
Maximal entanglement (J very large)	$S_{EE} = \ln 2$	$\frac{SU(2)}{\mathbb{Z}_2} = \mathbb{R}\mathrm{P}^3$	
Intermediate entanglement		$\mathbb{C}\mathrm{P}^1 \times \mathbb{R}\mathrm{P}^3$	

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Two-spin system: Berry phase and type I Von Neumann algebra

Berry phase from symplectic volume of entanglement orbit

Reduced density matrix from $P = \begin{bmatrix} \sqrt{\frac{1-\sin\alpha}{2}} & 0 \\ 0 & \sqrt{\frac{1+\sin\alpha}{2}} \end{bmatrix}$

Other points in the orbit $Q = uP, \quad u = e^{-\mathrm{i}\frac{\phi}{2}\sigma_z} e^{-\mathrm{i}\frac{\theta}{2}\sigma_y} e^{\mathrm{i}\frac{\phi}{2}\sigma_z}$

Connection on the orbit $A = \mathrm{i}\,\mathrm{tr}\left(Q^\dagger \mathrm{d}Q\right) = \frac{\sin\alpha}{2}(1-\cos\theta)\mathrm{d}\phi$

Symplectic form $\Omega = \mathrm{d}A = \frac{\sin\alpha}{2}\sin\theta \mathrm{d}\theta \wedge \mathrm{d}\phi$

Berry phase

$$V_{\text{symp}} = \int \Omega = \frac{\sin\alpha}{2} V(S^2) = 2\pi \sin\alpha = \Phi_G$$

vanishes for maximally entangled state

Two-spin system: Berry phase and type I Von Neumann algebra

Trace functional

$$f(ca) = cf(a) \quad \text{for} \quad c \in \mathbb{C}, a \in \mathcal{A} \quad \text{and}$$
$$f(a+b) = f(a) + f(b) \quad \text{for} \quad a, b \in \mathcal{A}.$$

Cyclicity?

$$f(ab) = f(ba)$$

$$f_0(a_L) = \langle \psi_0 | a_L | \psi_0 \rangle \qquad a_L = a_{L,n}\sigma_n, \quad b_L = b_{L,n}\sigma_n, \quad n \in \{0, x, y, z\}, \quad a_{L,n}, b_{L,n} \in \mathbb{R}.$$

$$f_0([a_L, b_L]) = 2\mathrm{i}\sin\alpha\big(a_{L,y}b_{L,x} - a_{L,x}b_{L,y}\big)$$

$$f_0([a_L, b_L]) \propto \Phi_G$$

Trace functional on commutator proportional to geometric phase

vanishes for maximally entangled state

Banerjee, Dorband, J.E., Weigel arXiv:2306.00055

Infinitely many spins

Using $|\lambda\rangle = \frac{1}{\sqrt{1+\lambda}}\left(|\downarrow\downarrow\rangle + \sqrt{\lambda}|\uparrow\uparrow\rangle\right)$ with $0 < \lambda \leq 1$

consider

$$|\Psi\rangle = \lim_{N\to\infty} \bigotimes_{n=1}^{N} |\lambda_n\rangle = \lim_{N\to\infty} \bigotimes_{n=1}^{N} \frac{1}{\sqrt{1+\lambda_n}}\left(|\downarrow\downarrow\rangle_n + \sqrt{\lambda_n}|\uparrow\uparrow\rangle_n\right)$$

Sum over pairs of spins, phases $\Phi_G^{(n)} = 2\pi \frac{1-\lambda_n}{1+\lambda_n}$

TFD state for two copies of an infinite collection of spins,

temperature given by entanglement temperature

In general type III von Neumann algebra

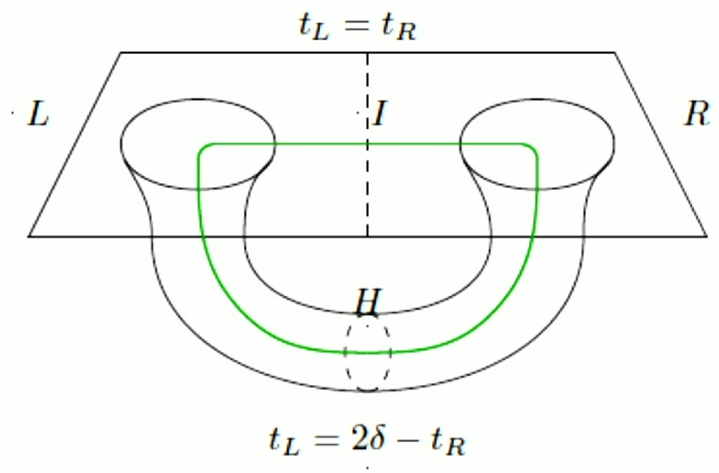
Geometric phases vanishes for maximal entanglement -> type II vN algebra

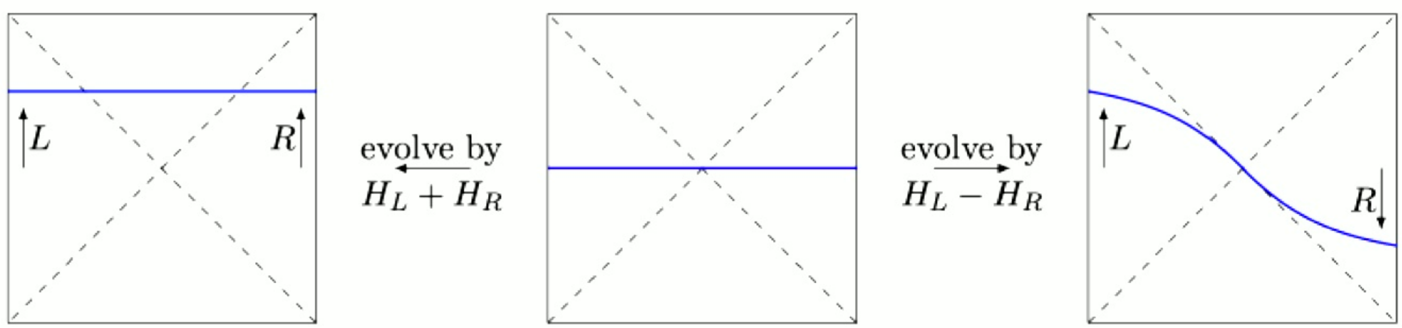
Berry phase and Von Neumann algebra for eternal black hole

No global Killing vector in
the presence of a wormhole

related to mass/temperature
of eternal black hole

Leads to non-exact symplectic form





$$\left|\mathrm{TFD}_{\delta}\right\rangle = e^{-i(H_L + H_R)\delta}\,|\,\mathrm{TFD}\rangle$$

Symmetry:
does not transform state

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Wormhole Berry Phase

Time translations at each boundary $\mathrm{U}(1) \times \mathrm{U}(1)$

Bulk isometry from *H_L - H_R* $\mathrm{U}(1)$

Bulk moduli space of classical solutions $\dfrac{\mathrm{U}(1) \times \mathrm{U}(1)}{\mathrm{U}(1)} \sim \mathrm{U}(1) \sim S^1$

parametrized by δ

Berry connection $A_\delta = \mathrm{i}\langle \mathrm{TFD}|U^\dagger \partial_\delta U|\mathrm{TFD}\rangle$ $U = e^{\mathrm{i}(H_L + H_R)\delta}$

$\Phi_G^{(\mathrm{TFD})} = \int^{\zeta} \mathrm{d}\delta\, A_\delta \neq 0$ proportional to geometric phase of bulk moduli space

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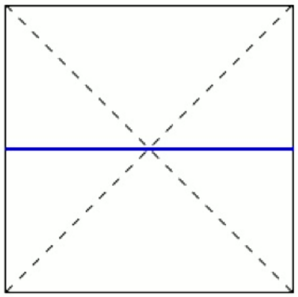
Type II vs. type III von Neumann algebra for eternal black hole

In geometric phase approach:

Non-factorization -> non-zero geometric phase -> no trace definition -> type III vN algebra

Maximally entangled state -> geometric phase vanishes -> type II vN algebra



Banerjee, Dorband, J.E., Weigel arXiv:2306.00055

Relation to black hole micro state counting

S. Banerjee, J.E,, J. Karl
2411.09616

Generalised phase-shifted TFD states $|\mathrm{TFD}_\alpha\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{\mathrm{i}\alpha_n} e^{-\beta \frac{E_n}{2}} |n\rangle_L |n\rangle_R^*$

Same entanglement orbit as TFD state

Berry connection $A_\delta = \mathrm{i} \langle \mathrm{TFD}_\alpha | \partial_\delta | \mathrm{TFD}_\alpha \rangle = \frac{2}{Z} \sum_n E_n e^{-\beta E_n}$

These states can be thought of as microstates of the eternal black hole

Black hole entropy from overlaps of phase-shifted states shows cancellation such that only discrete spectrum remains

(Matrix model calculation similar to Balasubramanian, Lawrence, Magan, Sasieta 2212.02447)

IV. Non-locality induces isometry and factorisation

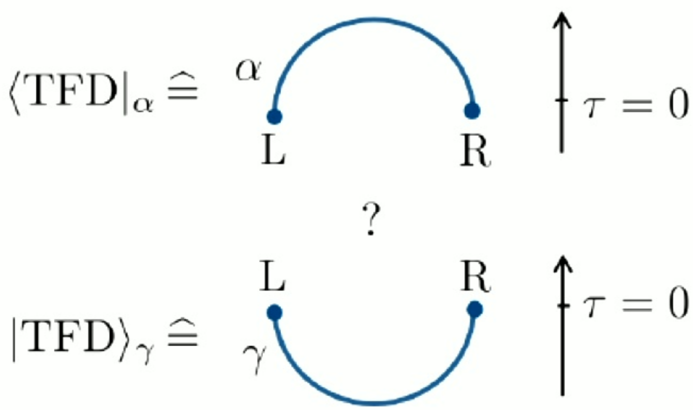
Banerjee, J.E., Karl arXiv:2411.09616

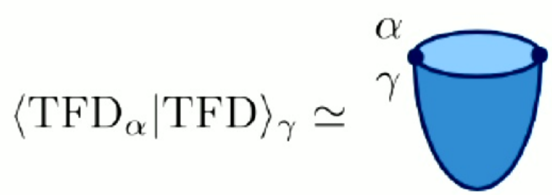
Black hole micro state counting

In semiclassical, or large N limit the energy spectrum is highly random (approx. continuous)
[Verlinde H. '20]

$$\Rightarrow \quad \langle \mathsf{TFD}_\alpha | \mathsf{TFD} \rangle_\gamma = \frac{1}{Z(\beta)} \sum_n e^{-\beta E_n} e^{i(\gamma_n - \alpha_n)} = \delta_{\alpha\gamma} + \mathcal{O}(e^{-S_{\mathbf{BH}}/2})$$

This may also be calculated from the **gravitational path integral**
$\Rightarrow$ includes a **sum over geometries** consistent with given boundary condition





$\Rightarrow$ Due to orthogonality states form infinite basis of bulk Hilbert space

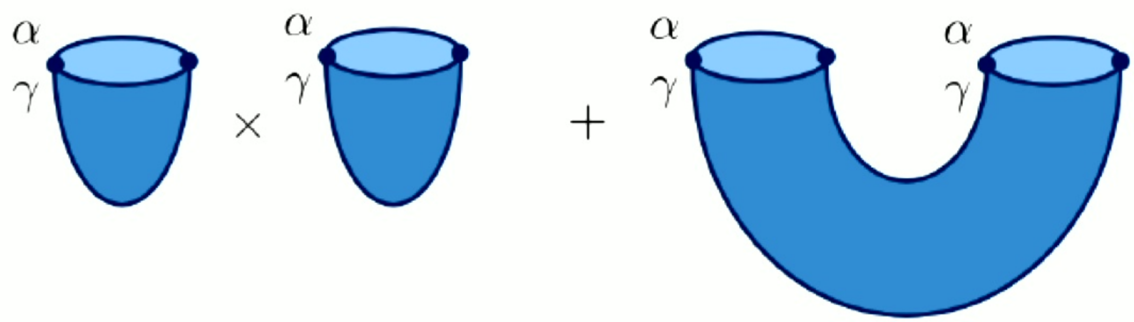
Black hole micro state counting

Problem is solved by calculating overlaps in microstates from higher moments

$$\frac{1}{\mathcal{N}}\sum_{\gamma}|\langle \text{TFD}_\alpha|\text{TFD}\rangle_\gamma|^2 = \frac{1}{\mathcal{N}} + \frac{1}{Z^2(\beta)}\sum_n e^{-2\beta E_n} = \frac{1}{\mathcal{N}} + \frac{Z(2\beta)}{Z^2(\beta)}$$

Correction term understood from appearance of **replica wormholes** in gravitational path integral [Verlinde H. '20, Verlinde H. '21]

$$|\langle \mathrm{TFD}_\alpha|\mathrm{TFD}\rangle_\gamma|^2 \simeq$$



⇒ **non-perturbative** and **non-local** correction

Black hole micro state counting

Appearance of non-trivial overlaps in higher moments understood from an averaging procedure

$$\langle \mathsf{TFD}_\alpha | \mathsf{TFD} \rangle_\gamma = \overline{\mathcal{M}_{\alpha\gamma}} \quad \text{with} \quad \mathcal{M}_{\alpha\gamma} = \delta_{\alpha\gamma} + e^{-S_{\mathbf{BH}}/2}\,\mathcal{R}_{\alpha\gamma}$$

and $\mathcal{R}_{\alpha\gamma}$ is random matrix with mean zero $\Rightarrow$ Higher moments contain variance of $\mathcal{R}$

- Degrees of freedom of fundamental theory encoded in $\mathcal{R}$
- Path integral averages over fundamental DoF [Penington, Shenker, Stanford, Yang '19]
- Here this is a state average over Hilbert space of phase-shifted states

The Euclidean replica wormholes arise from an average over states corresponding to Lorentzian wormholes of different length.
This connects two seemingly different notions of non-locality in QG

Black hole micro state counting

Counting only linearly independent microstates: **Strategy** [Balasubramanian et al. '22, Emparan et al. '24]

- Consider Hilbert space $\mathcal{H}_\Omega$ spanned by Ω microstates
- Calculate $d_\Omega = \text{dim}(\mathcal{H}_\Omega) \Rightarrow$ Dimension of full HS given by limit of d_Ω
- d_Ω given by rank of Gram matrix $G_{ij} := \langle \text{TFD}_{\alpha_i} | \text{TFD} \rangle_{\alpha_j} \quad i, j = 1, ..., \Omega$

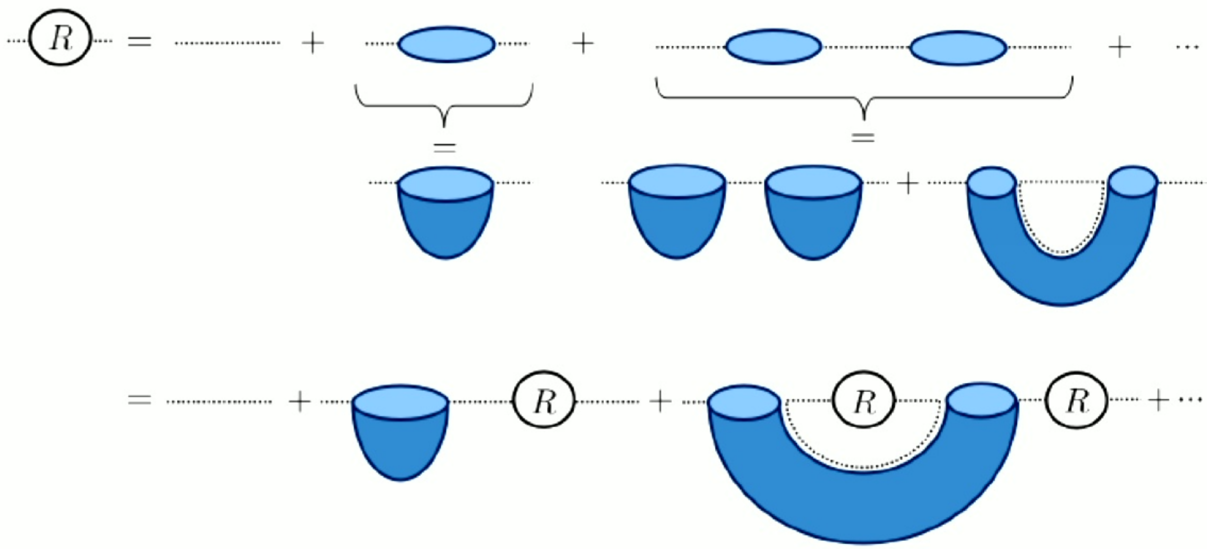
$$d_\Omega = \lim_{\epsilon \to 0^+} \int_\epsilon^\infty d\lambda \, D(\lambda) = \Omega - \text{Ker}(G) \quad \text{with} \quad D(\lambda) := \text{Tr} \left(\delta(\lambda \mathbb{1} - G) \right) = \sum_{i=1}^{\Omega} \delta(\lambda - \lambda_i)$$

- Higher powers G^n encode overlaps $\Rightarrow$ **resolvent method** [Penington, Shenker, Stanford, Yang '19]
- Eigenvalue density $D(\lambda)$ given in terms of trace of resolvent

$$D(\lambda) = \frac{1}{2\pi i} \lim_{\epsilon \to 0} \left(R(\lambda - i\epsilon) - R(\lambda + i\epsilon) \right)$$

Black hole micro state counting

Resolvent is defined as $R_{ij}(\lambda) = \left(\frac{1}{\lambda\mathbb{1} - G}\right)_{ij} = \frac{\delta_{ij}}{\lambda} + \sum_{n=1}^{\infty} \frac{(G^n)_{ij}}{\lambda^{n+1}}$ or pictorially:



$$\Longrightarrow R_{ij}(\lambda) = \frac{\delta_{ij}}{\lambda} + \frac{1}{\lambda}\sum_{n=1}^{\infty} \frac{Z_n}{Z_1^n} R^{n-1}(\lambda)\, R_{ij}(\lambda) \quad \text{with} \quad R(\lambda) = \sum_i R_{ii}(\lambda)$$

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Black hole micro state counting

$R(\lambda)$ is solution to quadratic equation and we find the EV density of G

$$D(\lambda) = \frac{e^{S_{\mathsf{BH}}}}{2\pi\lambda}\sqrt{\left[\lambda - \left(1 - (\Omega\, e^{-S_{\mathsf{BH}}})^{1/2}\right)^2\right]\left[\left(1 + (\Omega\, e^{-S_{\mathsf{BH}}})^{1/2}\right)^2 - \lambda\right]}$$
$$+ \delta(\lambda)\left(\Omega - e^{S_{\mathsf{BH}}}\right)\Theta\left(\Omega - e^{S_{\mathsf{BH}}}\right)$$

From this we find

$$d_\Omega = \min\left(\Omega, e^{S_{\mathsf{BH}}}\right)$$

Hilbert space dimension bounded from above

Algebraic perspective on factorisation

- In the type II description all microstates are orthogonal, and the entropy is infinite
- We showed explicitly that the inclusion of **non-perturbative** wormhole corrections imply: $\Rightarrow$ microstates span finite dimensional Hilbert space $\Rightarrow$ Type I
- Non-perturbative corrections yield discrete energy spectrum

- Consequently all microstates can be written as an element of a **factorized Hilbert space**

$$\mathcal{H}_l \otimes \mathcal{H}_r = \text{span}(\ |E_n\rangle_l \otimes |E_m\rangle_r\ )$$

Non-locality induces isometry and factorisation

The holographic map seems to be inconsistent in the semiclassical limit
Problem 1: Bulk Hilbert space has way too many states

- Infinite family of generalized TFD states are orthogonal for $G_N \to 0$
- Including **non-local** effects in the gravitational path integral leads to non-trivial overlaps
  $\Rightarrow$ linearly independent states span a HS consistent with holographic entropy bounds

Problem 2: Factorisation puzzle (type I vs type III/II)

- Reduction in size corresponds to a transition from type III/II to type I
- Transition is understood in terms of a discrete energy spectrum
  $\Rightarrow$ Energy eigenstates span **factorized Hilbert space**

A consistent formulation of a quantum theory of gravity has to be non-local

Conclusions and outlook

- Non-Factorization of wormhole Hilbert space due to non-exact symplectic form that results in non-zero Berry phase
- Mathematical structure also present in quantum mechanics and in CFT
- Relation to von Neumann algebras
- Inherently local quantum gravity corrections yield type I von Neumann algebra
- Relation between entanglement and geometry

Thanks to .....

Coraline Bacq, Souvik Banerjee, Pablo Basteiro, Rathindra Nath Das, Giuseppe Di Giulio, Haye Hinrichsen, Jonathan Karl, Henri Scheppach, Yanick Thurn, Zhuo-Yu Xian

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