Title: Quantum Spacetime: from Speculation to Numbers

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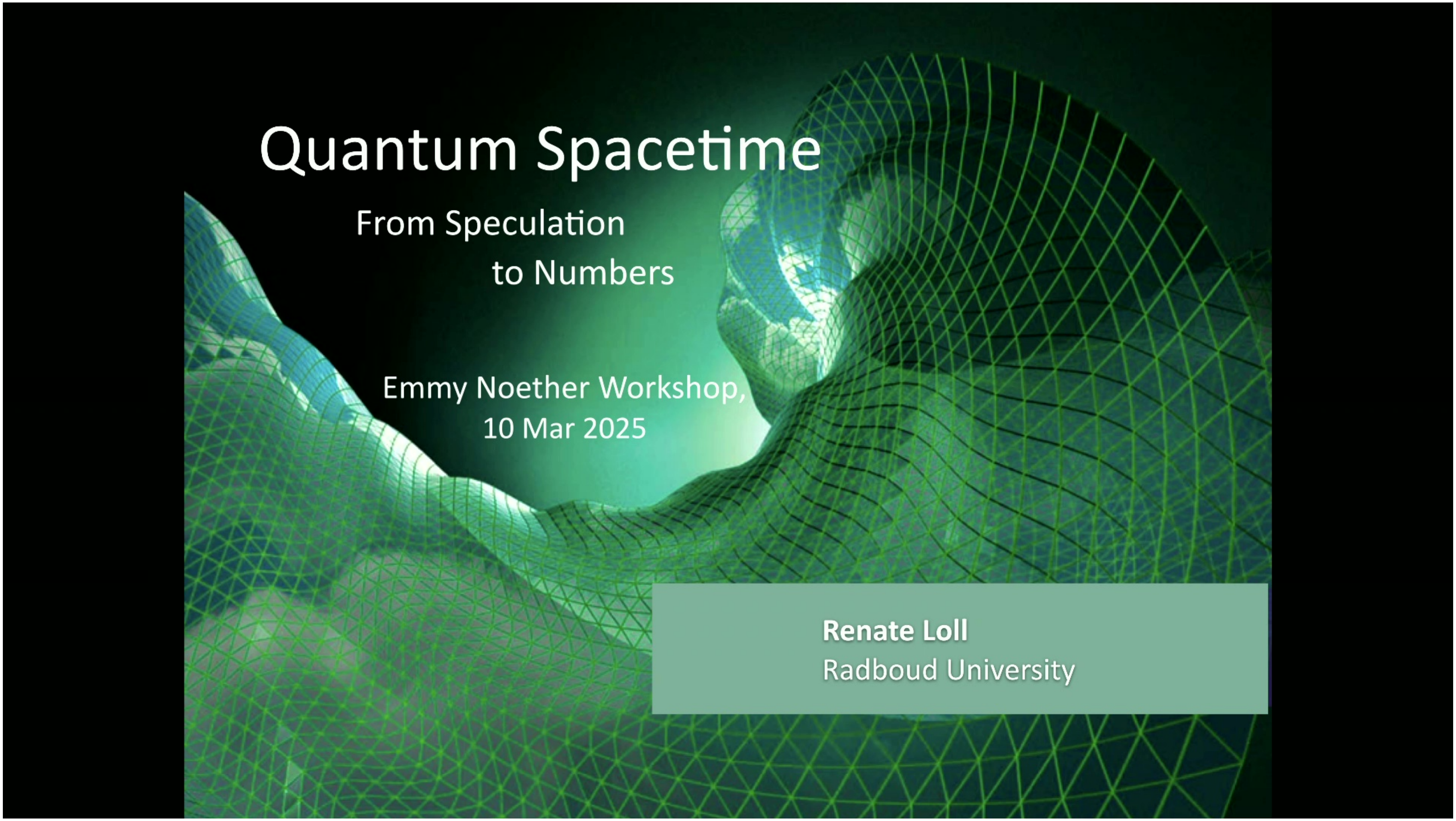
Abstract:

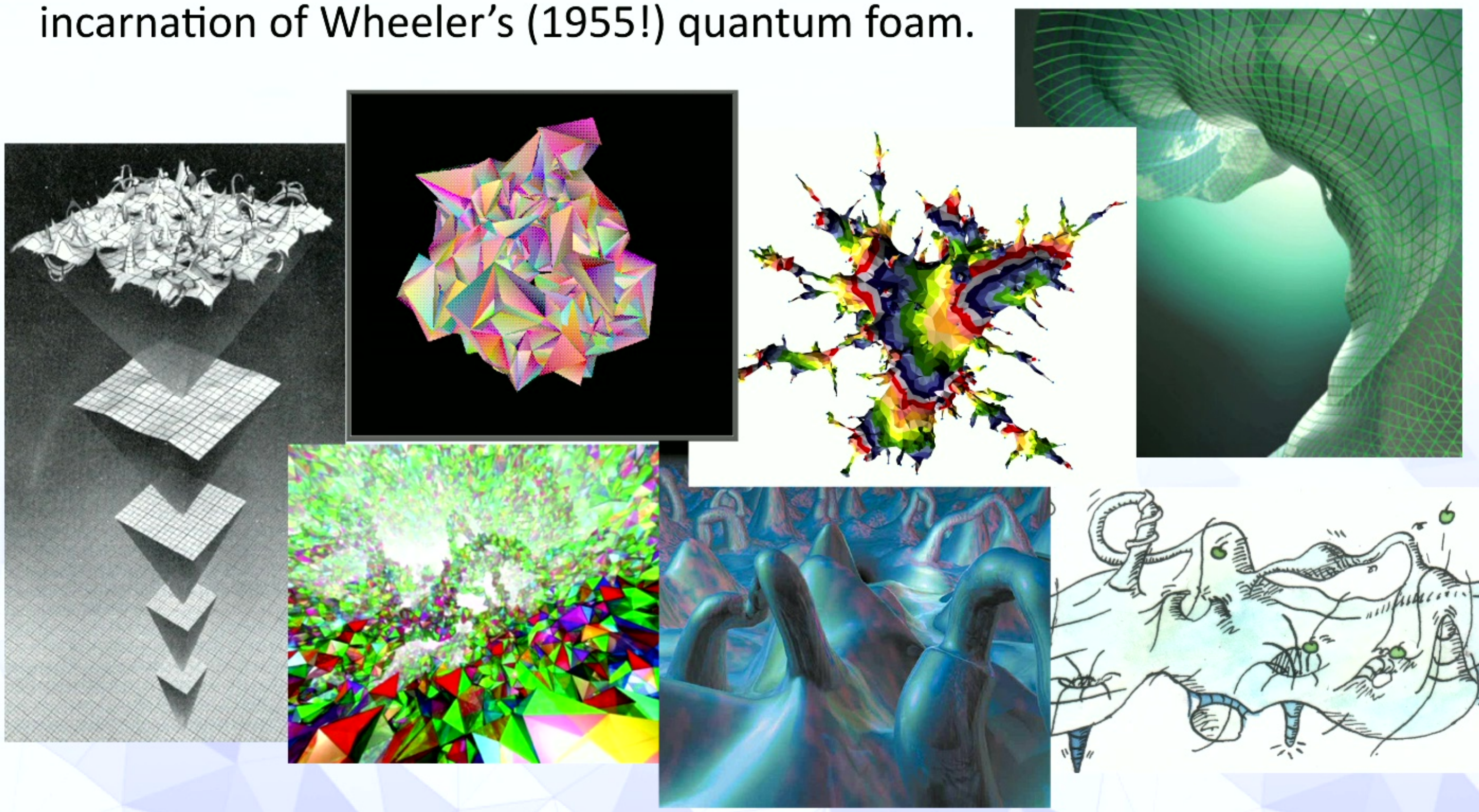
Many researchers in quantum gravity favour the notion of a quantum foam, coined by Wheeler 70 years ago to capture "whatever becomes of spacetime at the Planck scale". The underlying idea is that the quantum fluctuations of spacetime are so large that a description based on smooth metrics is no longer adequate. Equally popular is the notion that spacetime as we know it should "emerge" from this primordial quantum foam, alongside interesting quantum-gravitational effects.

These ideas are enticing, but remain speculative unless backed up by quantitative analysis and modelling within a coherent, nonperturbative formulation of quantum gravity. Fully nonperturbative computational tools are available in the form of 'lattice quantum gravity 2.0', based on causal dynamical triangulations. The power and beauty of this methodology lies in its use of curved, dynamical lattices, incorporating the principles of quantum field theory and general relativity from the outset. This has produced quantitative blueprints of both quantum foam and spacetime emergence, and a concrete perspective on what it means to “solve" quantum gravity. [arXiv:2501.17972]


Quantum Spacetime
From Speculation
to Numbers
Emmy Noether Workshop,
10 Mar 2025
Renate Loll
Radboud University

What *is* quantum spacetime (QST)?

"Whatever becomes of spacetime near the Planck scale ℓ_{Pl}"; some incarnation of Wheeler's (1955!) quantum foam.



(artistic impressions and toy models as placeholders of our ignorance)

Quantum spacetime (or, mystery no.1)

Near the Planck scale, the quantum fluctuations of spacetime itself become so large that

- spacetime develops wormholes/handles (nontrivial topology)?
- it becomes spiky, with singularities (not everywhere smooth)?
- it is “torn apart” (with holes/boundaries, disconnected)?
- it is “spikes only” (nowhere differentiable)?
- it falls apart into small disconnected “bits”, is discrete?
- it becomes something altogether different (non-geometric)?

It is unclear whether any of these concepts make sense in a quantum theory of spacetime, i.e. ***quantum gravity beyond perturbation theory***. Are they “properties” of quantum states or of spacetime histories in a quantum ensemble? Are they ***observable***? Are they put in by hand (“postulated”) or obtained dynamically? How are they modelled?

Emergence of classicality (or, mystery no.2)

Despite the highly nonclassical character of "quantum foam", one then also assumes that spacetime-as-we-know-it is recovered from it on sufficiently large scales (because, what else could happen?).

This turns out to be very difficult to realize and explain theoretically. It is an important part of why ***quantum gravity is not easy***.

- This is the "problem of the classical limit" of nonperturbative formulations (aka once the spacetime metric $g_{\mu\nu}$ is gone, how to get it back?)
- How do the microscopic, Planckian degrees of freedom "conspire" to produce classical features — not $g_{\mu\nu}$ per se — macroscopically?
- Is there a (universal?) dynamical mechanism that gives rise to such an ***emergence in the sense of statistical mechanics***?

Although hard, the primary aim and potential reward of understanding quantum foam and emergence is *not just* to retrieve classical general relativity, but to discover ***interesting, new quantum signatures.***

Preview of results

The ideas of quantum foam and spacetime (re-)emergence are very enticing, but remain speculative unless backed up by ***quantitative analysis and modeling in full 4D nonperturbative quantum gravity***.

Such quantitative, computational tools are now ***available*** in the framework of nonperturbative lattice QFT, applied to gravity: ***lattice quantum gravity "2.0"*** (key: dynamical, curved, Lorentzian lattices).

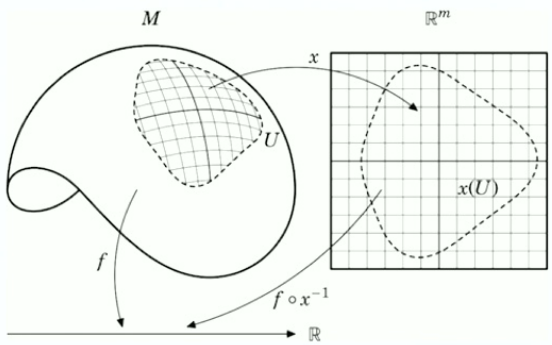
J. Ambjørn & R. Loll, in Encyclopedia of Mathematical Physics, 2nd edition [arXiv: 2401.09399]

Being able to compute nonperturbatively from first principles is a game changer and has delivered ***blueprints of QST and emergence***.

Quite apart from the concrete details (numbers!), it is the nature of these outcomes that is highly informative and radically different from our classically trained speculation and "intuition", despite the absence of any exotic ingredients.

==> to understand quantum gravity @ℓ_{Pl}, get rid of $g_{\mu\nu}$! <==

What is wrong with $g_{\mu\nu}$?



differentiable manifold *M* and a coordinate chart

Smooth manifolds *($M, g_{\mu\nu}$)* provide convenient, powerful models of spacetime, but already classically,

- $g_{\mu\nu}(x)$ is tied to an unphysical coordinate language: we never observe the local metric, but establish the presence of curvature e.g. through its effect on geodesics (rods and clocks);

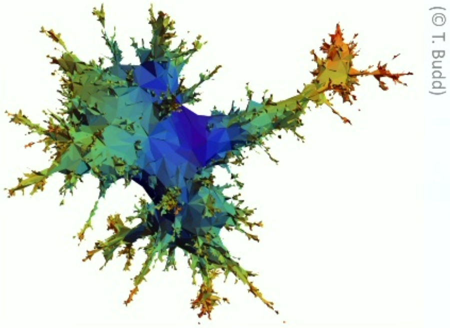
- GR seems to be a very good theory in its domain of applicability; this does not mean each of its textbook ingredients has a correlate in physical reality (although we are very attached to them):

- in postulating $ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$ Riemann was guided by "experience" and physical (Newtonian) considerations, which "may not apply in the immeasurably small". **B. Riemann, Habilitation Thesis 1854**

What is wrong with $g_{\mu\nu}$ in quantum gravity?

Claim: $g_{\mu\nu}(x)$ is not a good starting point for a quantum theory of gravity(*), because

- smooth tensor fields do not capture the physics of "quantum foam", we need to go beyond 19th-century notions of geometry;
- perturbation theory is not renormalizable;



typical nonperturbative quantum space(time)

- the standard continuum formulation has a huge redundancy (4D diffeomorphisms), leading to infinities e.g. in the path integral.

However, most QG literature uses metrics in an essential way, as basic fields and/or background, finding ever more conundrums.

Nonperturbative lessons: don't trust $g_{\mu\nu}$ @ ℓ_{Pl} ! don't insist that $g_{\mu\nu}$ should "emerge"; only *observables* of GR must be recovered in classical limit! don't insist that perturbative QG must be recovered!

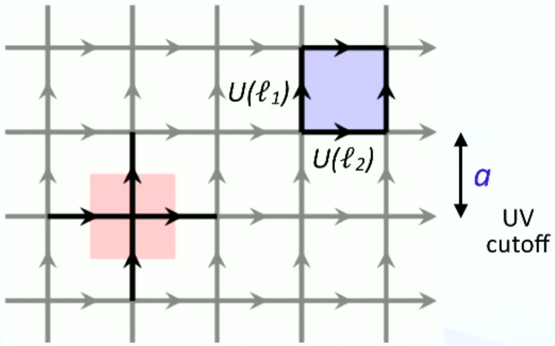
(*) using vierbeins, connections, etc. has similar issues

What to do instead: lattice quantum gravity!

- go-to methodology for nonperturbative QFT; try to emulate the formidable successes of lattice QCD

- <u>breakthrough</u>: use edge holonomies $U(\ell)=P\exp\int_\ell A$, still transforming under $SU(N)$ at their end points — ***exact gauge group action*** despite discretization!

  K. Wilson, PRD 10 (1974) 2445



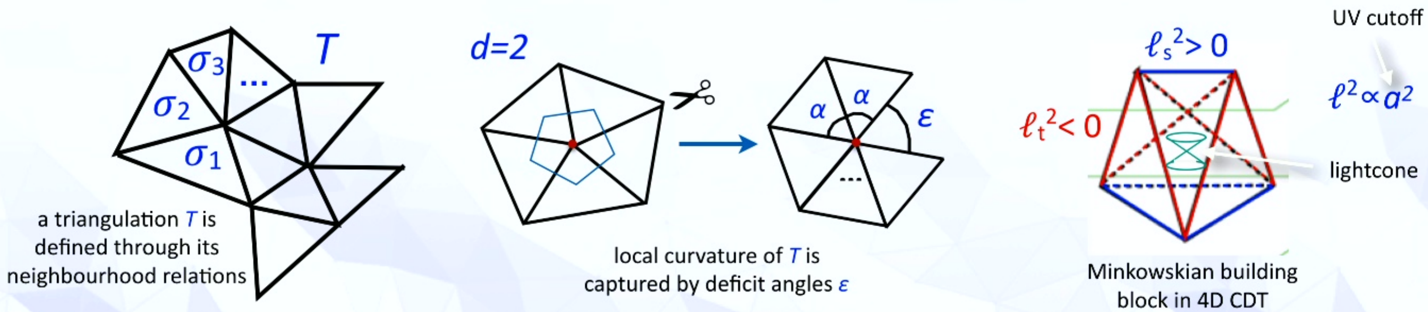
cubic lattice representing flat spacetime, with gauge fields living on edges

- **lattice QG 1.0**: make gravity look like a gauge theory and apply Wilson's idea **L. Smolin, NPB 148 (1979) 333, many others ...**
- ***but*** nothing interesting ever found in Monte Carlo (MC) simulations
- ***lattice QG is not easy***! different from (and harder than) QCD
- the nonperturbative nature of quantum gravity and quantum spacetime clashes with the rigidity of the fixed spacetime lattice

Game changer: lattice quantum gravity 2.0

- breakthrough: use ***curved, dynamical lattices***, originally 2D Euclidean ("DT"), now 4D Lorentzian ("CDT" — causal dynamical triangulations)(*)

- simplicial lattices, representing (regularized) intrinsically curved space-time configurations, can be thought of as "gluings" of identical 4D triangular building blocks (four-simplices); ***no coordinates are needed.***



- triangulations carry an ***exact lattice analogue of coordinate trans-formations*** in the continuum, namely relabelling of their simplicial building blocks; unlabelled triangulations represent pure geometry

(*) **CDT reviews: J. Ambjørn, A. Görlich, J. Jurkiewicz, R.Loll, Phys. Rept. 519 (2012) 127, R.Loll, CQG 37 (2020) 1**

Lattice quantum gravity à la CDT in a nutshell

geometries = Lor(M)/Diff(M) → $\mathcal{G}(M)$

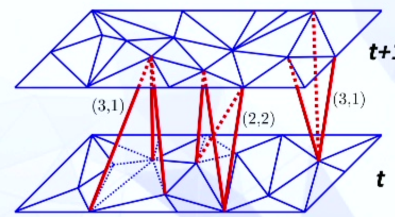
$$Z=\int_{\mathcal{G}(M)} \mathcal{D}g\, \mathrm{e}^{iS[g]} \quad \xrightarrow{\text{CDT}} \quad Z=\lim_{a\to 0} \sum_{\substack{causal\\ triang.\, T}} \frac{1}{C(T)}\, \mathrm{e}^{iS^{\mathrm{R}}[T]}$$

bare lattice action → S^{R}; # symmetries of T → $C(T)$

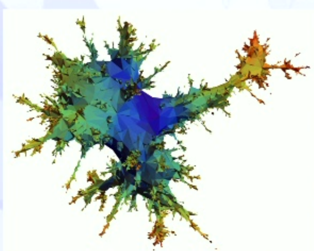
[formal continuum path integral] **[CDT: well-defined regularized path integral + cont. limit]**

==> Obtaining QG as continuum limit of a path integral of dynamical lattices incorporates the principles of general relativity *and* quantum field theory —often considered incompatible— from the outset. <==

- CDT configurations obey "global hyperbolicity":
- Z^{CDT} is amenable to powerful Monte Carlo simulations, after using CDT's ***Wick rotation***



"time layer" of a CDT configuration in 3D


typical history, 2D path integral

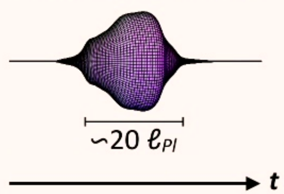
- typical path integral histories in CDT are highly nonclassical and "nowhere differentiable"

Breakthrough result: "emergent classicality"

Measurements of global Hausdorff and spectral dimensions, the shape $\langle V_3(t) \rangle$ and average curvature of the dynamically generated quantum universe (NP ground state of the path integral) match those of a classical 4D de Sitter space, ***although no background geometry was ever put in***.

J. Ambjørn, A. Görlich, J. Jurkiewicz, R. Loll, PRL 100 (2008) 091304; N. Klitgaard & R. Loll, Eur. Phys. J. C80 (2020) 990

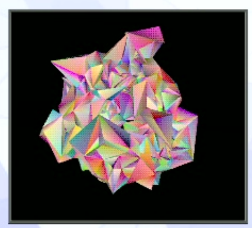
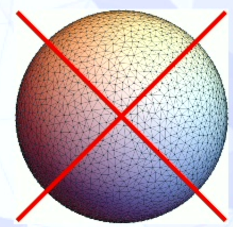
MC snapshot of the shape $\langle V_3(t) \rangle$ of the universe



Remarkable, but note that this quantum spacetime does ***not*** approximate a (Euclidean) de Sitter universe

$$ds^2 = dt^2 + c^2 \cos^2(t/c) d\Omega^2$$

in any local sense, where $a(t) = c \cos(t/c)$ is the scale factor of a homogeneous and isotropic FLRW cosmology.

The quantum universe looks like  and not  !

What about quantum spacetime locally?

CDT lattice gravity has discovered a genuine ***quantum signature***, a dynamical reduction $4 \rightarrow 2$ of the (average) ***spectral dimension*** of spacetime $@\ell_{Pl}$, impossible to predict perturbatively **J. Ambjørn, J. Jurkiewicz, R. Loll, PRL 95 (2005) 171301** , which may well be universal **S. Carlip, CQG 34 (2017) 193001.**

==> there is no corresponding $g_{\mu\nu}$!

Observables characterizing local quantum fluctuations $@\ell_{Pl}$ are ***manifestly diffeomorphism-invariant correlation functions***:

typical configuration in 2D CDT

- of local (curvature) scalars $\mathcal{O}(x)$, rather than gauge-fixed $g_{\mu\nu}(x)$
- must integrate over insertion points x, y, …
- correlators must depend on <u>geodesic</u> distances $d_g(x,y)$

e.g. two-point correlator:

geodesic distance

$$G[\mathcal{O}](r) = \frac{1}{Z}\int \mathcal{D}g\, \mathrm{e}^{-S[g]} \int dx \sqrt{g(x)} \int dy \sqrt{g(y)}\, \mathcal{O}(x)\mathcal{O}(y)\delta(d_g(x,y) - r)$$

2D prototype of a ***curvature correlator*** **J. van der Duin & R.Loll, Eur. Phys. J. C84 (2024) 7**

Nonperturbative insights and outlook

- Lattice QG is a computational lab for Planckian physics: it provides ***reality checks*** for our speculations and produces ***numbers.***
- In a nonperturbative realm $@\ell_{Pl}$, ***there are no local coordinates or reference frames*** and tensor calculus is inapplicable. This is a ***very different world*** from classical GR, where local frames always exist.
- ***Nevertheless***, QG and QST can be analyzed quantitatively with suitable observables involving distance and volume measurements.
- We must be creative in ***designing experiments*** with "rods and clocks", like in astrophysics, and subject to numerical limitations.
- **QG**$@\ell_{Pl}$ is characterized by universal behaviour, scaling relations and averages, reminiscent of other strongly coupled systems — this is ***a new perspective on what it means to solve quantum gravity***
- ***Roadmap to early-universe phenomenology,*** trying to derive its ad-hoc assumptions (symmetries, fluctuations) from first principles.

R.Loll, "Nonperturbative quantum gravity unlocked through computation", arXiv: 2501.17972