Title: Lecture - Strong Gravity, PHYS 777

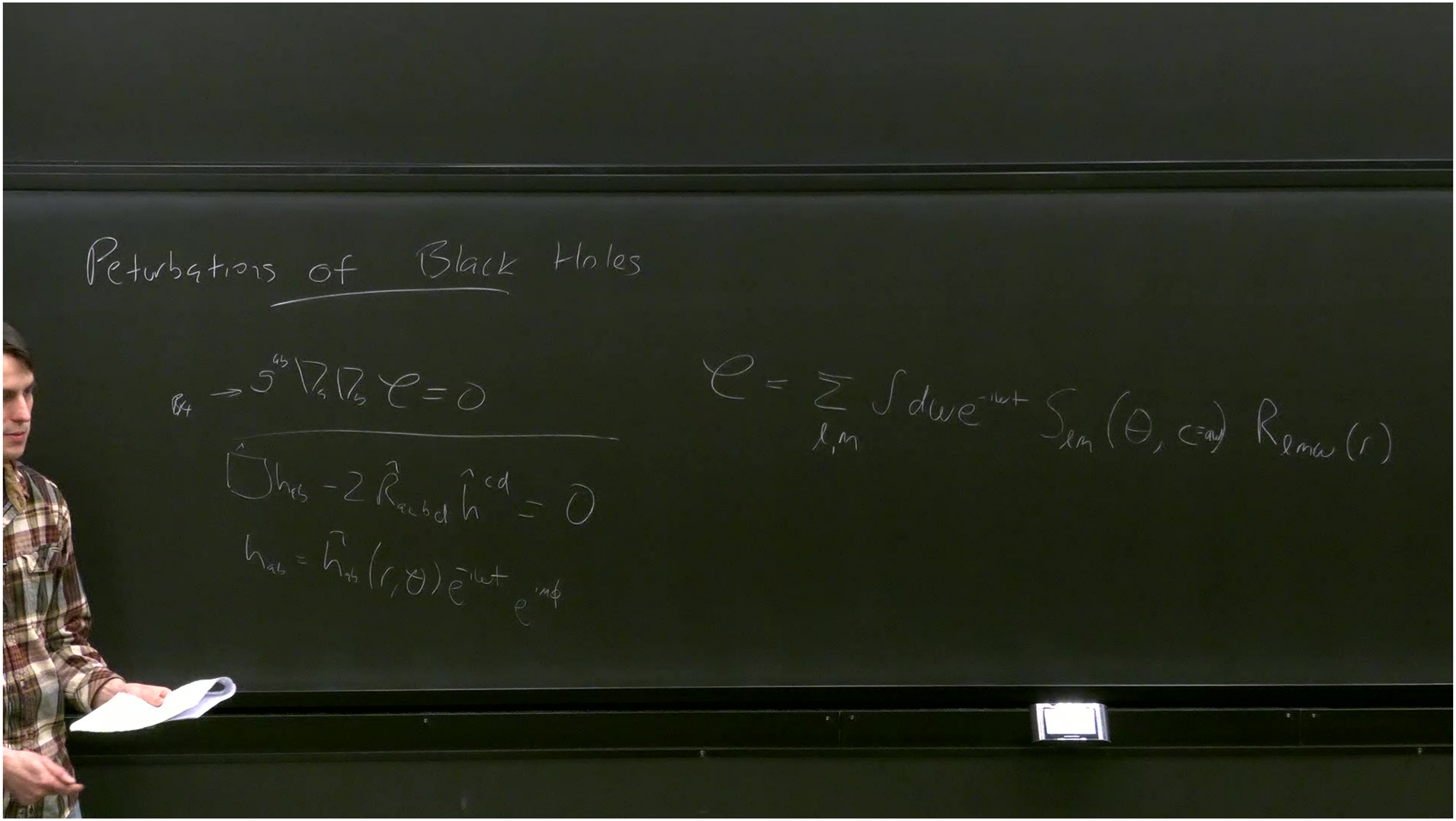
Speakers: William East

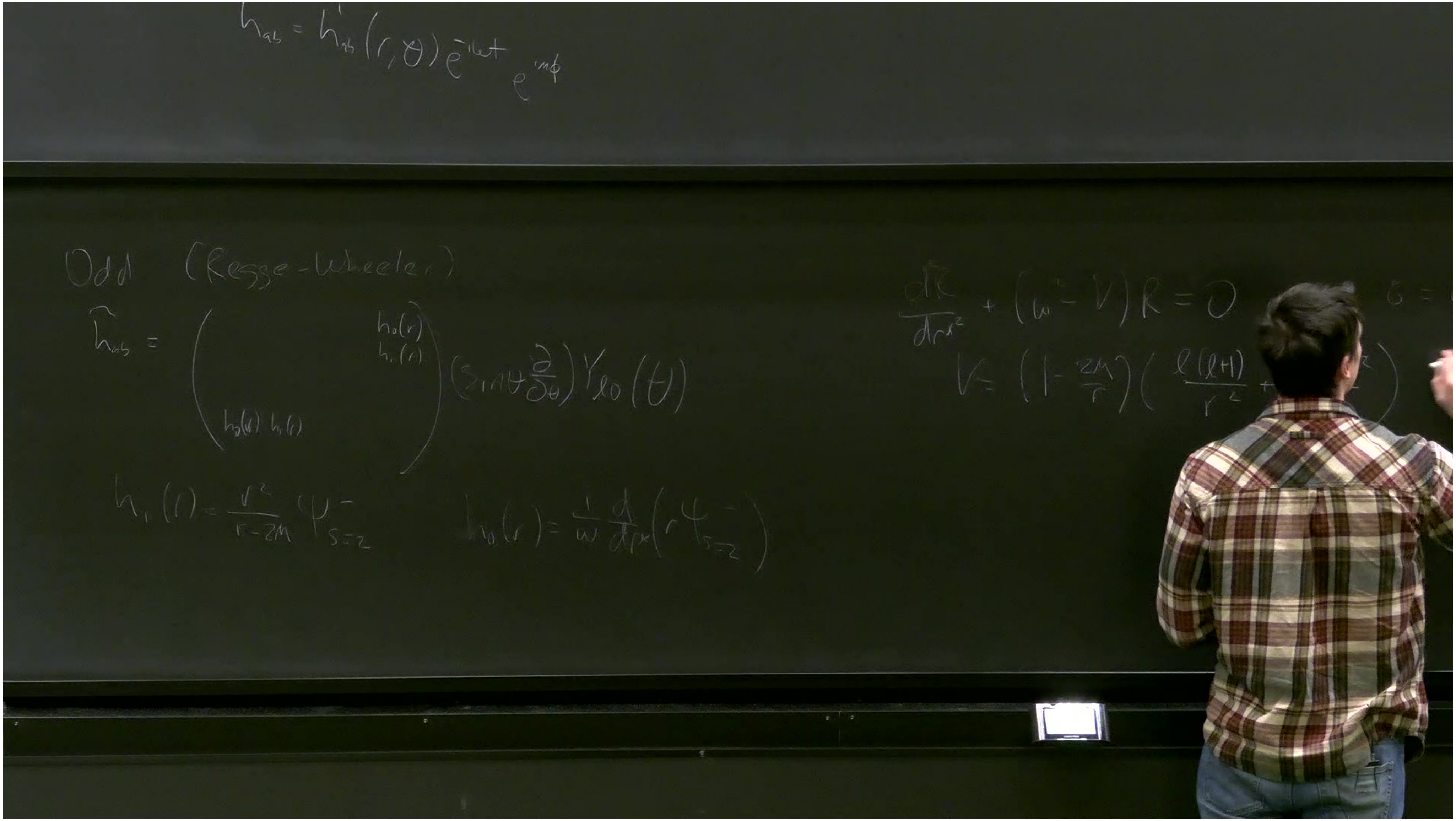
Collection/Series: Strong Gravity (Elective), PHYS 777, February 24 - March 28, 2025

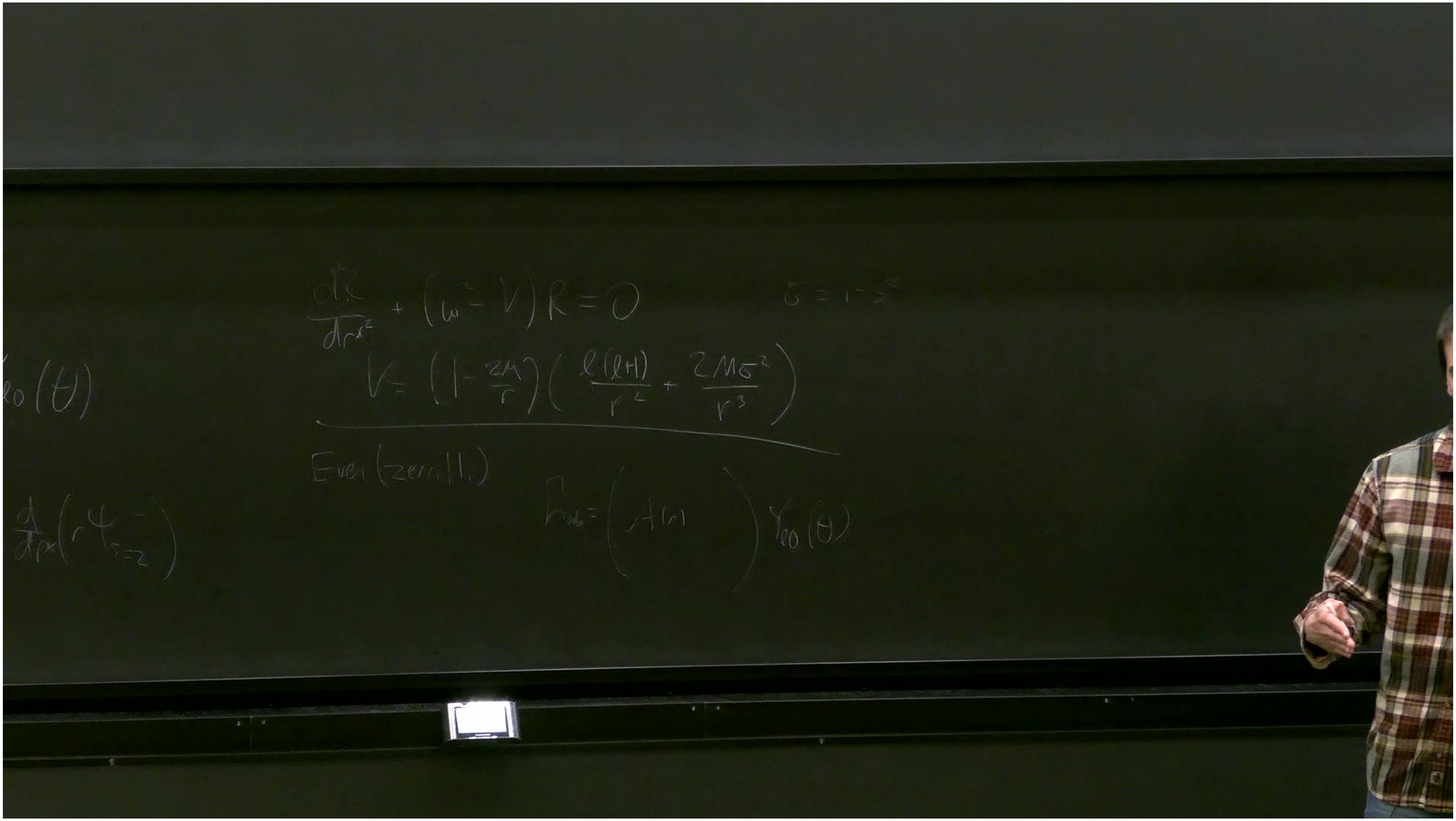
Subject: Strong Gravity

Date: March 27, 2025 - 10:15 AM

URL: https://pirsa.org/25030053


Peturbations of Black Holes
BH $\rightarrow g^{ab}\nabla_a\nabla_b \varphi = 0$
$\hat{\Box} h_{ab} - 2\hat{R}_{acbd}\hat{h}^{cd} = 0$
$h_{ab} = \hat{h}_{ab}(r,\theta)\, e^{-i\omega t}\, e^{im\phi}$
$\varphi = \sum_{l,m} \int d\omega\, e^{-i\omega t}\, S_{lm}(\theta, c = a\omega)\, R_{lm\omega}(r)$


Odd (Regge-Wheeler)


$\frac{d^2R}{dr_*^2} + (\omega^2 - V)R = 0$
$\sigma = 1 - s^2$
$V = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} + \frac{2M\sigma^2}{r^3}\right)$
$Y_{\ell 0}(\theta)$
Even (zerilli)
$h_{ab} = \begin{pmatrix} A(r) \end{pmatrix} Y_{\ell 0}(\theta)$
$\frac{d}{dr_*}\left(r\,\psi_{s=2}\right)$

$\sigma = 1 - s^2$

$$\ldots 1) \over \ldots 2} + \frac{2M\sigma^2}{r^3} \Big)$$

$$A(r) \Big) Y_{\ell 0}(\theta)$$

Boundary Conditions

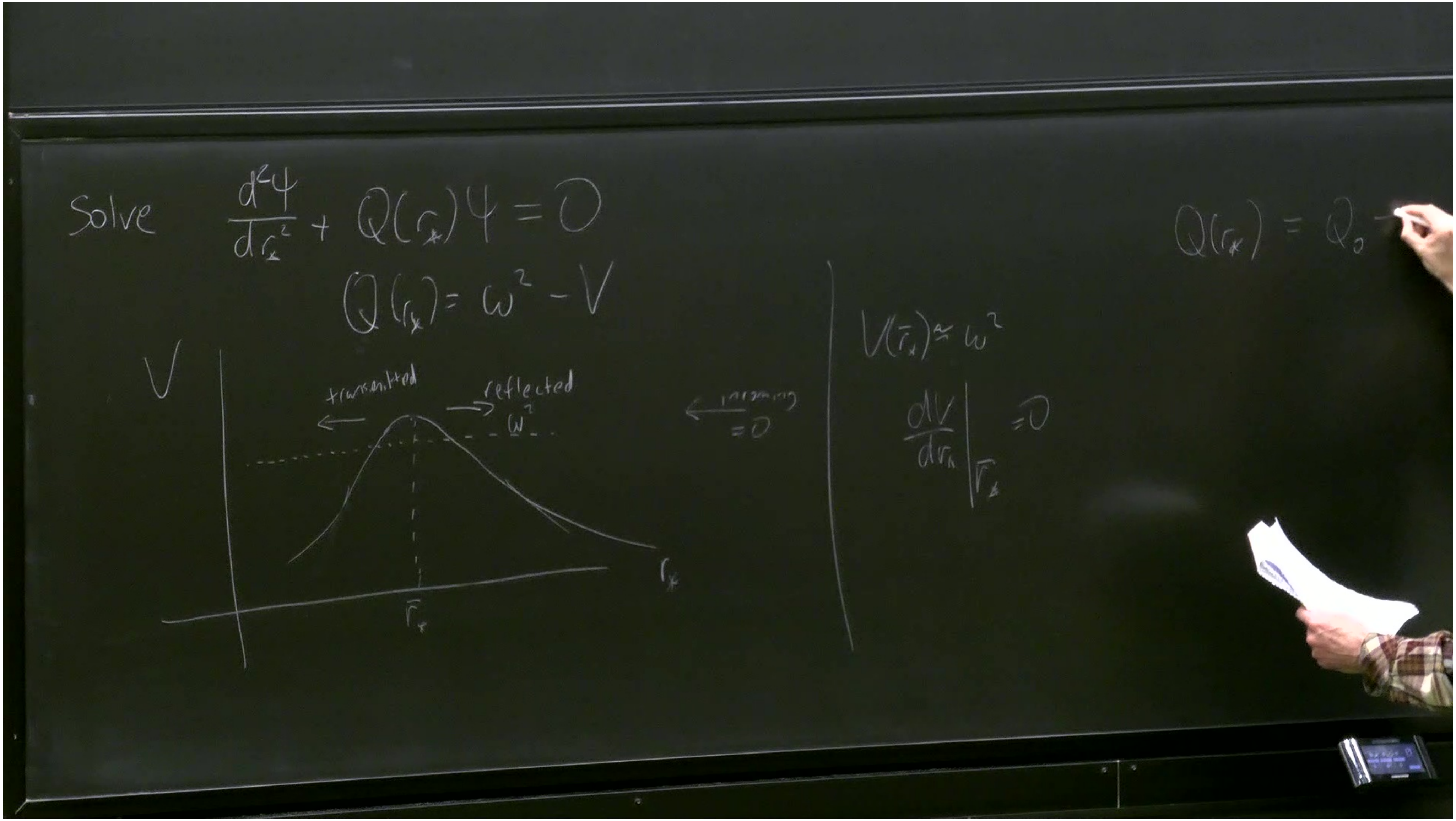
Ingoing at horizon

$r \to r_+$, $r_* \to -\infty$

$R \sim \psi \sim e^{-i\omega r_*}$

Outgoing Radiation

$r_*, r \to \infty$ $\quad R, \psi \to \sim e^{i\omega r_*}$


Solve d²Ψ/dr*² + Q(r*)Ψ = 0
Q(r*) = ω² − V
V
transmitted
reflected
ω²
r̄*
r*
incoming = 0
V(r̄*) ≃ ω²
dV/dr* |r̄* = 0
Q(r*) = Q₀

V

$\xleftarrow{\text{incoming}} = 0$

r_*

$V(\bar{r}_*) \simeq \omega^2$

$\left.\frac{dV}{dr_*}\right|_{\bar{r}_*} = 0$

$$Q(r_*) = Q_0 + \frac{1}{2} Q_0''(r_* - \bar{r}_*)^2$$

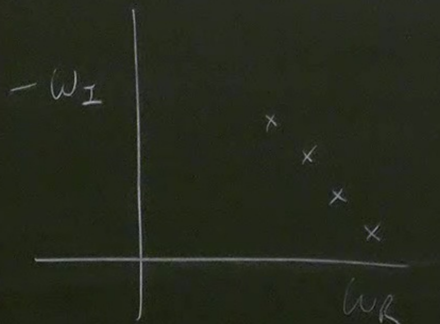
$$\Psi = A D_\nu(z) + B D_{-\nu-1}(iz)$$

$$\nu = \frac{-iQ_0}{\sqrt{2Q_0''}} - \frac{1}{2}, \quad z = (2Q_0'')^{1/4} e^{i\pi/4}(r_* - \bar{r}_*)$$

Can satisfy BCs iff $\nu = 0, 1, 2, \ldots = n$ $\quad l = 2,\ n = 0$

$$(M\omega_n)^2 = V_l(\bar{r}_*) - i\left(n + \tfrac{1}{2}\right)\left[-2\frac{d^2V_l}{dr_*^2}\right]^{1/2} \qquad M\omega \simeq 0.37 - 0.09i$$

Quasinormal modes (QNMs)



$$\sim e^{-i\omega t} = e^{-i\omega_R t + \omega_I t}$$

Weyl tensor

$$R_{abcd} = C_{abcd} + \frac{1}{2}\Big[g_{a[c} R_{d]b} - g_{b[c} R_{d]a} - \frac{1}{3} R\, g_{a[c}\, g_{d]b} \Big) \Big]$$

$$\Psi_4 = C_{abcd}\, n^a m^{*b} n^c m^{*d}$$

$$n^a = \frac{1}{2\Sigma}\left(r^2+a^2, -\Delta, 0, a\right)$$

$$m^a = \frac{\rho^*}{\sqrt{2}}\left(ia\sin\theta, 0, 1, i(\sin\theta)^{-1}\right) \sim \frac{1}{\sqrt{2}}($$

$\rho^{-1} = r - ia\cos\theta$

$r \to \infty$ in TT gauge

$$= \frac{1}{2}\left(\ddot{h}_{\hat\phi\hat\phi} - \ddot{h}_{\hat\theta\hat\theta}\right) - i\ddot{h}_{\hat\theta\hat\phi} = \ddot{h}_+ - i\ddot{h}_\times$$

General Soln

$$\Psi_4 = \rho^{-4}\sum_{l,m}\int d\omega\, e^{-i\omega t}\, {}_{-2}S_{lm}(\theta, c)\, e^{im\phi} R_{lm\omega}(r)$$

$s=-2$ spheroidal harmonics

QNMs (n, l, m)

Weyl tensor

$$R_{abcd} = C_{abcd} + \frac{1}{2}\left[g_{a[c}R_{d]b} - g_{b[c}R_{d]a} - \frac{1}{3}R\, g_{a[c}\, g_{d]b} \right]$$

$$\Psi_4 = C_{abcd}\, n^a m^{*b} n^c m^{*d}$$

$$n^a = \frac{1}{2\Sigma}\left(r^2+a^2, -\Delta, 0, a\right)$$

$$m^a = \frac{\rho^*}{\sqrt{2}}\left(ia\sin\theta, 0, 1, i(\sin\theta)^{-1}\right) \sim \frac{1}{\sqrt{2}}\left(\hat{\theta} + i\hat{\phi}\right)$$

$$\rho^{-1} = r - ia\cos\theta$$

$r \to \infty$ in TT gauge

$$= \frac{1}{2}\left(\ddot{h}_{\hat{\phi}\hat{\phi}} - \ddot{h}_{\hat{\theta}\hat{\theta}}\right) - i\ddot{h}_{\hat{\theta}\hat{\phi}} = \ddot{h}_+ - i\ddot{h}_\times$$

General Soln

$$\Psi_4 = \rho^{-4} \sum_{l,m} \int d\omega\, e^{-i\omega t}\, {}_{-2}S_{lm}(\theta, c)\, e^{im\phi} R_{lm\omega}(r)$$

$s=-2$ spheroidal harmonics

QNMs (n, l, m) for a given BH spin

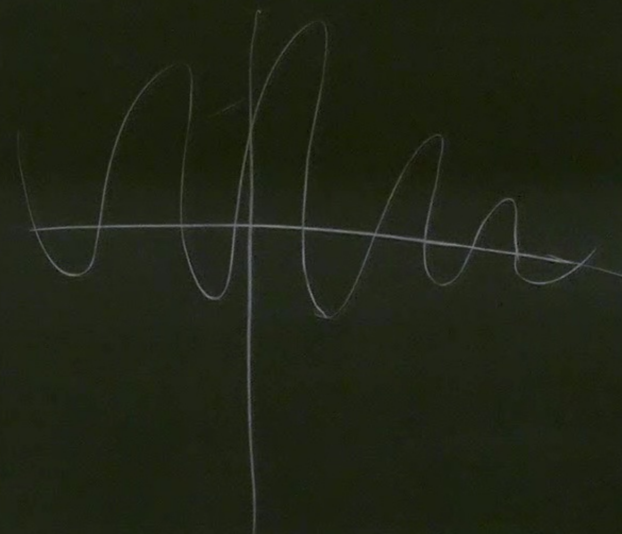
$l = 2 = m$ $\quad n = 0$

$$M\omega_{22} \simeq 1.53 - (1.16)\left(1-\frac{a}{M}\right)^{0.53}$$

$$Q_{22} \simeq \frac{\omega_R}{2\omega_I} \simeq 0.70 + 1.42\left(1-\frac{a}{M}\right)^{-0.5}$$

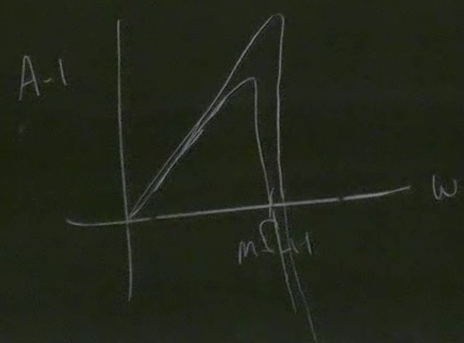
$a/M = 0.7$

$\omega^R_{22} \simeq 0.53$ $\quad$ $\omega_I \simeq 0.08$

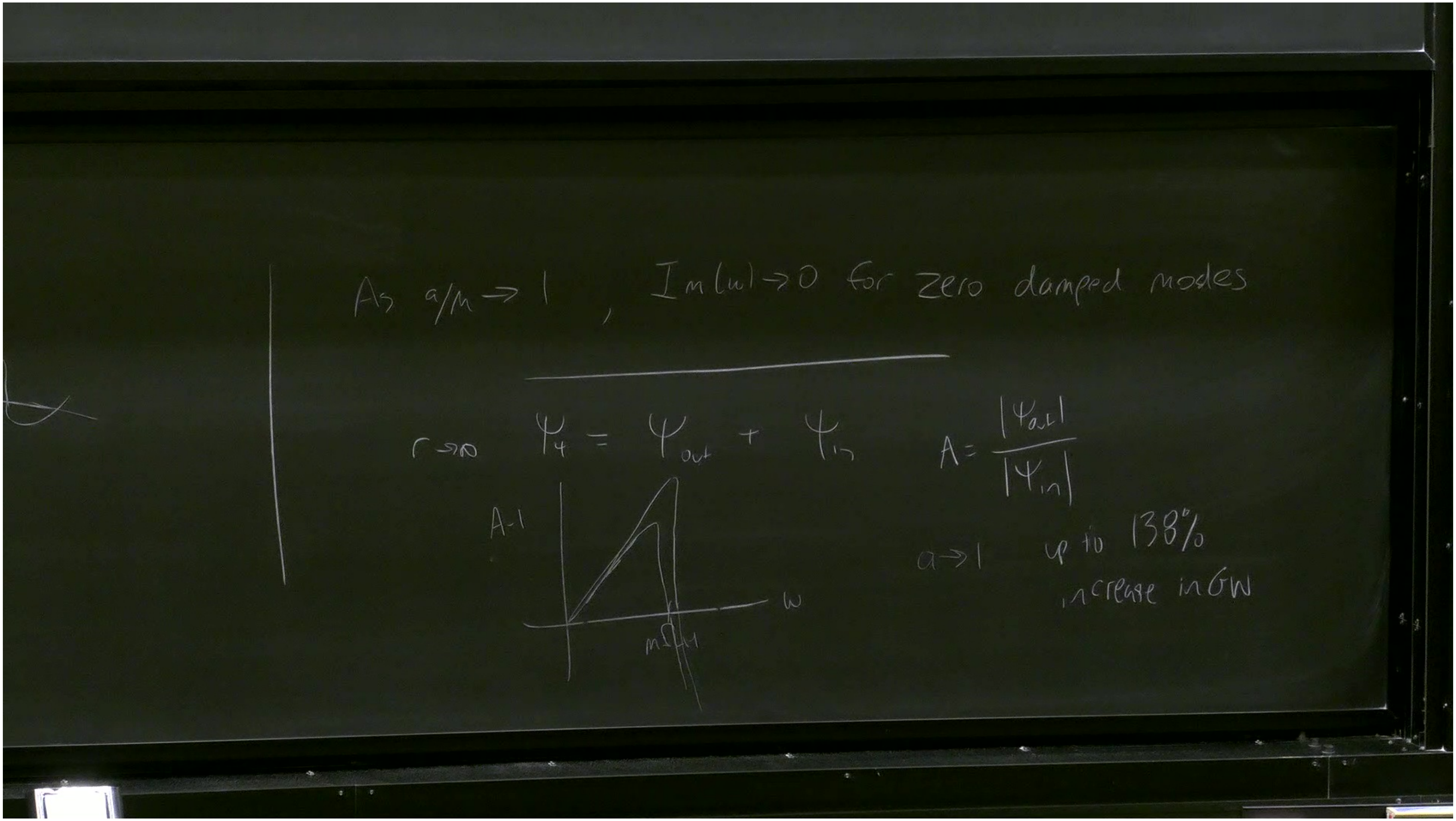


As $a/M \to 1$, $\mathrm{Im}(\omega) \to 0$ for zero damped modes

$r \to \infty \quad \Psi_4 = \Psi_{out} + \Psi_{in} \qquad A = \dfrac{|\Psi_{out}|}{|\Psi_{in}|}$



$a \to 1$ up to 138% increase in GW


As a/M → 1 , Im(ω) → 0 for zero damped modes
r → ∞ Ψ4 = Ψout + Ψin
A = |Ψout| / |Ψin|
A-1
ω
mΩH
a → 1 up to 138%
increase in GW