Title: Lecture - Strong Gravity, PHYS 777

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Collection/Series: Strong Gravity (Elective), PHYS 777, February 24 - March 28, 2025

Subject: Strong Gravity

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Generalized harmonic Formulation

$R_{ab} = 0 \longrightarrow R_{ab} - \nabla_{(a} C_{b)} = 0$ $\quad C^a = H^a - \Box x^a$

Maxwell Eqns

$\partial_a F^{ab} = \partial^a \partial_a A_b - \partial^a \partial_b A_a = 0$

+ component $\quad -\cancel{\partial_t^2 A_t} + \partial_i \partial^i A_t + \cancel{\partial_t^2 A_t} - \partial^i \partial_t A_i = 0$

$\partial^i (\partial_i A_t - \partial_t A_i) = 0$

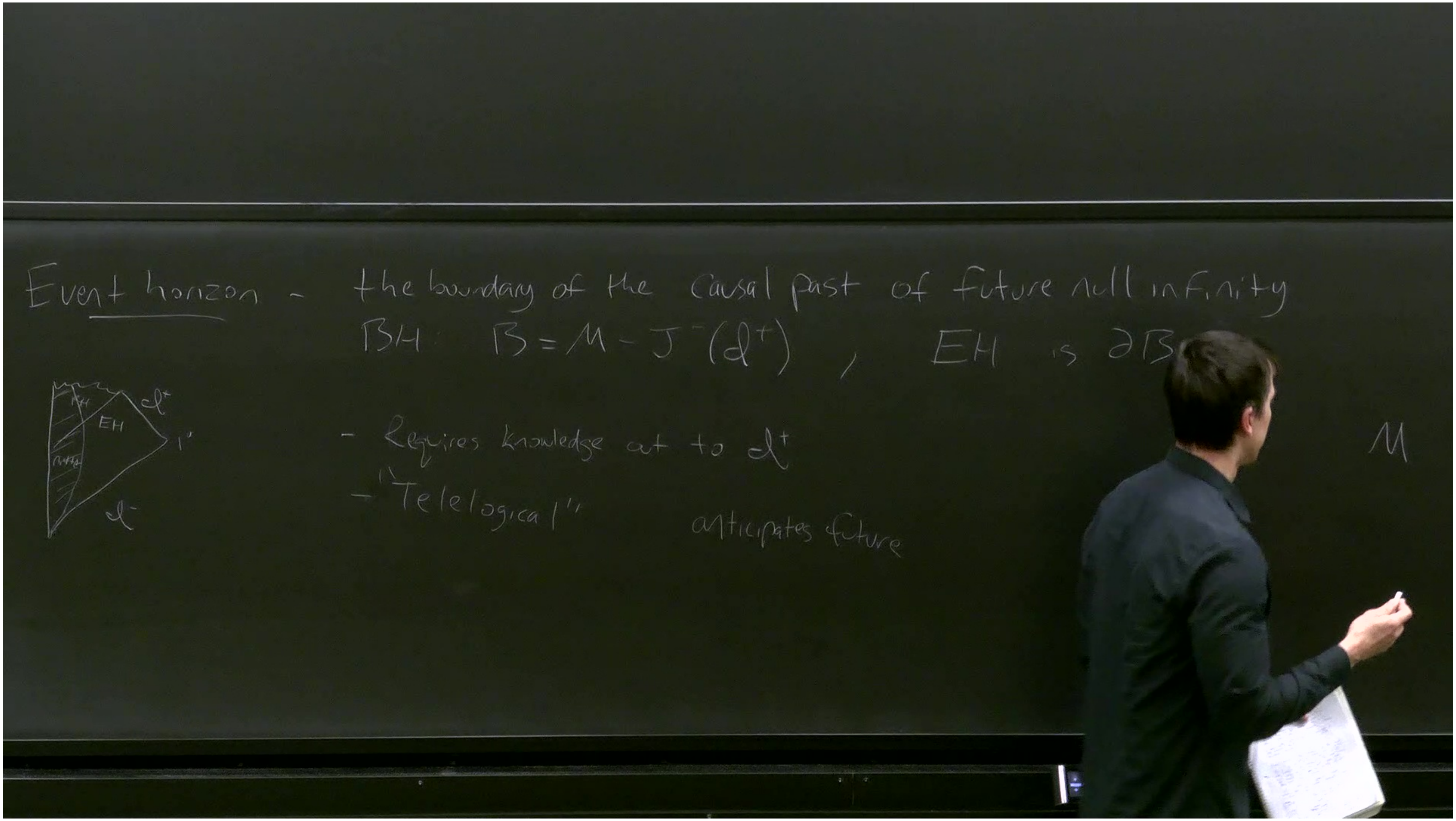
$\vec{\nabla} \cdot \vec{E} = 0$

Lorenz Gauge

$\partial_a A^a = 0$

$\partial_a \partial^a A_b = 0$

$\partial^b (\partial^a \partial_a A_b) = \partial_a \partial^a (\partial^b A_b) = 0$


Event horizon - the boundary of the causal past of future null infinity
BH: $B = M - J^-(\mathscr{I}^+)$, EH is ∂B
AH
EH
$\mathscr{I}^+$
i^0
$\mathscr{I}^-$
- Requires knowledge out to $\mathscr{I}^+$
- "Telelogical"
anticipates future
M

e null infinity

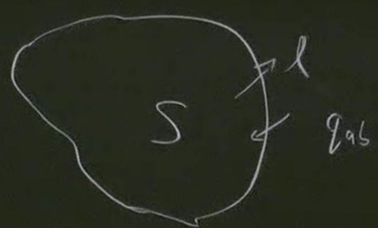
is ∂B

$$\frac{2M}{r} \leq 1$$

$n_a n^a = -$

Apparent horizons

Σ_t - 3 dim



Expansion

$$\theta := \mathcal{L}_\ell \left(\log\left(\sqrt{q}\right) \right) = q^{ab} \nabla_a \ell_b$$

Take ℓ to be null

Outward/inward null $\quad \ell^{\pm} = n^a \pm s^a$

unit normal to Σ_t

unit spacelike vector normal to

$-1 \quad s_a s^a = 1 \quad \ell_\pm^a \ell_a^\pm = 0 \quad \ell_-^a \ell_a^+ = -2$

Outward null expansion

$$\theta_{\ell^+} = q^{ab} \nabla_a \ell_b^+ = D_i s^i + K_{ij} s^i s^j - K \quad \text{(in 3+1 variables)}$$

$$\frac{1}{r^2}\partial_r(r^2) = \frac{2}{r}$$

If $\theta_{\ell^+} \leq 0$ on S $\quad S$ is outer trapped surface

$\theta_{\ell^+} = 0 \quad S$ is marginally " " " (MOTS)

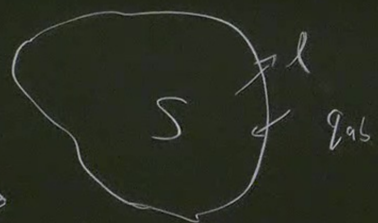
Outermost MOTS = Apparent horizon

nal to S

Apparent horizons

$n_a n^a = -$

Σ_t - 3 dim



Trapped surfaces $\subset B$

(assume CC, NEC)

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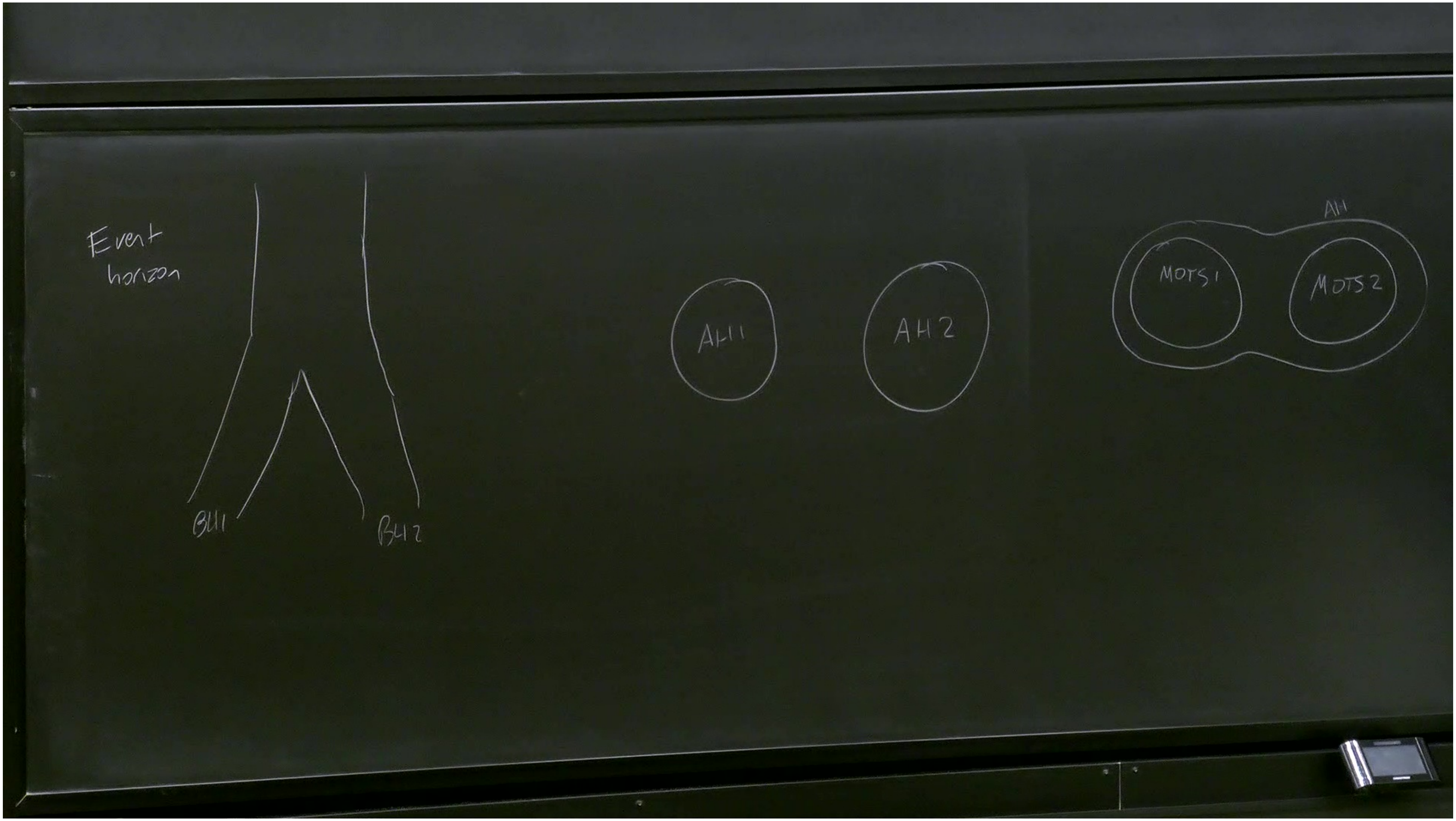

Event horizon
BH1
BH2
AH1
AH2
AH
MOTS1
MOTS2

Gravitational Waves ref. • arXiv gr-qc/0501041

Linear perturbations of metric (vacuum $R_{ab}=0$)

$$g_{ab} = \hat{g}_{ab} + h_{ab} \qquad |h_{ab}| \ll \hat{g}_{ab}$$

$$g^{ab} = \hat{g}^{ab} - h^{ab} \qquad \text{(to linear order)}$$

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$$\Gamma^a_{bc} = \hat\Gamma^a_{bc} + \frac{1}{2}\hat g^{ad}\left(\partial_b h_{cd} + \partial_c h_{bd} - \partial_d h_{bc}\right) - \frac{1}{2} h^{ad}\left(\partial_b \hat g_{cd} + \partial_c \hat g_{bd} - \partial_d \hat g_{bc}\right)$$

$\hat g^{bc}$

$$\delta\Gamma^a_{bc} = \Gamma^a_{bc} - \hat\Gamma^a_{bc} = \frac{1}{2}\hat g^{ad}\left(\hat\nabla_c h_{bd} + \hat\nabla_c h_{bd} - \hat\nabla_d h_{bc}\right)$$

$$\left(-\hat\Box x^a = -\Box x^a + h^{bc}\hat\Gamma^a_{bc} = \hat\Gamma^a + \hat\nabla_b\left(h^{ab} - h\hat g^{ab}\right), \quad h = \hat g^{ab} h_{ab}\right)$$

$$R^a{}_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc} + \Gamma^a_{ce}\Gamma^e_{bd} - \Gamma^a_{de}\Gamma^e_{bc}$$

$$= \partial_c \hat\Gamma^a_{bd} - \partial_d \hat\Gamma^a_{bc} + \partial_c \delta\Gamma^a_{bd} - \partial_d \delta\Gamma^a_{bc} \quad \text{in locally flat coordinates}$$

$\hat\Gamma^a_{bc} = 0$

$$= \hat R^a{}_{bcd} + \hat\nabla_c \delta\Gamma^a_{bd} - \hat\nabla_d \delta\Gamma^a_{bc} \quad \text{in general}$$