Title: Lecture - Quantum Gravity, PHYS 644

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Subject: Quantum Gravity

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$$\mathcal{F} = \mathrm{Riem}_{1,3}(M)$$

$$\mathcal{G} = \mathrm{diff}(M) \simeq \mathfrak{X}(M)$$

$$\rightsquigarrow$$

$$\mathcal{P}_{con} = T^*\mathrm{Riem}_3(\Sigma)$$

symmetries ?

↳ (certainly not 4-diffeos!)

"Best case scenario" (YM)

$$\mathbb{I}_{\rho(\xi)} \Omega_\Sigma = -\mathbb{d} Q_\Sigma(\xi)$$

$$Q_\Sigma(\xi) = \text{constraints} + \cancel{\text{corner}}$$

$\partial\Sigma = \phi$

gauge syms are Hamiltonian generated by constr.

Symmetries descend to

$$(\mathcal{P}_{con}, \omega_{con}) \to \mathbb{I}_{\rho(\xi)} \omega_{con} = -\mathbb{d} Q(\xi)$$

$$Q \approx 0$$

$$\{Q(\xi), Q(\eta)\} = Q([\xi, \eta])$$

constraint algebra represents the gauge algebra.

$$\mathcal{F} = \mathrm{Riem}_{1,3}(M) \ni g_{ab}$$

$$\mathcal{G} = \mathrm{diff}(M) \simeq \mathfrak{X}(M)$$

$$\rightsquigarrow$$

$$\mathcal{P}_{\mathrm{con}} = T^*\mathrm{Riem}_3(\Sigma) \ni (\pi^{ij}, h_{ij})$$

symmetries ?

↳ (certainly not 4-diffeos!)

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Constraints in GR

Vector constraints: $\mathcal{V}_i = \bar{\nabla}_j \pi^j{}_i$

Hamiltonian constraints: $\mathcal{H} = \frac{1}{2\sqrt{h}} \underbrace{\mathbb{G}_{ijkl}\, \pi^{ij} \pi^{kl}}_{\text{kinetic}} - \underbrace{\frac{1}{2}\sqrt{h}\left(\bar{R} - 2\Lambda\right)}_{\text{potential}}$

$$\mathbb{G}_{(ij)(kl)} = h_{ik} h_{jl} - \frac{1}{2} h_{ij} h_{kl}$$

DeWitt supermetric on $\mathrm{Riem}(\Sigma)$

$$V(X) := \int_\Sigma \mathcal{V}_i(\pi, h)\, X^i \qquad V : \mathfrak{X}'(\Sigma) \to C^\infty(P_{can})$$

$$H(N) = \int_\Sigma \mathcal{H}(\pi, h)\, N \qquad H : C^\infty(\Sigma) \to C^\infty(P_{can})$$

To understand syms on P_{can}, consider

$$\{V(X), V(Y)\} = V([X,Y]) \equiv V(L_X Y) \leftarrow \mathcal{V}_i \text{ vector density under 3 diffeos}$$

$$\{V(X), H(N)\} = H(L_X N) \leftarrow \mathcal{H} \text{ scalar density} \text{ — " —}$$

$$\{H(N), H(M)\} = \pm V(Z), \quad Z^i = \boxed{h^{ij}} (N \nabla_j M - M \nabla_j N)$$

$\uparrow$ depending on signature of g_{ab}

Dirac's hypersurface deformation algebra (HDA)

(is not a repn of 4-diffeos!)

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In, GR first computed by Kuchař 1972

(DeWitt 1967)

Dirac already discovered the HDA in 1950
studying Hamilt. field theory (on Minkowski)
in arbitrary foliation

$\leadsto$ "parametrized (scalar) field" $\quad C_a = P_a + h_a(X, \pi, \phi)$

conjug

Two options: either

(1) GR is secretly a parametrized f.t., or ✗

(2) the HDA is "universal" ✓ (Teitelboim 1974, Kuchař, Hojman, ...)

$$\{ \nu_i \, \overset{R}{\overset{\downarrow}{\delta N^i}} , \cdot \} = \delta N^i \{ \nu_i , \cdot \} + \overset{\approx 0}{\overset{\downarrow}{\nu_i}} \{ \delta N^i , \cdot \}$$

IMPORTANT:

in GR, this "integrability" makes sense

<u>only</u> $\boxed{\text{on shell}}$ of the constraints

b.c. of $\mathcal{Z}^{i} = h^{ij}\left(N \overleftrightarrow{\nabla}_{j} M\right)$

$\rightarrow$ "A 4d sp.t. exists only on shell"

in a sense $\Pi^{ij} \sim K_{ij} = \frac{1}{2} L_{n} h_{ij}$ only on shell.

In GR,

$(\mathcal{H}, \mathcal{V}_i) \approx 0 \longleftrightarrow$ Einstein's eqs $+$ ($\exists$ spacetime)

Fact ($\underbrace{\text{Cartan, Weyl}}_{}^{\text{1920s}}$, Lovelock):

GR (EH action or Einstein field eqs)

are fully pinned down by a couple of assumptions.

1. gravity is fully described by g_{ab} $(-+++)$
2. coord indep, 2nd order eom. $\left(G_{ab}(g, \partial g, \partial^2 g) \approx 0\right)$
3. $\exists$ action principle or $\nabla^a G_{ab} \equiv 0$

ell.

Is there a Hamiltonian
version of this uniqueness
pinning down functional form of $\mathcal{H}, V_i$?

(HKT 1973) yes

(1) $T^*\mathrm{Riem}(\Sigma)$, 4d signature.

(2b) ✓ (Ham eqs over a ph sp.)

(2a) HDA generates deformations

Mathy note

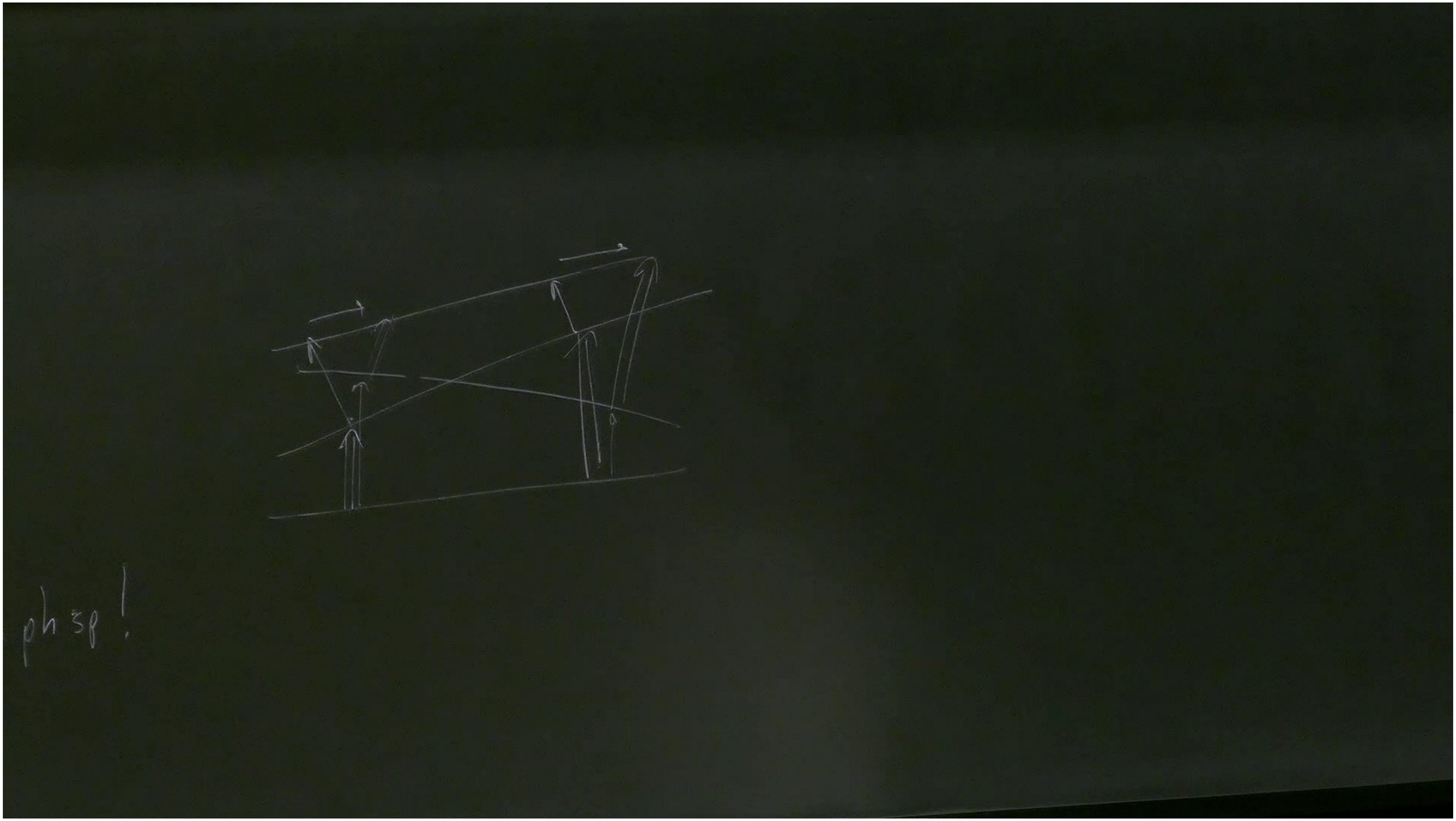
$$V : \mathfrak{X}^1(\Sigma) \longrightarrow C^\infty(P_{con})$$

$$H : C^\infty(\Sigma) \longrightarrow C^\infty(P_{con})$$

however

$$\{ H(\cdot), H(\cdot) \} = V(h\,\cdot)$$

is not eating a "simple" vector but a vector-valued function on ph sp!


ph sp!