Title: Lecture - Quantum Gravity, PHYS 644

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Subject: Quantum Gravity

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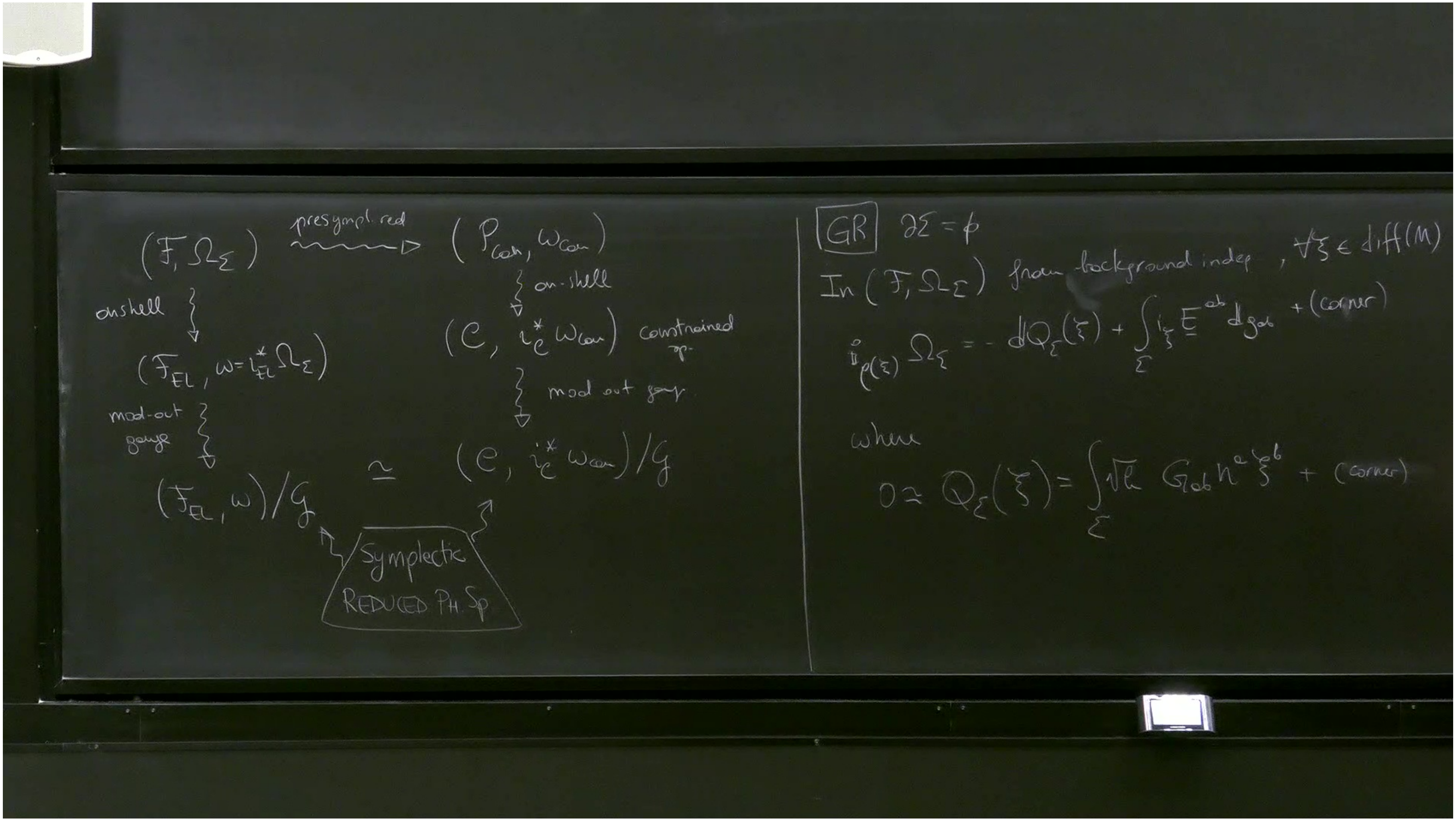
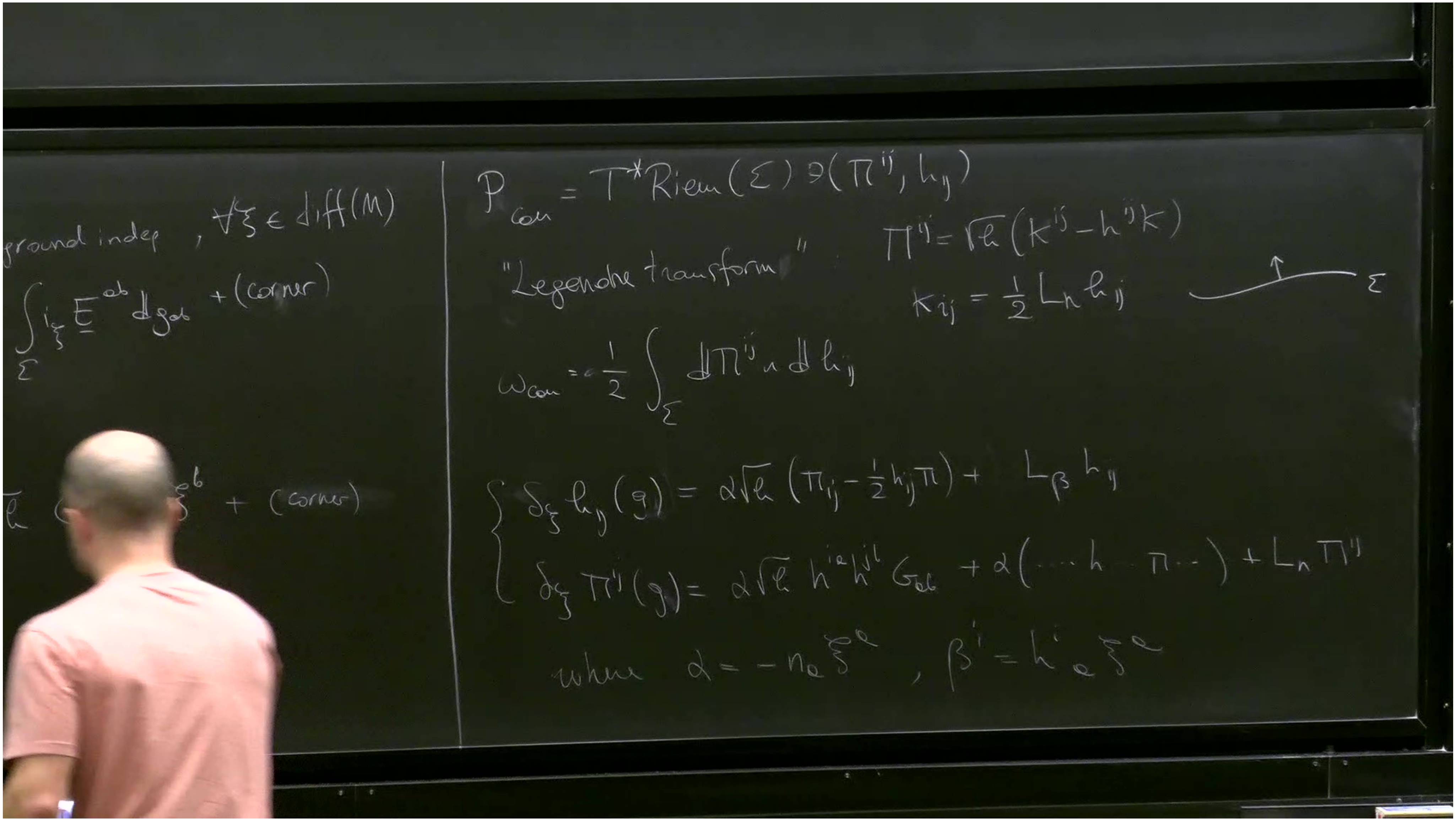
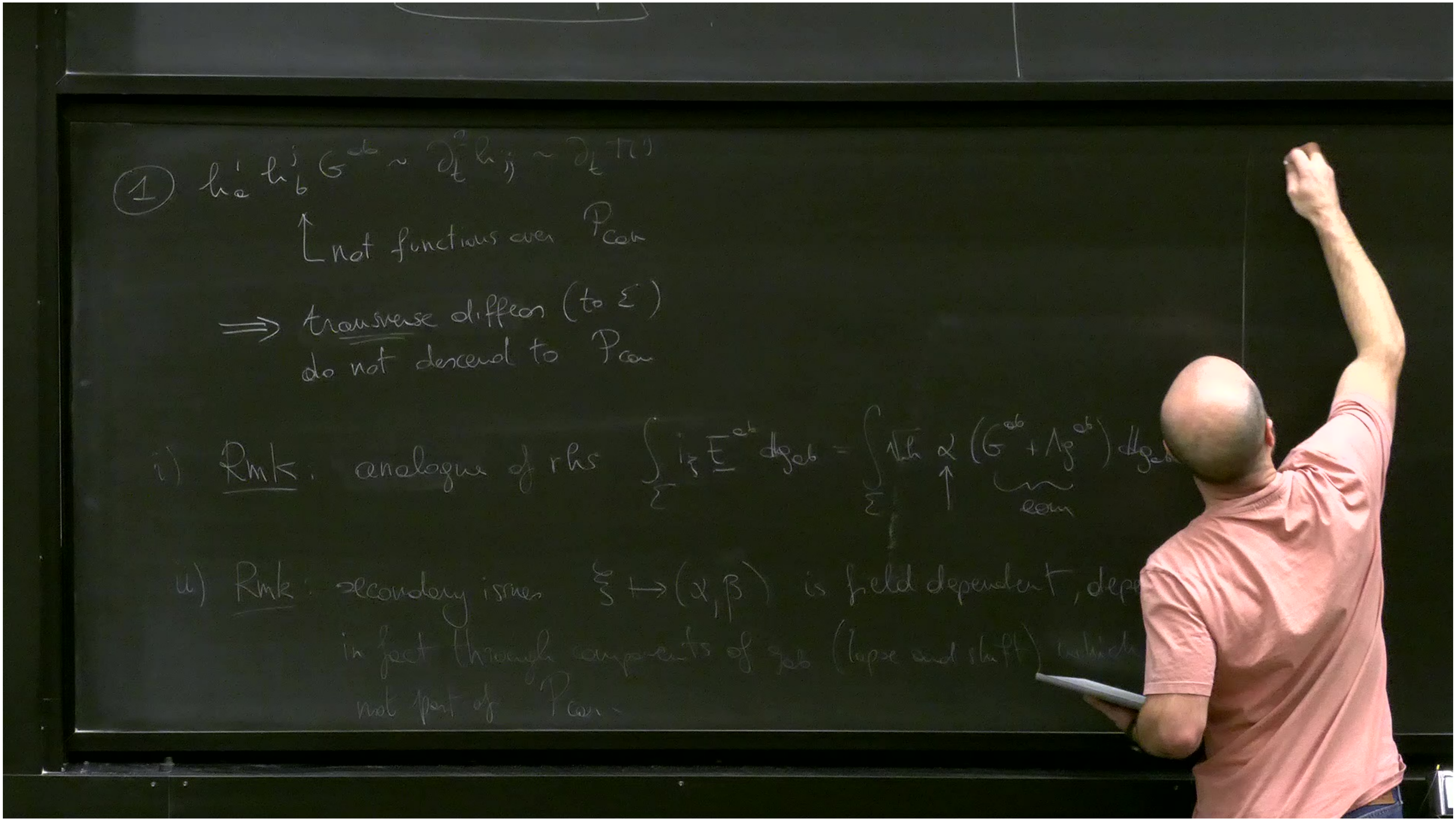

$(\mathcal{F}, \Omega_\Sigma)$ presympl. red (P_{con}, ω_{con})
on shell
$(\mathcal{F}_{EL}, \omega = \iota^*_{EL}\Omega_\Sigma)$
mod-out gauge
$(\mathcal{F}_{EL}, \omega)/\mathcal{G}$
on-shell
$(\mathcal{C}, \iota^*_{\mathcal{C}}\omega_{con})$ constrained sp.
mod out gauge
$\simeq$ $(\mathcal{C}, \iota^*_{\mathcal{C}}\omega_{con})/\mathcal{G}$
Symplectic
REDUCED PH. Sp
GR $\partial\Sigma = \phi$
In $(\mathcal{F}, \Omega_\Sigma)$ from background indep, $\forall \xi \in \mathrm{diff}(M)$
$\mathbb{I}_{\rho(\xi)}\Omega_\Sigma = -\mathrm{d}Q_\Sigma(\xi) + \int_\Sigma i_\xi E^{ab}\,\mathrm{d}g_{ab} + (\text{corner})$
where
$0 \simeq Q_\Sigma(\xi) = \int_\Sigma \sqrt{h}\, G_{ab} n^a \xi^b + (\text{corner})$


ground indep , $\forall \xi \in \mathrm{diff}(M)$
$\int_\Sigma i_\xi \underline{E}^{ab}\, d g_{ab} + (\text{corner})$
$\xi^b + (\text{corner})$
$P_{can} = T^*\mathrm{Riem}(\Sigma) \ni (\pi^{ij}, h_{ij})$
"Legendre transform" $\pi^{ij} = \sqrt{h}\,(K^{ij} - h^{ij}K)$
$K_{ij} = \frac{1}{2} L_n h_{ij}$
Σ
$\omega_{can} = \frac{1}{2}\int_\Sigma d\pi^{ij} \wedge d h_{ij}$
$\delta_\xi h_{ij}(g) = \alpha\sqrt{h}\,(\pi_{ij} - \frac{1}{2} h_{ij}\pi) + L_\beta h_{ij}$
$\delta_\xi \pi^{ij}(g) = \alpha\sqrt{h}\, h^{ia}h^{jb} G_{ab} + \alpha(\cdots h \cdots \pi \cdots) + L_n \pi^{ij}$
where $\alpha = -n_a \xi^a$, $\beta^i = h^i{}_a \xi^a$


① $h^i_a h^j_b G^{ab} \sim \partial_t^2 h_{ij} \sim \partial_t \pi^{ij}$
not functions over P_{cov}
$\Longrightarrow$ transverse diffeos (to Σ)
do not descend to P_{cov}
i) Rmk: analogue of rhs $\int_\Sigma i_\xi E^{ab} \delta g_{ab} = \int_\Sigma \sqrt{h}\, \alpha\, (G^{ab} + \Lambda g^{ab})\, \delta g_{ab}$
eom
ii) Rmk: secondary issues $\xi \mapsto (\alpha, \beta)$ is field dependent, depe
in fact through components of g_{ab} (lapse and shift) which
not part of P_{cov}.

where $\alpha = -n_a \xi^a$, $\beta^i = h^i{}_a \xi^a$

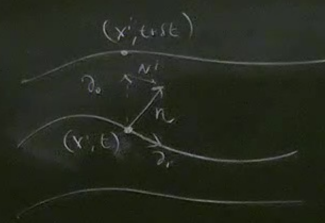
<u>Def</u> (lapse & shift):

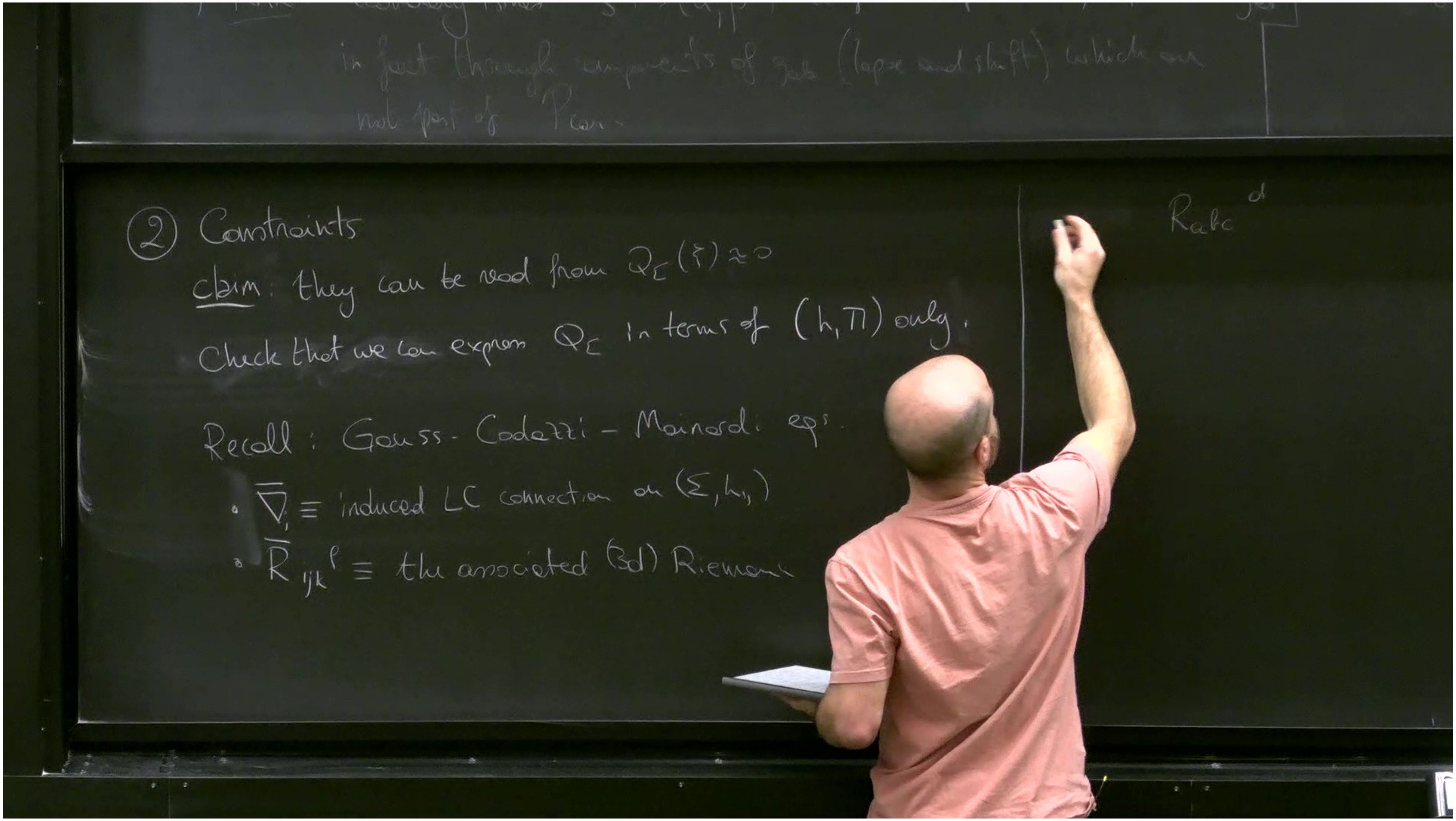
$$g_{ab} = \begin{pmatrix} -N^2 + h^{ij} N_i N_j & N_j \\ N_i & h_{ij} \end{pmatrix}$$

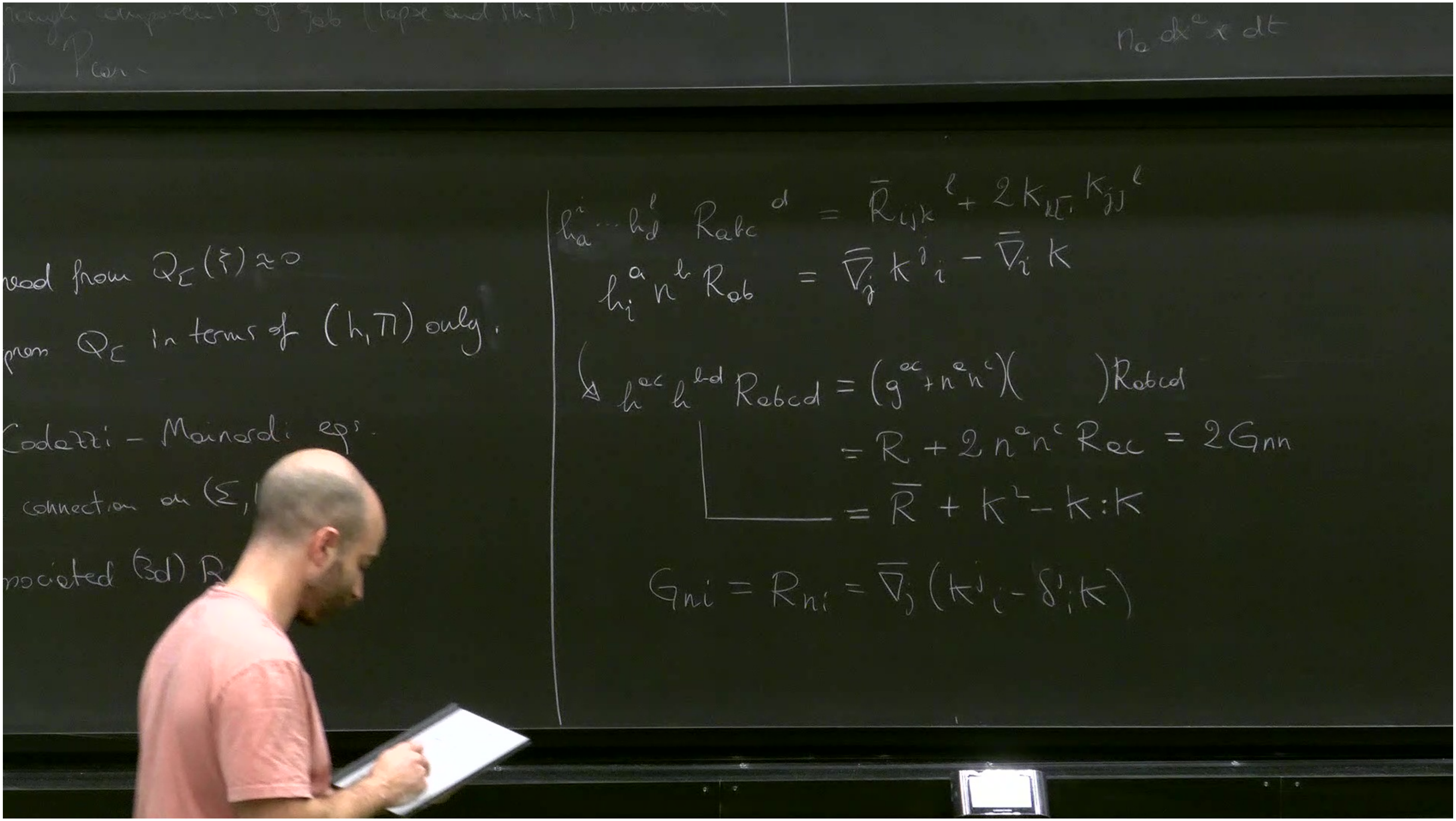
$$-N^2 = \frac{1}{g^{00}} \quad \& \quad N^i = N^2 g^{0i} = h^{ij} g_{0j}$$

$\boxed{N = \text{lapse}}$ $\qquad$ $\boxed{N^i = \text{shift vector}}$

if $\Sigma = \{t = 0\}$ $\quad n^a \partial_a = \frac{1}{N}\left(\partial_0 - N^i \partial_i\right)$




in fact through components of g_{ab} (lapse and shift) which are
not part of P_{can}.
② Constraints
claim: they can be read from $Q_\xi(\xi) \approx 0$
Check that we can express Q_ξ in terms of (h, π) only.
Recall: Gauss - Codazzi - Mainardi eqs.
• $\bar{\nabla}_i \equiv$ induced LC connection on (Σ, h_{ij})
• $\bar{R}_{ijk}{}^{l} \equiv$ the associated (3d) Riemann
$R_{abc}{}^{d}$


$n_a\, dx^a \propto dt$
...ed from $Q_\Sigma(\xi) \approx 0$
...rom Q_Σ in terms of (h,π) only.
Codazzi – Mainardi eqs.
connection on $(\Sigma,$
...sociated (3d) R
$h_a^i \cdots h_d^l R_{abc}{}^d = \bar{R}_{ijk}{}^l + 2K_{k[i}K_{j]}{}^l$
$h_i^a n^b R_{ab} = \bar{\nabla}_j K^j{}_i - \bar{\nabla}_i K$
$h^{ac} h^{bd} R_{abcd} = (g^{ac} + n^a n^c)(\quad) R_{abcd}$
$= R + 2 n^a n^c R_{ac} = 2 G_{nn}$
$= \bar{R} + K^2 - K:K$
$G_{ni} = R_{ni} = \bar{\nabla}_j (K^j{}_i - \delta^j_i K)$

$n_a \, dx^a \wedge dt$

$\ell + 2K_{k[i}K_{j]}{}^{\ell}$

$_i - \bar{\nabla}_i K$

$n^a n^c)(\qquad)R_{abcd}$

$+ 2 n^a n^c R_{ac} = 2G_{nn}$

$+ K^2 - K:K$

$(K^j{}_i - \delta^j_i K)$

Thus:

$$Q_\Sigma(\xi) = \int_\Sigma \sqrt{h}\,\left(\bar{R} + K^2 - K:K - \Lambda\right)\alpha + \sqrt{h}\,\beta^j \bar{\nabla}_j (K^j{}_i - \delta^j_i K)$$

$\downarrow$ express in terms of (h, π)

$$= \int_\Sigma \mathcal{H}\,\alpha + \int_\Sigma \mathcal{V}_i \beta^i \equiv H(\alpha) + V(\beta^i)$$

depend on lapse & shift

$$\begin{cases} \mathcal{V}_i = \bar{\nabla}_j \pi^j{}_i & (\sim \text{Gauss}) \quad \text{vector constraint} \\ \mathcal{H} = \dfrac{1}{2\sqrt{h}}\left(\pi:\pi - \dfrac{1}{2}\pi^2\right) - \dfrac{1}{2}\sqrt{h}\left(\bar{R} - 2\Lambda\right) & \text{Hamiltonian constraint} \end{cases}$$

$\uparrow (3+1)d$

Rmk $\mathcal{H}$, V_i are constraints on P_{can}

Hower $Q_\Sigma(\xi) = H(\alpha) + V(\beta)$ is not per se a function on P_{can}, b.c. normal decomp.

$\xi \mapsto (\alpha, \beta)$ depends on (N, N^i)