Title: Lecture - Quantum Gravity, PHYS 644

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Collection/Series: Quantum Gravity (Elective), PHYS 644, February 24 - March 28, 2025

Subject: Quantum Gravity

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BACKGROUND INDEPENDENCE

A (k,ℓ) - mixed form $\underline{\alpha} \in \Omega^{k,\ell}(M \times F)$

is said B.I. if

$$\forall \xi \in \mathrm{diff}(M) \simeq \mathfrak{X}^1(M) \qquad \mathbb{L}_{\rho(\xi)} \underline{\alpha} = L_\xi \underline{\alpha}$$

Lie dragging in F along the action of a diffeo on the DYNAMICAL FIELDS

Lie dragging of $\underline{\alpha}$ as a k-form on M

D

B

Counterex: $\mathcal{L}_{SF}(\varphi) = \sqrt{g}\, \frac{-1}{2} g^{ab} \partial_a \varphi \partial_b \varphi \, d^4x$

$\mathbb{d} g \equiv 0$ since $F = C^\infty(M) \ni \varphi$

g of
form on M

$\mathcal{G}$ gauge grp

$\mathcal{G}_1$ the id component of $\mathcal{G}$

$D = \mathcal{G}/\mathcal{G}_1$ discrete grp of components of $\mathcal{G}$.

Offshell

B.I. $\Rightarrow$ $\mathbb{i}_{\rho(\xi)} \underline{\Omega} = -\, d\, \underline{J}(\xi) + i_\xi \underline{E} + d i_\xi \underline{\Theta}$

where $\underline{J}(\xi) = \underset{\substack{\wr\wr \\ 0}}{\underline{C}_a} \xi^a + d \underline{j}(\xi)$

①

$+ d i_\xi \underline{\Theta}$

$\xi) = \underset{\approx 0}{\underline{C}_a} \xi^a + d \underline{j}(\xi)$

① $\int_\Sigma i_\xi \underline{E} \neq 0$ offshell and for $\xi \perp \Sigma$

↳ "timelike diffeos"

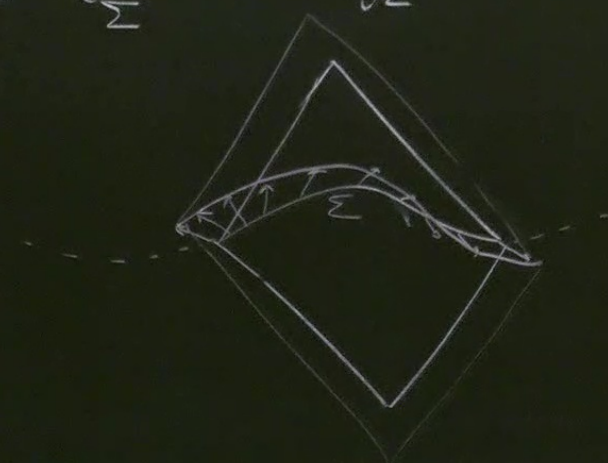
⇝ dynamics (eom)

② $\int_\Sigma d i_\xi \underline{\Theta} = \int_{\partial\Sigma} i_\xi \underline{\Theta} \neq 0$ if $\xi \perp \partial\Sigma$

& survives on shell.

$\Downarrow$

difficulties in defining notions of subsystems in B.I. theories.



$d i_\xi \underline{\Theta}$

$\underset{\approx 0}{\underline{C}_a} \xi^a + d\underline{j}(\xi)$

① $\int_\Sigma i_\xi \underline{E} \neq 0$ offshell and for $\xi \perp \Sigma$

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$\Downarrow \xi$

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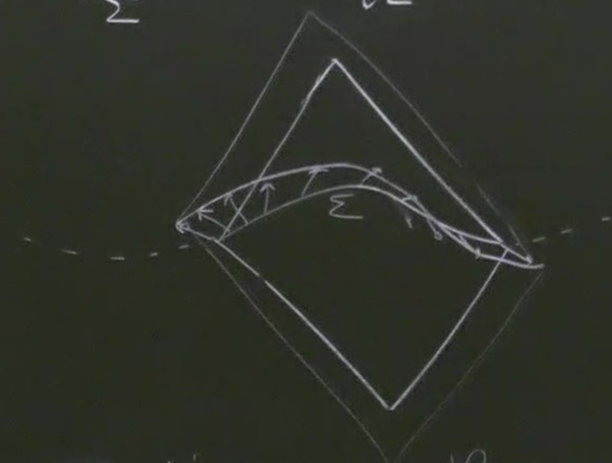


(Spacetime is pointless ⤳ hole argument)

Komar 2-current

$$\underset{\sim}{j}(\xi) = *_g \, d \, \underset{\sim}{g}(\xi)$$

$$g_{ab}\,\xi^a\,dx^b$$

$$\tilde{j}^{[ab]}(\xi) = \frac{1}{2?}\nabla^{[a}\xi^{b]}$$

In asymptotically flat sp.t.,
at spatial infinity we find (approx, asympt)
Killing vf

$$\xi^a \xrightarrow{\infty} \begin{cases} t^a & \sim \text{time transl} \\ \phi^a & \sim \text{rotations} \end{cases}$$

$$\int_{S^2_\infty} \underset{\sim}{j}(\phi) \sim \text{angular mom}$$

$$\int_{S^2_\infty} \underset{\sim}{j}(t) = \frac{1}{2}\,\text{ADM mass}$$

$$\int_{S^2_\infty} i_t\,\Theta \sim \frac{1}{2}\,\delta\left(\text{ADM mom}\right)$$

- on shell & $\partial\Sigma = \emptyset$

$$\xi_D \left(\mathcal{F}_{EL},\ \omega = \iota^{*}_{EL} \int_\Sigma \Omega \right)$$

$$\mathring{I}_{\rho(\xi)}\, \omega \approx 0 \longrightarrow$$ diffeos need to be mod-out to get a non deg sympl structure.

- $N2 \Rightarrow {}^{\dagger}D^I_\alpha E_I \equiv 0$ for α an index of the g.t. ξ^α and ${}^{\dagger}D$ is the adj of $\delta_\xi \varphi^I = D^I_\alpha \xi^\alpha$

For GR: $\delta_\xi g_{ab} = L_\xi g_{ab} = 2\nabla_{(a}\xi_{b)} \rightarrow {}^{\dagger}D = \nabla = \partial + \Gamma(g)$

Noether id is $0 \equiv \nabla_a E^{ab} \equiv \nabla_a \left(G^{ab} + \Lambda g^{ab} \right)$

) = angular mom

$= \frac{1}{2}$ ADM mass

$\sim \frac{1}{2} \delta$(ADM mom)

$$\boxed{\begin{aligned} {}^{\dagger}DE &= \partial_a E^a \\ &= \partial_a(\partial_b F^{ab}) \end{aligned}}$$

• on shell & $\partial\Sigma = \phi$

$$\hookrightarrow \left(\mathcal{F}_{EL},\ \omega = \imath_{EL}^{*} \int_{\Sigma} \Omega \right)$$

$\mathring{I}_{\rho(\xi)}\, \omega \approx 0 \longrightarrow$ diffeos need to be mod-out to get a non deg sympl structure

• N2 $\Rightarrow {}^{\dagger}D^{I}_{\alpha} E_{I} \equiv 0$ for α an index of the g.tr. ξ^{α} and ${}^{\dagger}D$ is the adj of $\delta_{\xi}\varphi^{I} = D^{I}_{\alpha}\xi^{\alpha}$

For GR: $\delta_{\xi} g_{ab} = L_{\xi} g_{ab} = 2\nabla_{(a}\xi_{b)} \to {}^{\dagger}D = \nabla = \partial + \Gamma(g)$

Noether id is $0 \equiv \nabla_a E^{ab} \equiv \nabla_a (G^{ab} + \Lambda g^{ab})$

$$\int_M E_I \, \delta_\xi \varphi^I = \int_M E^{ab} \, 2 D_{(a} \xi_{b)}$$

From COVARIANT to CANONICAL picture

$$\begin{tikzcd} (\mathcal{F}, \Omega_\Sigma) \arrow[d, squiggly, "\text{on shell}"'] \arrow[drr, squiggly, "\text{presympl red } /\ker(\Omega_\Sigma)"] & & \\ (\mathcal{F}_{EL}, \omega) \arrow[rr, phantom, "\simeq"] & & (P_{can}, \omega_{can}) \end{tikzcd}$$

off shell

initial conds $\Leftrightarrow$ sols eom

• scalar field:

$\mathcal{F} = C^\infty(M)$

$P_{can} = T^* C^\infty(\Sigma) \ni (\pi, \phi)$

$\omega_{can} = \int_\Sigma d\pi \wedge d\phi$

$\Omega_\Sigma = \int_\Sigma (\partial_t d\varphi) \wedge d\varphi \,, \quad \Sigma = \{t=0\} \subset \mathbb{R}^{1,3}$

NONICAL picture

$(\ , \Omega_\Sigma)$

off shell

presympl red $/\ker(\Omega_\Sigma)$

on shell

$(EL, \omega) \simeq (\mathcal{P}_{can}, \omega_{can})$

initial conds $\Leftrightarrow$ sols eom

$\Sigma = \{t=0\} \subset \mathbb{R}^{1,3}$

presympl. red.

$$\ker(\Omega_\Sigma) = \{ \mathbb{X} : \mathbb{i}_{\mathbb{X}} \Omega_\Sigma = 0 \}$$

$$\mathbb{X} = \int_M X(x) \frac{\delta}{\delta\varphi(x)}$$

$$0 \overset{!}{=} \mathbb{i}_{\mathbb{X}} \Omega_\Sigma = \int_\Sigma \partial_t X \, \mathbb{d}\varphi - X \, \partial_t \mathbb{d}\varphi$$

functionally indep.

$$\Longrightarrow \begin{cases} \partial_t X|_\Sigma = 0 \\ X|_\Sigma = 0 \end{cases}$$

Noether id is $0 \equiv \nabla_a E^{ab} \equiv \nabla_a\left(G^{ab} + \Lambda g^{ab}\right)$

red.

$) = \{ \mathbb{X} : i_{\mathbb{X}} \Omega_\Sigma = 0 \}$

$\frac{\delta}{\delta \varphi(x)}$

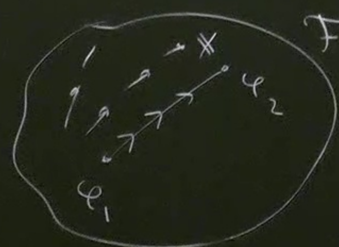
$\int_\Sigma \partial_t X \, \underbrace{d\varphi} - X \, \underbrace{\partial_t d\varphi}$

functionally indep.

$X|_\Sigma = 0$ (*)

$\dot{X}|_\Sigma = 0$

quotient out $\mathrm{Ker}(\Omega_\Sigma)$
means identifying
$\varphi_1 \sim \varphi_2$ if they can be connected by a flow of an $\mathbb{X} \in \mathrm{Ker}(\Omega_\Sigma)$



But by (*)

$\varphi_1 \sim \varphi_2$ iff $\begin{cases} \varphi_1 - \varphi_2 \big|_{t=0} = 0 \\ \dot{\varphi}_1 - \dot{\varphi}_2 \big|_{t=0} = 0 \end{cases}$

$\Rightarrow [\varphi] = (\phi, \pi) = \left(\varphi|_\Sigma, \dot{\varphi}|_\Sigma\right)$