Title: Lecture - Quantum Information, PHYS 635

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Subject: Quantum Information

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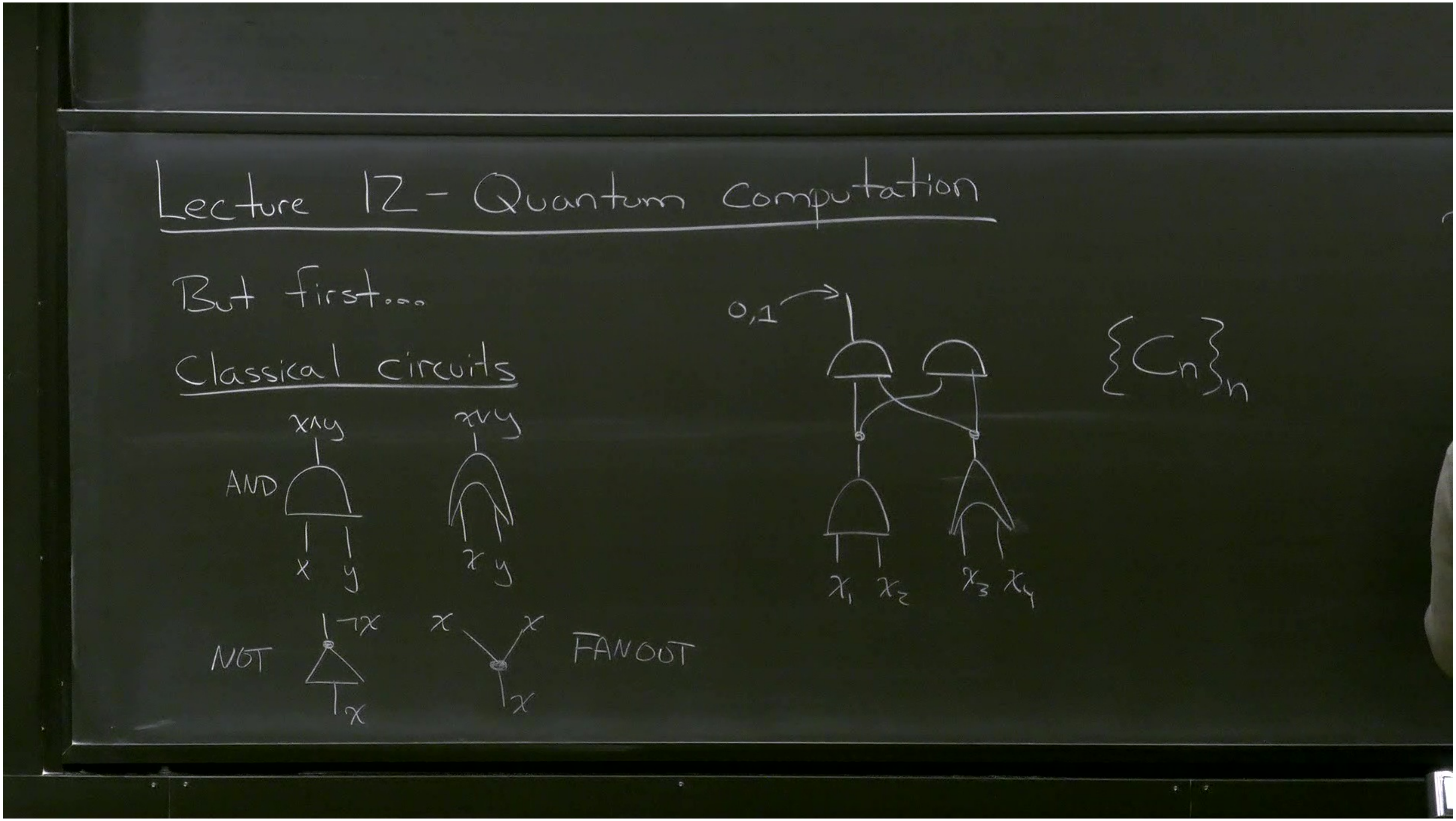
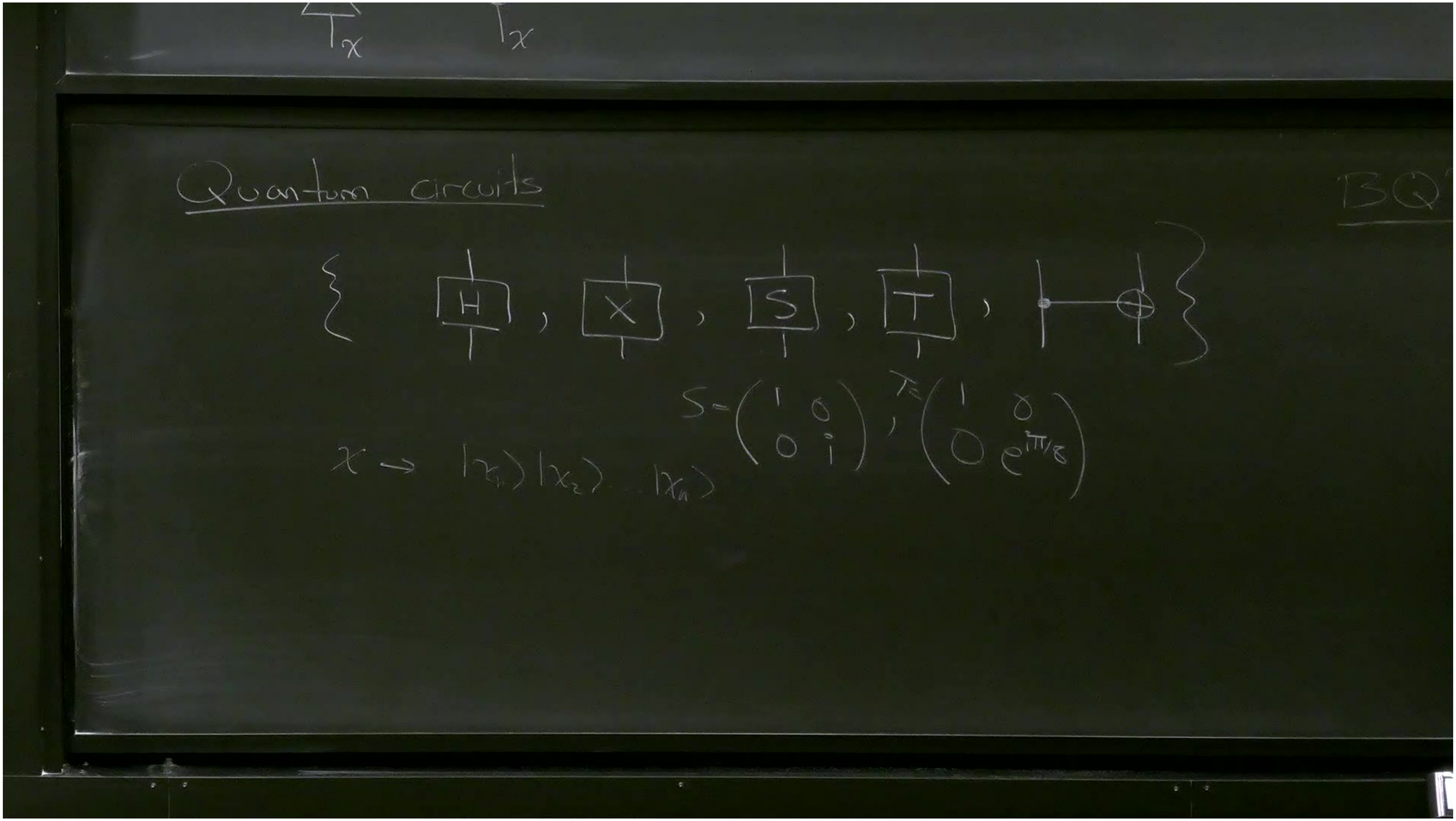
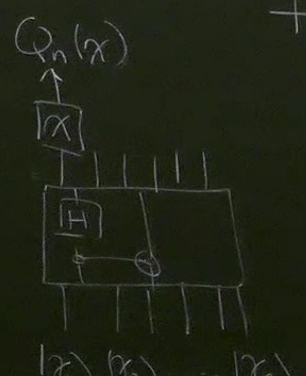

Lecture 12 - Quantum computation
But first...
Classical circuits
x∧y
AND
x y
x∨y
x y
NOT
¬x
x
x
x
x
FANOUT
0,1
x1 x2 x3 x4
{Cn}n

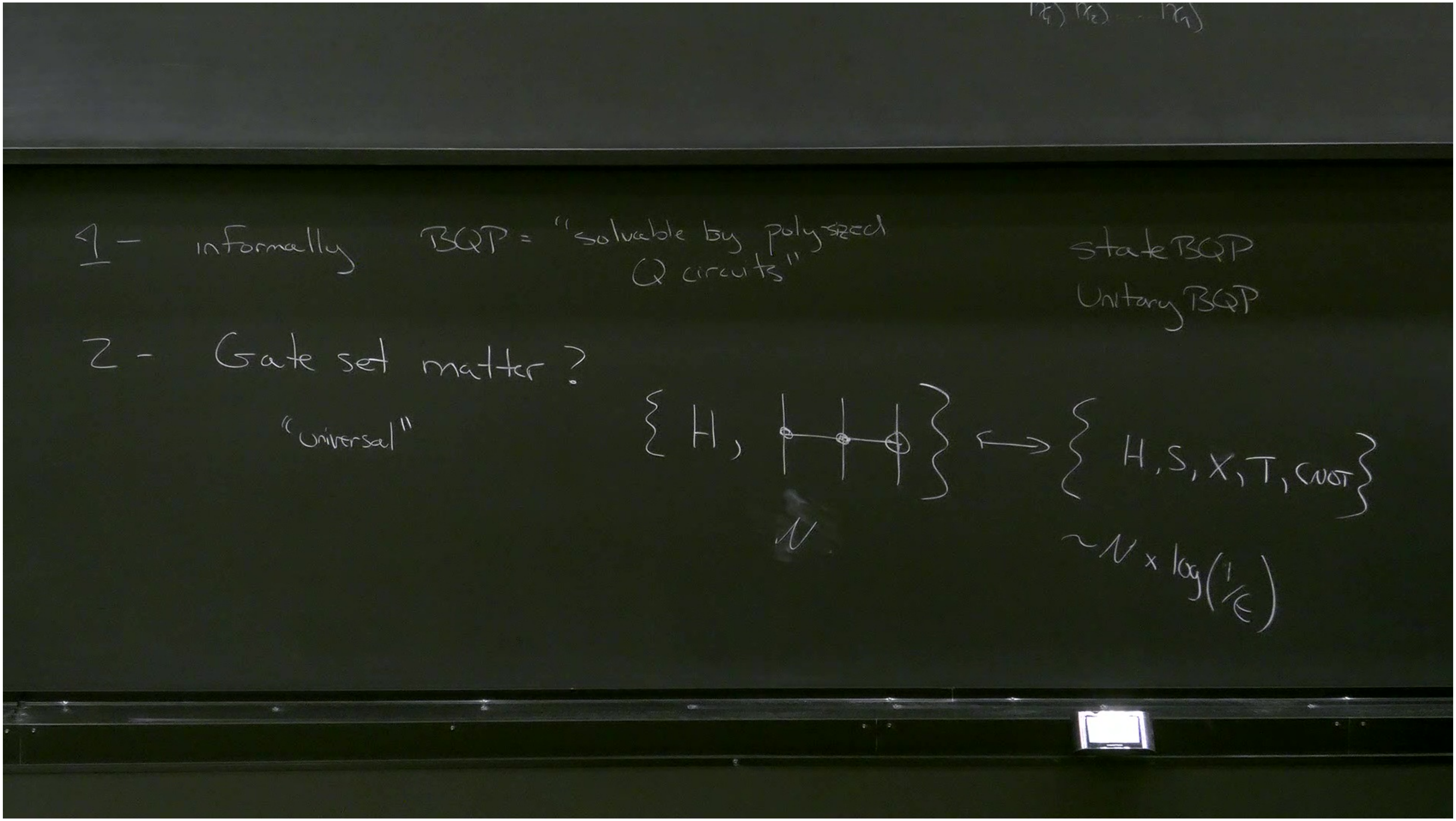

Quantum circuits
H
X
S
T
S = $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$, T = $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/8} \end{pmatrix}$
$x \to |x_1\rangle|x_2\rangle \dots |x_n\rangle$

BQP | A language $L \in BQP$ if $\exists$ a poly-time TM taking $n \in \mathbb{N}$ as input, outputting Q_n s.t.



1) $\forall x \in L$ of length n, $Pr[Q_n(x)=1] \geq 2/3$

2) $\forall x \notin L$ $Pr[Q_n(x)=0] \geq 2/3$



$|x_1\rangle |x_2\rangle \ldots |x_n\rangle$
1 - informally BQP = "solvable by poly-sized Q circuits"
state BQP
Unitary BQP
2 - Gate set matter?
"universal"
{ H, } ⟷ { H, S, X, T, CNOT }
N
$\sim N \times \log(1/\epsilon)$

Deutsch's problem "algorithmic interference"

"oracle"

$$O_f |z\rangle_O |x\rangle_X = |z+f(x)\rangle_O |x\rangle_X$$

$$O_f |z\rangle \left(|x_1\rangle + |x_2\rangle \right) = |z+f(x_1)\rangle |x_1\rangle + |z+f(x_2)\rangle |x_2\rangle$$

Deutsch's problem

Input: oracle access to unknow Boolean function f of 1 bit.

Output: $f(0) \oplus f(1)$

$$O_f(x) = f(x)$$

$f(0) \longrightarrow f(0) \oplus f(1)$

$f(1) \longrightarrow f(0) \oplus f(1)$

Classically: 2 calls!

Quantumly

$$\boxed{O_f |-\rangle |x\rangle = (-1)^{f(x)} |-\rangle |x\rangle}$$

$$O_f |0\rangle |x\rangle - O_f |1\rangle |x\rangle = |f(x)\rangle |x\rangle - |1 \oplus f(x)\rangle |x\rangle$$

$$= (-1)^{f(x)} \; |-\rangle \; |x\rangle$$

$$O_f |-\rangle_Q |+\rangle_x = \dots$$

$$\mathcal{O}_f |-\rangle|+\rangle = \mathcal{O}_f|-\rangle|0\rangle + \mathcal{O}_f|-\rangle|1\rangle$$

$$= (-1)^{f(0)}|-\rangle|0\rangle + (-1)^{f(1)}|-\rangle|1\rangle$$

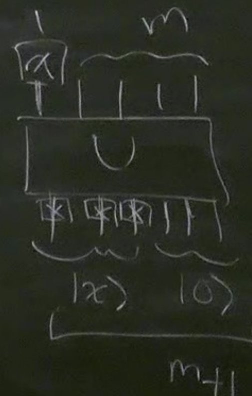
$$= |-\rangle\left(|0\rangle + (-1)^{f(1)\oplus f(0)} |1\rangle \right)$$

$$= \begin{cases} |-\rangle|+\rangle & f(0)\oplus f(1)=0 \\ |-\rangle|-\rangle & f(0)\oplus f(1)= \end{cases}$$

$BQP \subseteq PSPACE$

$PSPACE \subseteq EXP$

use $\{H,$  $\}$



$$U = G_T G_{T-1} \cdots G_2 G_1$$

$$p(1) = \sum_x \left| \langle 1 | \langle x |^{\otimes m} U |0\rangle |0\rangle \right|^2$$

1) Compute $|\langle 1|\langle x| U |00\rangle|^2$ for $x=0$.

2) Store in memory $\underline{\underline{S}}$

3) Compute .... for $x=1$

$$p(1) = \sum_x |\langle 1 | \langle x |^{\otimes m} U |0\rangle|0\rangle|^2$$

$$\langle 1x|U|00\rangle = \langle 1x| G_T G_{T-1} \dots G_2 G_1 |00\rangle$$

$$= \langle 1x| G_T \Big(\sum_{i_{T-1}} |i_{T-1}\rangle\langle i_{T-1}|\Big) G_{T-1} \Big(\sum_{i_{T-2}} |i_{T-2}\rangle\langle i_{T-2}|\Big) G_{T-2} \dots \quad G_1 |00\rangle$$

$$= \sum_{\{i_j\}_{j=1}^{T-1}} \langle 1x| G_T |i_{T-1}\rangle\langle i_{T-1}| G_{T-1} |i_{T-2}\rangle\langle i_{T-2}| \dots G_1 |00\rangle$$