Title: Lecture - Quantum Information, PHYS 635

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Subject: Quantum Information

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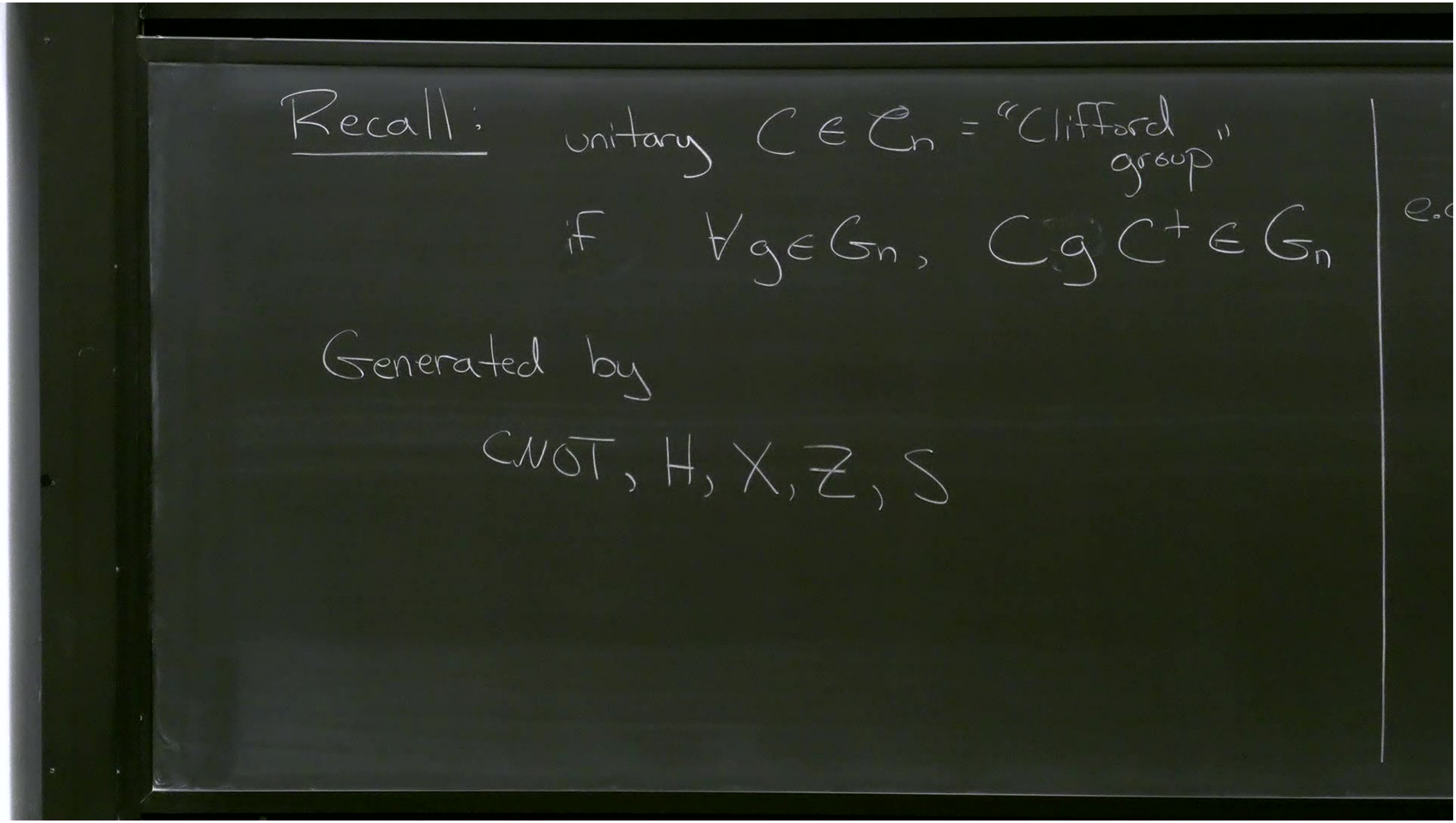
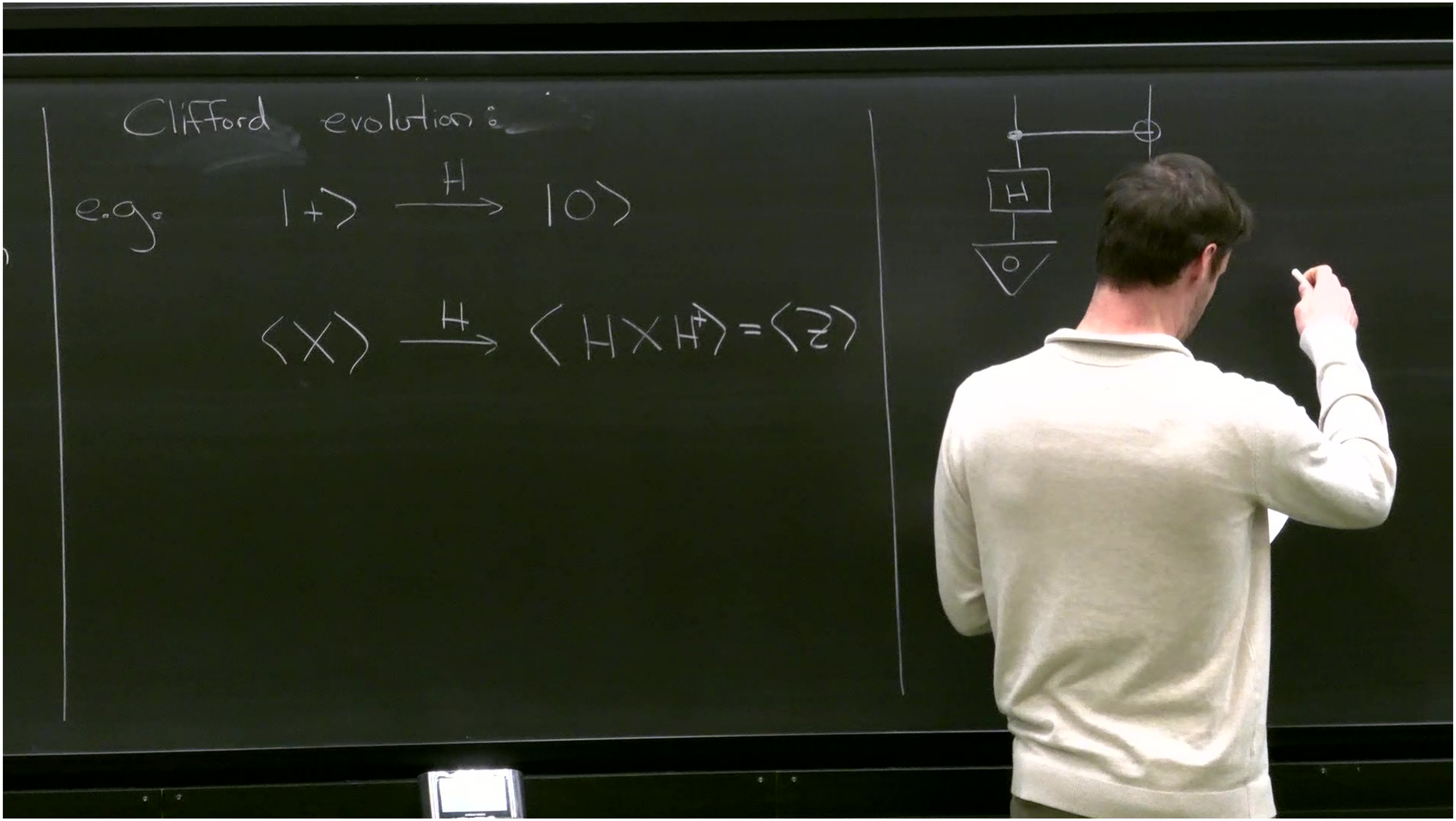
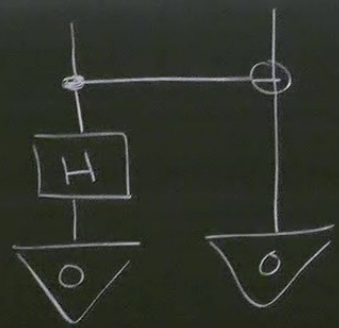

Recall: unitary $C \in \mathcal{C}_n$ = "Clifford group"
if $\forall g \in G_n$, $C g C^\dagger \in G_n$
Generated by
CNOT, H, X, Z, S


Clifford evolution:
e.g. $|+\rangle \xrightarrow{H} |0\rangle$
$\langle X \rangle \xrightarrow{H} \langle H X H^{\dagger} \rangle = \langle Z \rangle$
H
0

$\langle Z \rangle$



$$S_0 = \langle Z\mathbb{I}, \mathbb{I}Z \rangle$$

$$S_1 = \langle X\mathbb{I}, \mathbb{I}Z \rangle$$

$$S = \langle XX, ZZ \rangle$$

$\mathrm{CNOT}_{1\to 2}$

X_1	$X_1 X_2$
X_2	X_2
Z_1	Z_1
Z_2	$Z_1 Z_2$

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Stabilizer codes

$$S = \langle g_1, \dots, g_{n-k} \rangle$$

$\hookrightarrow$ 2^k dim subspace $\mathcal{H}_P$

$$V_{L\to P}|\psi\rangle_L = |\psi\rangle_P$$

$-\mathbb{I} \notin S$

$g_1 g_2 = g_2 g_1$

$S = \langle$

$-\mathbb{I} \notin S$

$g_1 g_2 = g_2 g_1$

$\text{span}\{|000\rangle, |111\rangle\}$

$$\mathbb{Z}^3 / \mathbb{Z}^2 = \mathbb{Z}$$

$$S = \langle Z_1 Z_2, Z_2 Z_3 \rangle$$

QECC for

$S = \langle g_1, \ldots$

1) $E = a_i^\dagger a_j$ anticom

$\langle \psi | E | \varphi \rangle$

QECC for stabilizer codes?

$$S = \langle g_1, \ldots, g_{n-k} \rangle \quad , \quad A = \{ a_i \}$$

1) $E = a_i^\dagger a_j$ anticommutes with g_* , $|\psi\rangle, |\varphi\rangle \in$ Codespace

$$\langle \psi | E | \varphi \rangle = \langle \psi | E g_* | \varphi \rangle = -\langle \psi | g_* E | \varphi \rangle = -\langle \psi | E | \varphi \rangle$$

$$\left(\langle \psi | a_i^\dagger \right) \left(a_j | \varphi \rangle \right) = 0$$

2) $E = a_i^\dagger a_j$ commutes with g_i,

a) $a_i^\dagger a_j \in S$ $\quad \langle \psi | a_i^\dagger a_j | \varphi \rangle = \langle \psi | \varphi \rangle$

b) $a_i^\dagger a_j \notin S$ bad ×

$$Z(S) = \{ g \in G_n : gh = hg \ \forall h \in S \}$$

$$N(S) = \{ g \in G_n : ghg^{-1} \in S \ \forall h \in S \}$$

for S stab. $Z(S) = N(S)$

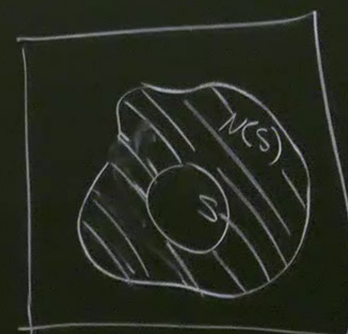
Theorem Given S, describing $C(S)$,
Then $A=\{a_i\}$ are correctable whenever

$$\forall\, a_i, a_j \in A, \quad a_i^\dagger a_j \notin N(S)\setminus S$$

Proof

① $a_i^\dagger a_j \in S$

② $a_i^\dagger a_j \in G_n \setminus N(S)$



① $\langle \psi | a_i^\dagger a_i | \varphi \rangle = \langle \psi | \varphi \rangle.$

② $\langle \psi | a_i^\dagger a_j g_* | \varphi \rangle = \ldots = -\langle \psi | a_i^\dagger a_j | \varphi \rangle$

$\langle \psi | a_i^\dagger a_j | \varphi \rangle = 0$

when i, j is s.t. ① pick $\alpha_{ij} = 1$

i, j is s.t. ② pick $\alpha_{ij} = 0$

Theorem Given S, describing $C(S)$,

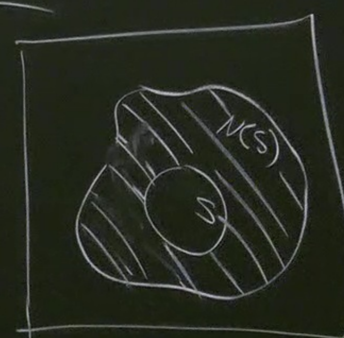
Then $A = \{a_i\}$ are correctable whenever

$$\forall a_i, a_j \in A, \quad \underline{a_i^{\dagger} a_j \notin N(S) \setminus S}$$

Proof

① $a_i^{\dagger} a_j \in S$

② $a_i^{\dagger} a_j \in G_n \setminus N(S)$



$$S = \langle Z_1 Z_2 , Z_2 Z_3 \rangle$$

Sol!

$$A = \{ X_1 , X_2 , X_3 \} \quad X_1 X_2 \qquad X_1 X_2 X_3$$

$$a_i^\dagger a_j = \{ \mathbb{I} , X_1 X_2 , X_2 X_3 , X_1 X_3 \}$$

$$g_1 = X Z Z X \mathbb{I}$$
$$g_2 = \mathbb{I} X Z Z X$$
$$g_3 = X \mathbb{I} X Z Z$$
$$g_4 = Z X \mathbb{I} X Z$$

$$a_i^\dagger a_j = \{ X X \mathbb{I}\mathbb{I}\mathbb{I}, \dots \}$$

$$h = (XZ)(X)(X)(XZ)(\mathbb{I}) \quad X$$

$$g_3 g_4 = -h$$

"Five qubit code"

$|0\rangle \longrightarrow (\dots$

$|1\rangle \longrightarrow$

"Logical operator"

$|\psi\rangle_L \to |\psi\rangle_p$

$X|0\rangle_L \to |1\rangle_L$

$|\psi\rangle_p \in C(S) \to |\phi\rangle_p \in C(S)$

logical $\in N(S) \setminus S$

$S = \langle Z_1 Z_2, Z_2 Z_3 \rangle$

$\bar{X}|000\rangle_p \to \bar{X}|111\rangle_p$

$\bar{X} = XXX$