Title: Lecture - Quantum Information, PHYS 635

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Subject: Quantum Information

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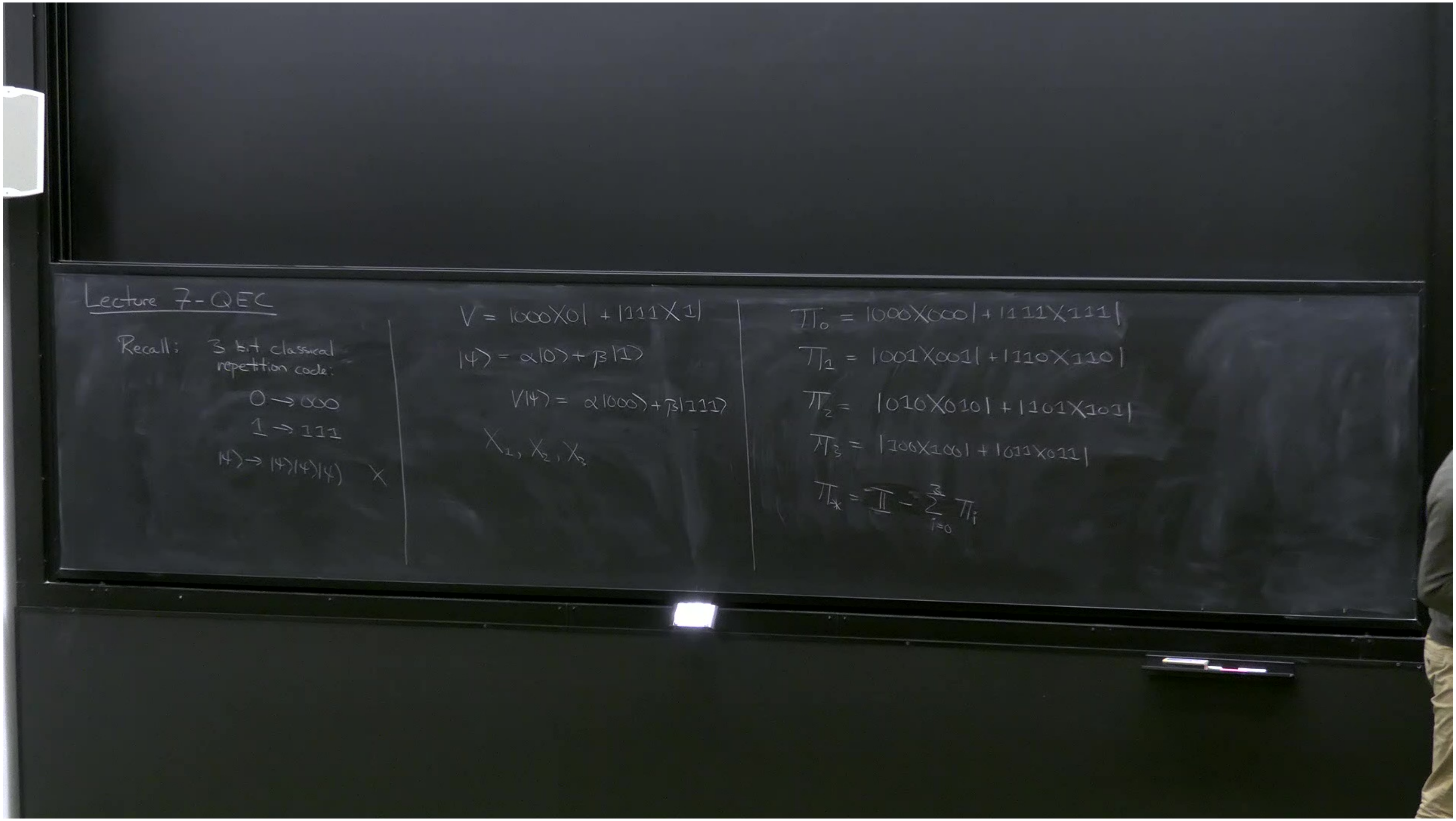
Lecture 7 - QEC
Recall: 3 bit classical repetition code:
$0 \to 000$
$1 \to 111$
$|\psi\rangle \to |\psi\rangle|\psi\rangle|\psi\rangle$ ✗
$V = |000\rangle\langle 0| + |111\rangle\langle 1|$
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$V|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$
X_1, X_2, X_3
$\Pi_0 = |000\rangle\langle 000| + |111\rangle\langle 111|$
$\Pi_1 = |001\rangle\langle 001| + |110\rangle\langle 110|$
$\Pi_2 = |010\rangle\langle 010| + |101\rangle\langle 101|$
$\Pi_3 = |100\rangle\langle 100| + |011\rangle\langle 011|$
$\Pi_* = \mathbb{I} - \sum_{i=0}^{3} \Pi_i$

Lecture 7 - QEC

Recall: 3 bit classical repetition code:

$0 \to 000$

$1 \to 111$

$|\psi\rangle \to |\psi\rangle|\psi\rangle|\psi\rangle$ ×

$V = |000\rangle\langle 0| + |111\rangle\langle 1|$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$V|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$

Lecture 7 - QEC

Recall: 3 bit classical repetition code:

$$0 \to 000$$

$$1 \to 111$$

$$|\psi\rangle \to |\psi\rangle|\psi\rangle|\psi\rangle \quad ✗$$

$$V = |000\rangle\langle 0| + |111\rangle\langle 1|$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$V|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$$

$$X_1, X_2, X_3$$

$$\Pi_0 = |000\rangle\langle 000| + |111\rangle\langle 111|$$

$$\Pi_1 = |001\rangle\langle 001| + |110\rangle\langle 110|$$

$$\Pi_2 = |010\rangle\langle 010| + |101\rangle\langle 101|$$

$$\Pi_3 = |100\rangle\langle 100| + |011\rangle\langle 011|$$

$$\Pi_* = \mathbb{I} - \sum_{i=0}^{3} \Pi_i$$

$$|\psi\rangle_p = \alpha|000\rangle + \beta|111\rangle$$

$$P_0 = \langle\psi|\Pi_0|\psi\rangle = 1$$

$$X_1|\psi\rangle$$

$$P_1 = \langle\psi|X_1\Pi_1X_1|\psi\rangle$$

$$= \langle\psi|X_1X_1\Pi_0X_1X_1|\psi\rangle$$

$$= 1$$

$$Z_i \; ?$$

$$|\psi\rangle = \alpha|000\rangle + \beta|111\rangle$$

$$\rightarrow \alpha|000\rangle - \beta|111\rangle$$

$$\hat{W} = |{+}{+}{+}\rangle\langle{+}| + |{-}{-}{-}\rangle\langle{-}|$$

$$|0\rangle \longrightarrow |000\rangle = \big(|+\rangle + |-\rangle\big)\big(|+\rangle + |-\rangle\big)\big(|+\rangle + |-\rangle\big)$$

$$\longrightarrow \big(|{+}{+}{+}\rangle + |{-}{-}{-}\rangle\big)^{\otimes 3}$$

$$|0\rangle \xrightarrow{H^{\otimes 9}} \big(|000\rangle + |111\rangle\big)^{\otimes 3} = |0\rangle_P$$

$$|1\rangle \longrightarrow \big(|000\rangle - |111\rangle\big)^{\otimes 3} = |1\rangle_P$$

$$|P\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$|M\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle - |111\rangle\right)$$

$$\Pi_0 = |PPP\rangle\langle PPP| + |MMM\rangle\langle MMM|$$

$$\Pi_1 = |PPM\rangle\langle PPM| + |MMP\rangle\langle MMP|$$

$$\Pi_2 \quad \vdots$$

$$\Pi_3$$

$$\Pi_*$$

$$|P\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

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$$\Pi_3 = |PPM\rangle\langle PPM| + |MMP\rangle\langle MMP|$$

$$\Pi_2 \quad \vdots$$

$$\Pi_1 = |MPP\rangle\langle MPP| + |PPM\rangle\langle PPM|$$

$$\Pi_*$$

$$Z_1|\psi\rangle_P = Z_1(\alpha|0\rangle_P + \beta|1\rangle_P)$$

$$= \alpha(Z_1|P\rangle)|P\rangle|P\rangle + \beta(Z_1|M\rangle)|M\rangle|M\rangle$$

$Z_2|P\rangle = |M\rangle$

$Z_2|M\rangle = |P\rangle$

$$\alpha|M\rangle|P\rangle|P\rangle + \beta|P\rangle|M\rangle|M\rangle$$

$Z_2(|000\rangle + |111\rangle)$

$(|000\rangle - |111\rangle)$

$$|0\rangle \longrightarrow |PPP\rangle_P \qquad |P\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$|1\rangle \longrightarrow |MMM\rangle_P \qquad |M\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle - |111\rangle\right)$$

$$Z_1 Z_2 |\psi\rangle_P = \alpha\left(Z_1 Z_2 |P\rangle\right)|PP\rangle + \beta\left(Z_1 Z_2 |M\rangle\right)|MM\rangle$$

$$Z_1\left(Z_2 |P\rangle\right) = Z_1 |M\rangle = |P\rangle$$

$$|P\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$$

$$|M\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle - |111\rangle\right)$$

$$\Pi_0 = |PPP\rangle\langle PPP| + |MMM\rangle\langle MMM|$$

$$\Pi_3 = |PPM\rangle\langle PPM| + |MMP\rangle\langle MMP|$$

$$\Pi_2 = |PMP\rangle\langle PMP| + |MPM\rangle\langle MPM|$$

$$\Pi_1 = |MPP\rangle\langle MPP| + |PPM\rangle\langle PPM|$$

$$\Pi_*$$

$$\alpha|PPP\rangle + \beta|MMM\rangle$$
$$\downarrow$$
$$\alpha|MMP\rangle + \beta|PPM\rangle$$
$$\downarrow$$
$$\alpha|MMM\rangle + \beta|PPP\rangle$$

Def | A QECC (V, A)

$\exists$ isometry, $V : \mathcal{H}_L \to \mathcal{H}_P$

logical

physical

A set of linear maps (errors) $A = \{a\}$

s.t. $\exists$ $D : \mathcal{H}_P \to \mathcal{H}_L$

$$D\left(a V_{L\to P}\, \rho_L\, V^{\dagger}_{L\to P}\, a^{\dagger}\right) = c\, \rho_L$$

$$\| \mathcal{A}_i - \mathbb{I} \|_\diamond < \epsilon$$

$$\| \mathcal{A}^i_x - \mathbb{I} \|_\diamond = \max_{|\Psi\rangle_{RX}} \| \mathcal{A}^i_x \left(|\Psi\rangle\langle\Psi|_{XR} \right) - |\Psi\rangle\langle\Psi|_{XR} \|_1$$

Thm 1| Suppose a code corrects X, Z on a set of t qubits. Then it corrects arbitrary errors on those t qubits.

Thm 2| Suppose we have $\mathcal{A}_p = \bigotimes_{i=1}^{n} \mathcal{A}^i$

$$\| \mathcal{A}^i - \mathbb{I} \| \leq \epsilon$$

Have a code corrects errors on t qubits.

$\to \| \mathcal{D} \circ \mathcal{A} \circ \mathcal{E} - \mathbb{I} \|_{\diamond}$

code corrects X, Z on
t qubits. Then it corrects
errors on those t qubits.

e have $\mathcal{A}_p = \bigotimes_{i=1}^{n} \mathcal{A}^i$

$\|\mathcal{A}^i - \mathbb{I}\| \leq \epsilon$

corrects errors on t qubits $\Rightarrow \|\mathcal{D} \circ \mathcal{A} \circ \mathcal{E} - \mathbb{I}\|_\diamond \leq 2\binom{n}{t}(e\epsilon)^{t+1}$

Z on

$t = 2$

it corrects

$Y = XZ$

$\mathcal{E}(\cdot) = V(\cdot)V^\dagger$

qubits.

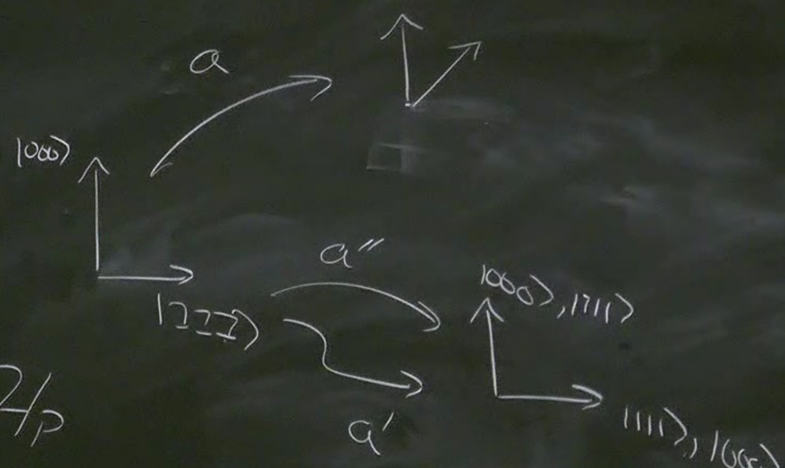
$\bigotimes_i \mathcal{A}^i$

$$\|\mathcal{D}\circ\mathcal{A}\circ\mathcal{E} - \mathbb{I}\|_\diamond \leq 2\binom{n}{t}(e\epsilon)^{t+1}$$

$$\lesssim \epsilon^{t+1}$$

n t qubits.

"Knill-LaFlamme" / QEC conditions



$\mathbb{Z}/p$

$$\mathrm{Im}\, V = \mathrm{span}\{|000\rangle, |111\rangle\}$$