Title: Lecture - Quantum Information, PHYS 635

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Subject: Quantum Information

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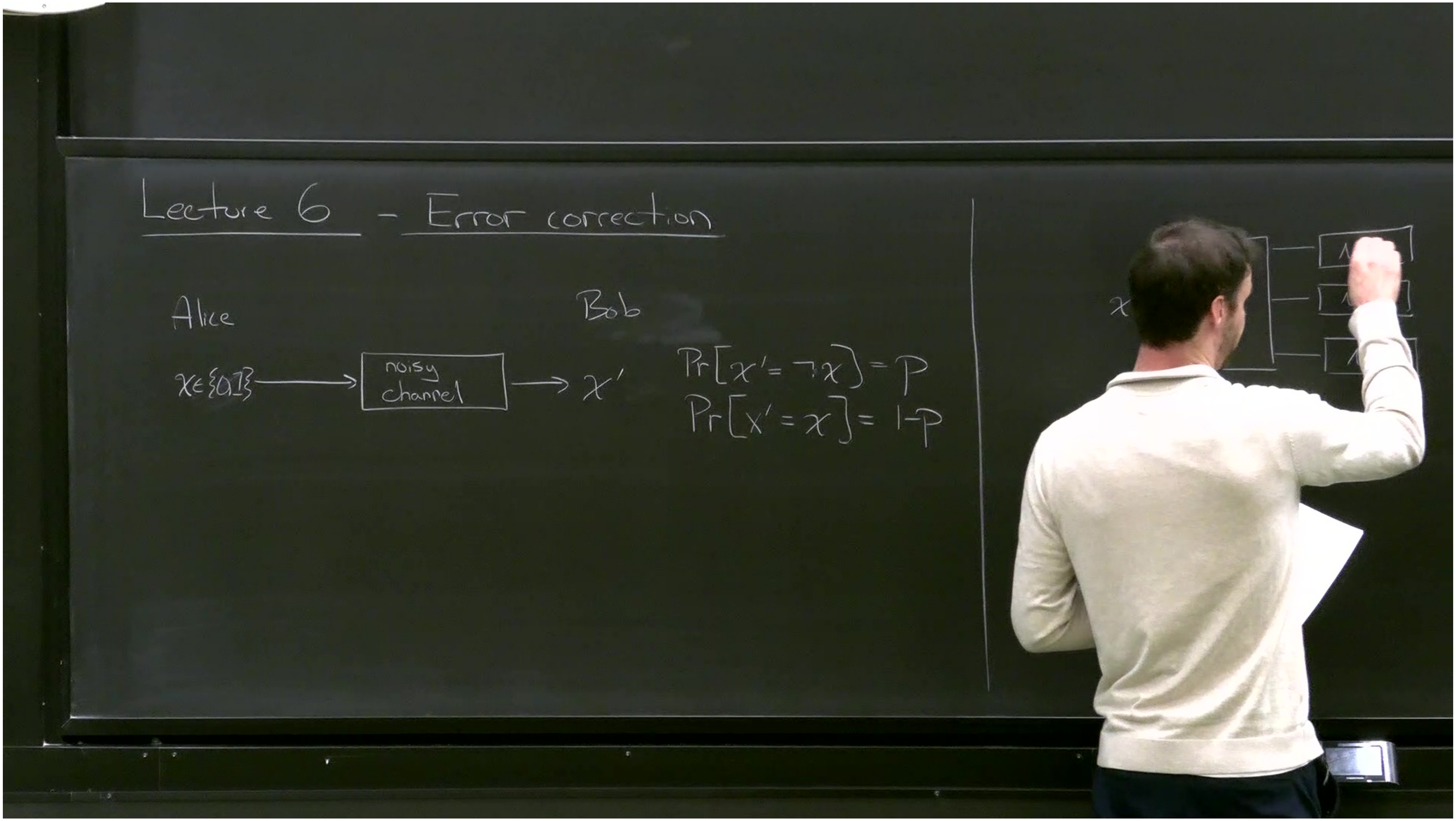

Lecture 6 - Error correction
Alice
Bob
x∈{0,1}
noisy channel
x'
Pr[x' = ¬x] = p
Pr[x' = x] = 1-p
x

$x \to xxx$

$0 \to 000 \xrightarrow{NC} 001 \xrightarrow{D} 0$

$0 \to 000 \longrightarrow 011 \to 1$

$$P_{fail} \simeq 3p^2(1-p) + p^3 \lesssim p$$

Def A classical ECC is an encoding map

$$E : L \to P$$

$L = \{0,1\}^k$

$P = \{0,1\}^n$

Along with set of errors $\{e_i\}$ s.t.

$$\exists D \; , \; D(e_i(E(x))) = x$$

$L =$ "logical space"

$P =$ "physical space" $y \in \mathrm{Im}\, E$, y is a "codeword"

$L = \{0,1\}^1$

$P = \{0,1\}^3$

$e_1(xxx) = (\neg x)x(\,.\,x)$

$e_2(\quad)$

$e_3(\quad)$

$001 \rightarrow 000$

$011 \rightarrow 111$

Linear codes

$L, P \qquad L = \mathbb{Z}_2^k$

$\qquad\qquad P = \mathbb{Z}_2^n$

E map $\rightarrow$ matrix G

$Gx = y$

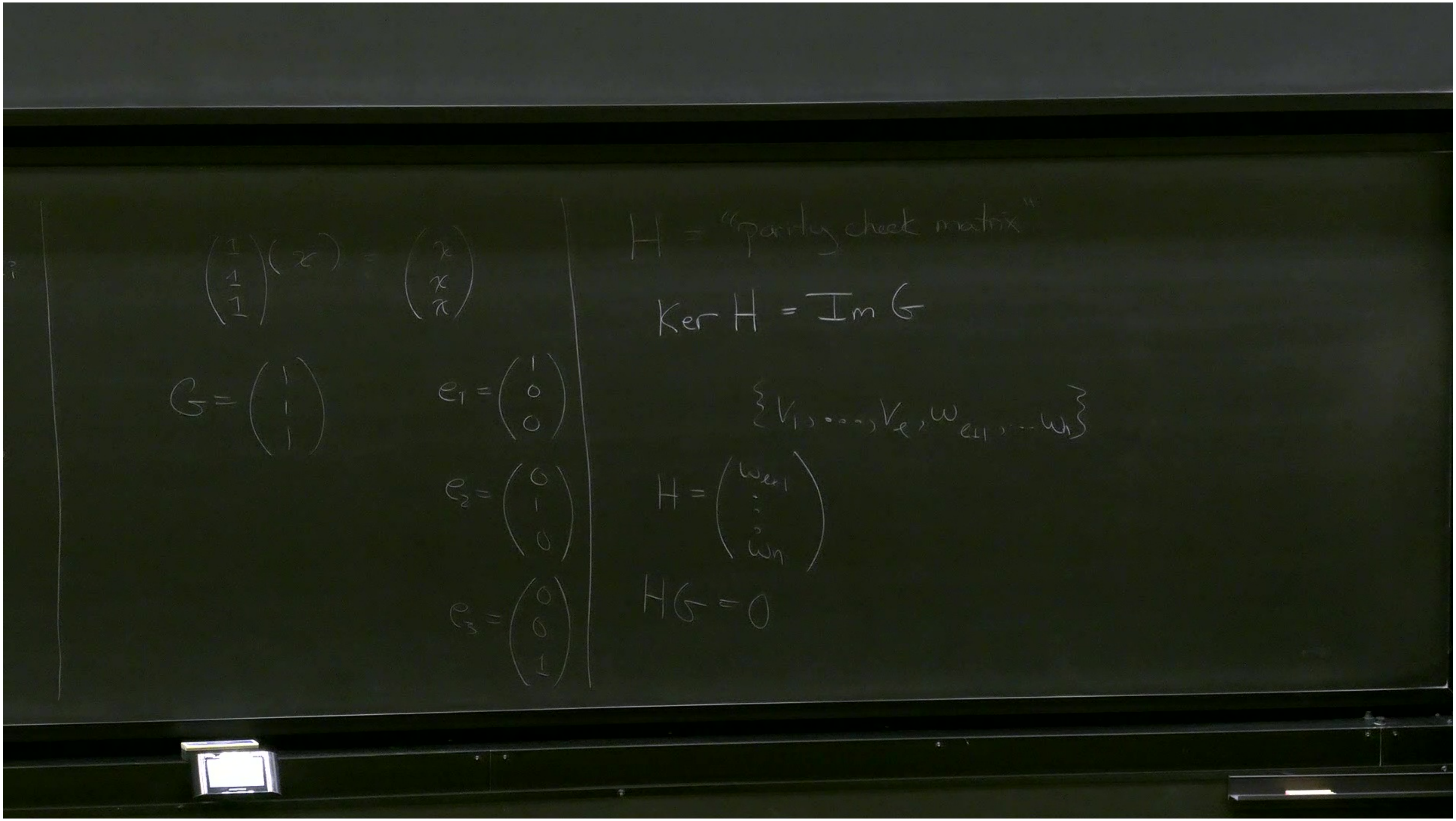
$x \in L \qquad y \in P$

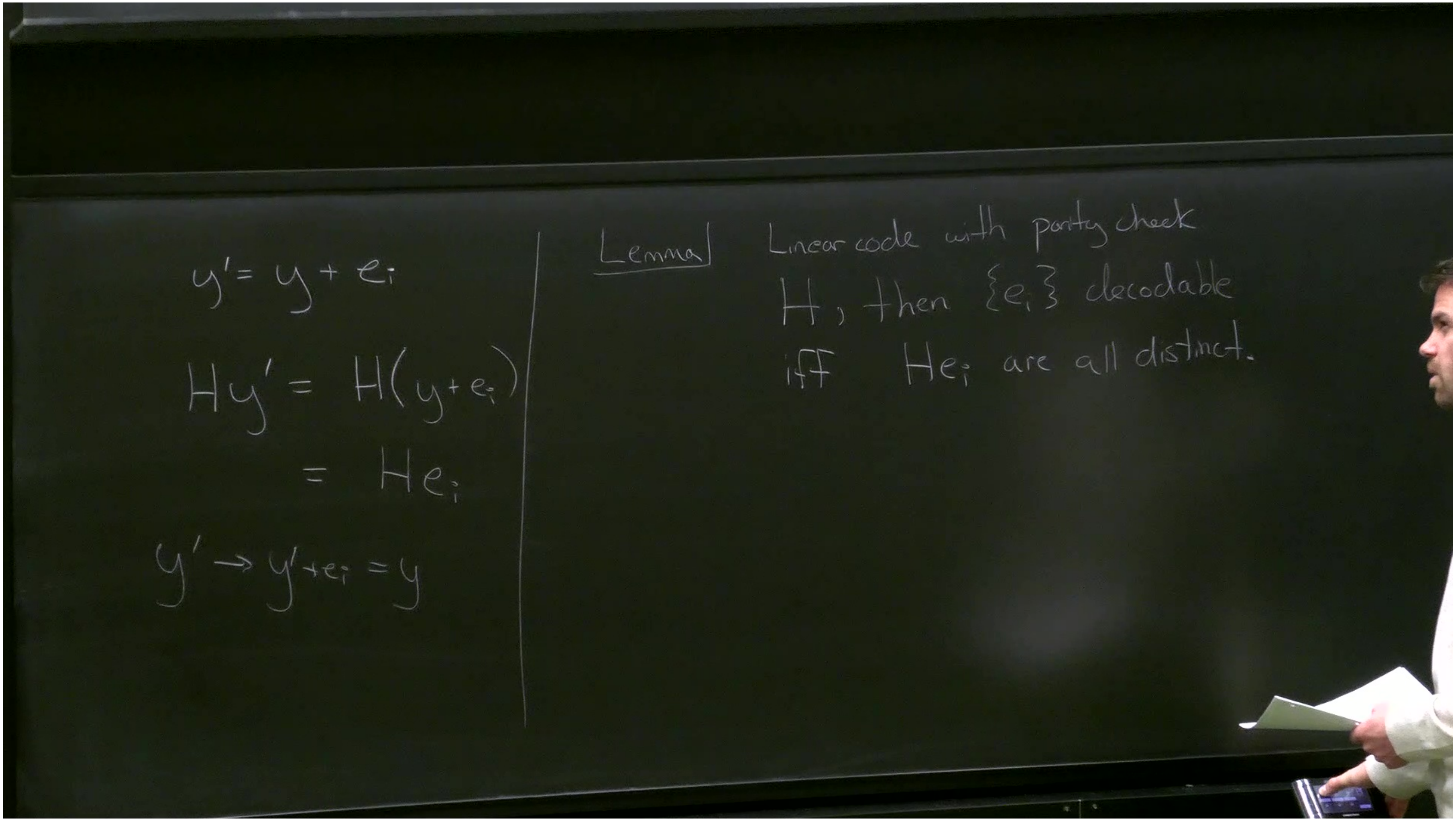
$e_i(y) = y + e_i$

Rep code.

what is G?

$e_i = ?$


$\begin{pmatrix}1\\1\\1\end{pmatrix}(x) = \begin{pmatrix}x\\x\\x\end{pmatrix}$
$G = \begin{pmatrix} | \\ | \\ | \end{pmatrix}$
$e_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}$
$e_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}$
$e_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$
H = "parity check matrix"
$\mathrm{Ker}\, H = \mathrm{Im}\, G$
$\{v_1, \ldots, v_\ell, w_{\ell+1}, \ldots w_n\}$
$H = \begin{pmatrix} w_{\ell+1} \\ \vdots \\ w_n \end{pmatrix}$
$HG = 0$



y' = y + e_i
Hy' = H(y + e_i)
= He_i
y' → y' + e_i = y
Lemma
Linear code with parity check H, then {e_i} decodable iff He_i are all distinct.

Proof suppose He_i are distinct

$$y' + e_i = y \checkmark$$

suppose $He_i = He_j$ for some $i \neq j$

$$0 = H(e_i + e_j) \longrightarrow e_i + e_j \in \mathrm{Im}\, G$$

$$y + (e_i + e_j) = \tilde{y}$$

$$z = y + e_i = \tilde{y} + e_j = z \Longrightarrow \text{can't decode!}$$

$$G = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad HG = 0$$

$$H = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$He_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$He_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$He_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Error-suppression

r se

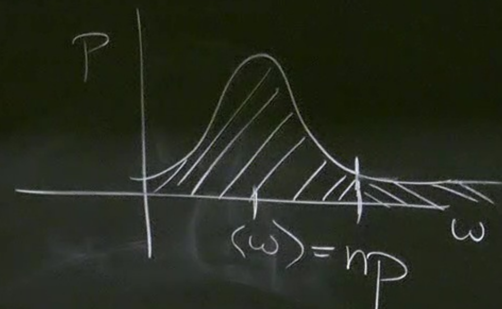
ω_n

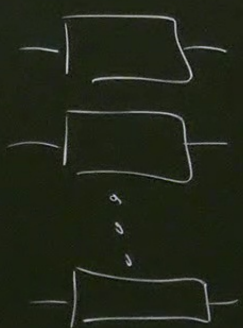
$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ | $HG = 0$

Error-suppression

1) Error set $\{e_i\}$

2) I.I.D. noise : for each bit, flip with prob p





$$P_\omega = \binom{n}{\omega} p^\omega (1-p)^{n-\omega}$$

Thm (Hoeffdings inequality)

- X_i independent RV, $0 \leq X_i \leq 1$
- $S_n = \sum_{i=1}^{n} X_i$

$$\mathrm{Prob}\left[S_n - \langle S_n \rangle \geq t \right] \leq \exp\left(-\frac{2t^2}{n} \right)$$

$$X_i = \begin{cases} 0 & \text{if } x_i \text{ is not flipped} \\ 1 & \text{if } x_i \text{ is flipped} \end{cases}$$

$$0 \le \gamma \le 1$$

$$\boxed{w_0 = \gamma n}$$

$$S_n = w$$

$$\langle S_n \rangle = np = \langle w \rangle$$

$$\Pr[w - \cancel{np} \geq w_0 - \cancel{np}] \leq \exp\left(-2n(\gamma - p)^2\right)$$