Title: Lecture - Quantum Information, PHYS 635

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Collection/Series: Quantum Information (Elective), PHYS 635, February 24 - March 28, 2025

Subject: Quantum Information

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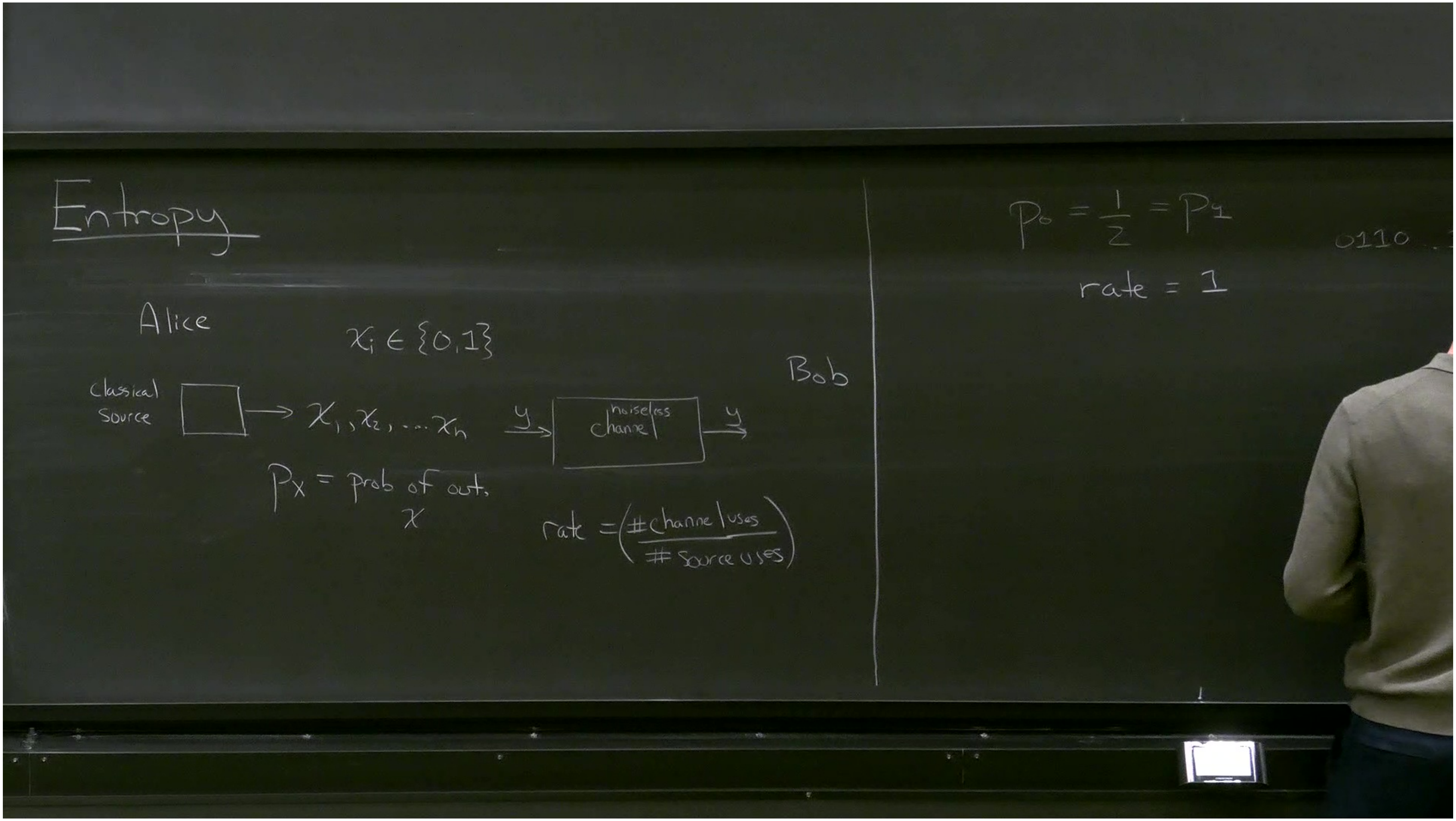

Entropy
Alice
xi ∈ {0,1}
Bob
Classical source
x1, x2, ... xn
y
noiseless channel
y
Px = prob of out. x
rate = (# channel uses / # source uses)
P0 = 1/2 = P1
0110
rate = 1

$p_0 = \frac{1}{2} = p_1$

$x = 0110 \ldots 10 \rightarrow \boxed{\text{channel}} \rightarrow x$

rate $= 1$

$p_0 = 3/4\ ,\ p_1 = 1/4$

$$\langle \text{channel uses} \rangle = \sum_{x_1 x_2} p_{x_1 x_2}\, m_{x_1 x_2}$$

$$= \frac{3}{4}\frac{3}{4} \times 1 + \frac{1}{4}\frac{3}{4} \times 3 + \frac{3}{4} \times \frac{1}{4} \times 3 + \overset{p_{11}}{\frac{1}{4}\frac{1}{4}} \times 3$$

$$= \frac{17}{9} < 2$$

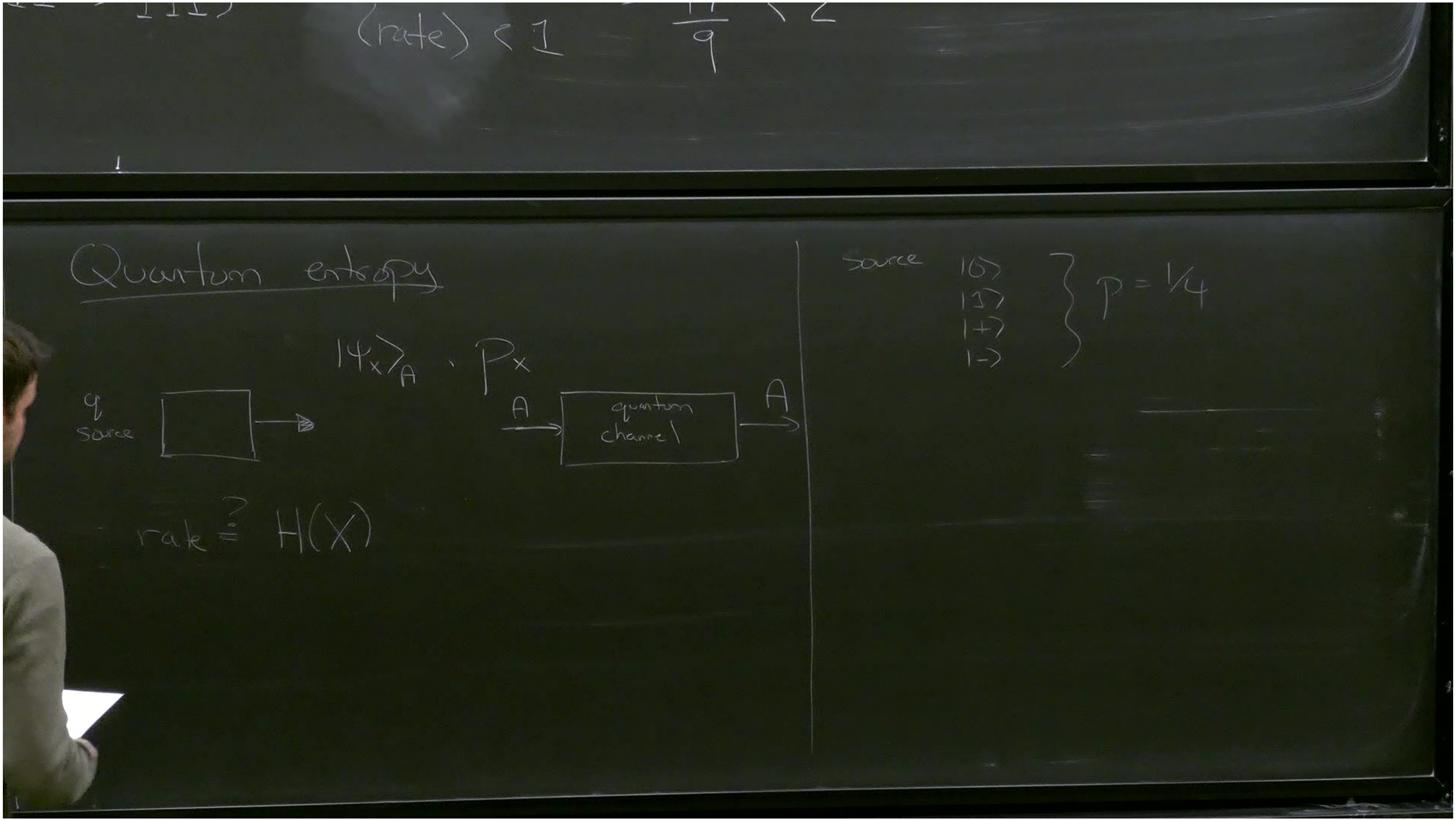
$00 \rightarrow 0$

$01 \rightarrow 101$

$10 \rightarrow 110$

$11 \rightarrow 111$

$\langle \text{rate} \rangle < 1$


(rate) < 1
Quantum entropy
|ψx⟩A · Px
q source
A
quantum channel
A
rate ≟ H(X)
source
|0⟩
|1⟩
|+⟩
|−⟩
p = 1/4

$(\text{rate}) < 1$

Quantum entropy

$|\psi_x\rangle_A \, , \, p_x$

q source $\boxed{} \longrightarrow$ $\xrightarrow{A}$ $\boxed{\text{quantum channel}}$ $\xrightarrow{A}$

$\text{rate} \overset{?}{=} H(X)$

source
$|0\rangle_A$, $|1\rangle_A$, $|+\rangle_A$, $|-\rangle_A$ $\Big\}\; p = 1/4$

$$H(X) = -4 \times \left(\frac{1}{4} \times \log\left(\frac{1}{4}\right)\right)$$
$$= 2$$

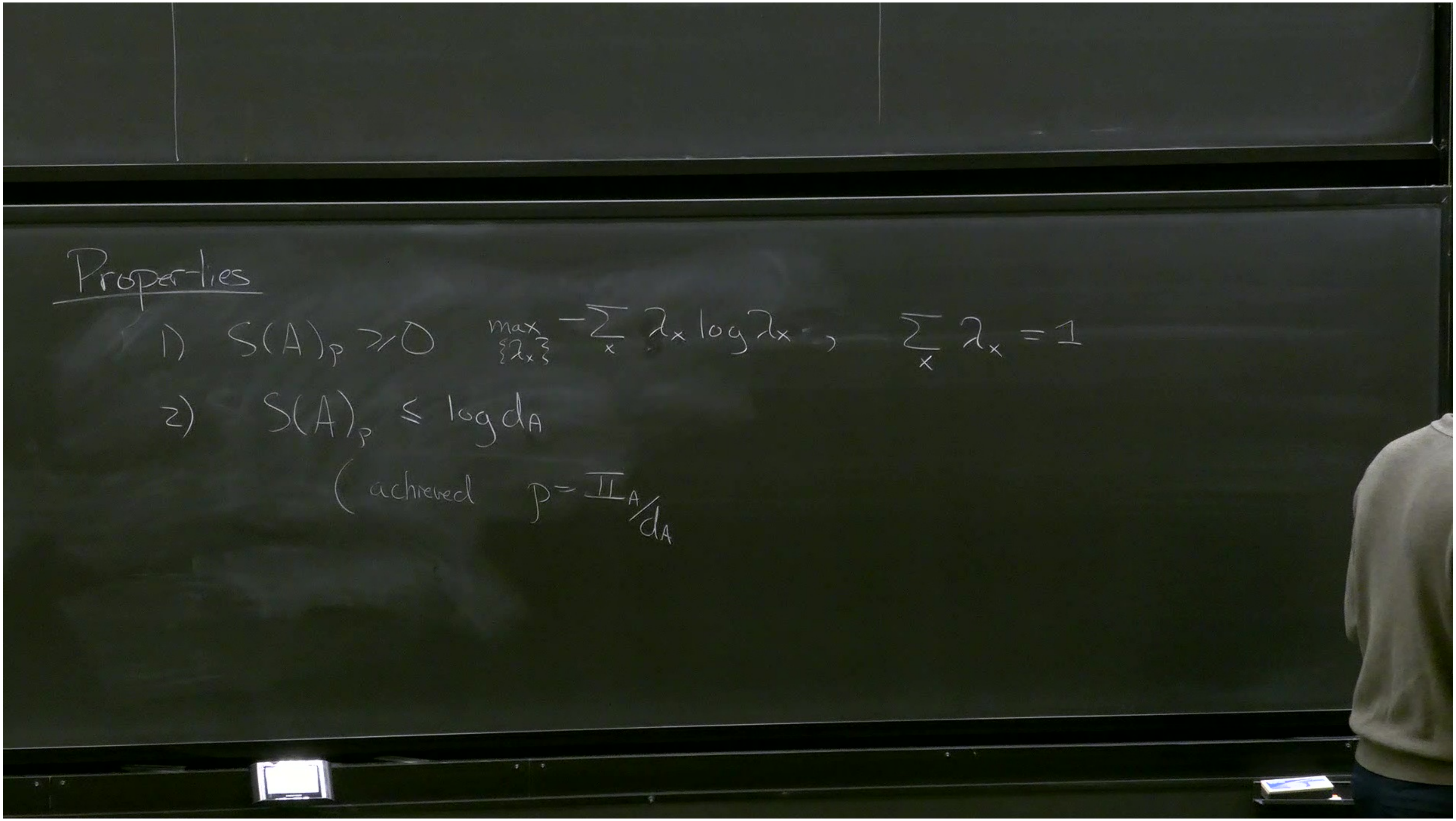
Von Neumann entropy:

$$\boxed{S(A)_\rho = -\mathrm{tr}(\rho \log \rho)}$$

$$\rho = \sum p_x |\psi_x\rangle\langle\psi_x|$$

$$\rho = \sum_i \lambda_i |i\rangle\langle i|$$

$$S(A)_\rho = \underbrace{-\sum_i \lambda_i \log \lambda_i}$$


Properties
1) $S(A)_\rho \geq 0$ $\max_{\{\lambda_x\}} -\sum_x \lambda_x \log \lambda_x$, $\sum_x \lambda_x = 1$
2) $S(A)_\rho \leq \log d_A$
(achieved $\rho = \mathbb{I}_A / d_A$

3) $\rho_{AB} = \rho_A \otimes \rho_B$

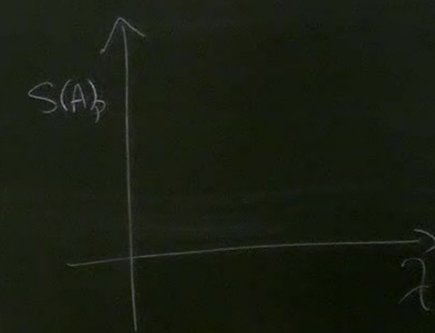
$$S(AB)_\rho = S(A)_\rho + S(B)_\rho$$

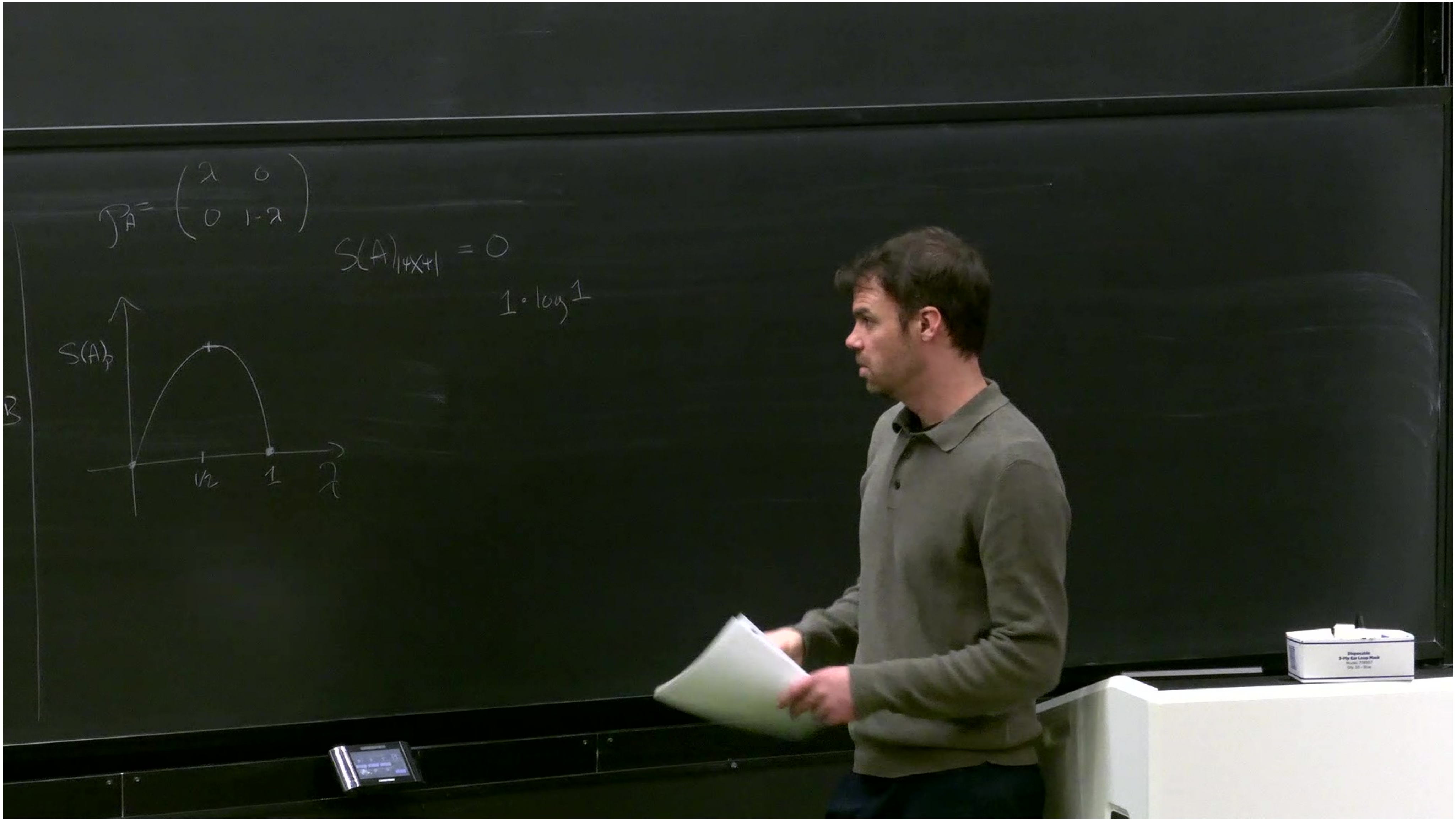
$$\log(\rho_A \otimes \rho_B) = (\log \rho_A) \otimes \mathbb{I} + \mathbb{I}_A \otimes \log \rho_B$$

4) Strong subadditivity

$$S(AB)_\rho + S(BC) \geq S(B)_\rho + S(ABC)_\rho$$

$$\rho_A = \begin{pmatrix} \lambda & 0 \\ 0 & 1-\lambda \end{pmatrix}$$





$\rho_A = \begin{pmatrix} \lambda & 0 \\ 0 & 1-\lambda \end{pmatrix}$
$S(A)_{|\psi\rangle\langle\psi|} = 0$
$1 \cdot \log 1$
$S(A)_\rho$
1/2
1
λ

Relative entropy

$$D(\rho\|\sigma) = \begin{cases} \mathrm{tr}(\rho\log\rho) - \mathrm{tr}(\rho\log\sigma) & \ker\sigma \subseteq \ker\rho \\ \infty & \ker\sigma \not\subseteq \ker\rho \end{cases}$$

1) $D(\rho\|\sigma) \geq 0$

2) $D(\rho\|\sigma) \geq D(\mathcal{N}(\rho)\|\mathcal{N}(\sigma))$

1) $D(\rho_{ABC} \| \rho_A \otimes \rho_{BC}) \geq D(\rho_{AB} \| \rho_A \otimes \rho_B)$

2) $\to$ SSA

$$\rho_{AB} = \rho_A \otimes \rho_B$$

$$S(AB)_\rho = S(A)_\rho + S(B)_\rho$$

$$\log(\rho_A \otimes \rho_B) = (\log \rho_A) \otimes \mathbb{1} + \mathbb{1}_A \otimes \log \rho_B$$

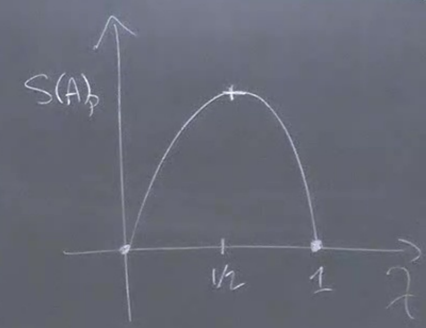
Strong subadditivity

$$S(AB)_\rho + S(BC) \geq S(B)_\rho + S(ABC)_\rho$$

$$\rho_A = \begin{pmatrix} \lambda & 0 \\ 0 & 1-\lambda \end{pmatrix}$$

$$S(A)_{|\psi\rangle\langle\psi|} = 0$$

$$1 \cdot \log 1$$



relative entropy

$$\operatorname{tr}(\rho \log \rho) - \operatorname{tr}(\rho \log \sigma) \quad \ker \sigma \subseteq \ker \rho$$

1) $D(\rho_{ABC} \| \rho_A \otimes \rho_{BC}) \geq \underbrace{D(\rho_{AB} \| \rho_A \otimes \rho_B)}$

$$\underbrace{\mathrm{tr}(\rho_{AB} \log \rho_{AB})}_{-S(AB)_\rho} - \mathrm{tr}\left(\rho_{AB} \log(\rho_A \otimes \rho_B)\right)$$

$$- \underbrace{\mathrm{tr}(\rho_A \log \rho_A)}_{+S(A)} - \underbrace{\mathrm{tr}(\rho_B \log \rho_B)}_{+S(B)}$$

$\boxed{\text{Def}}$ Mutual information.

$$I(A:B)_\rho = \min_{\sigma_A,\sigma_B} D(\rho_{AB} \,\|\, \sigma_A \otimes \sigma_B) = D(\rho_{AB} \,\|\, \rho_A \otimes \rho_B)$$

1) $I(A:B)_\rho = S(A)_\rho + S(B)_\rho - S(AB)_\rho$

2) Compute $I(A:B)_\rho$

$$\rho = |\Phi_{00}\rangle\langle\Phi_{00}|_{AB}$$

$$\rho = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11| \right)$$