Title: Lecture - Quantum Field Theory III - PHYS 777

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Subject: Quantum Fields and Strings

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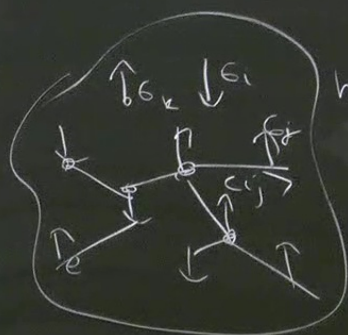
Phase transitions & critical exponents



$h, M = \sum_i \langle \sigma_i \rangle$

$$Z = \sum_s e^{-E(s)/T} \qquad \mathbb{P}(s) = \frac{1}{Z} e^{-\frac{E(s)}{T}}$$

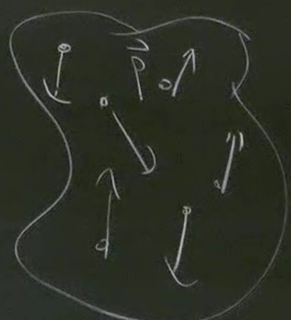
$$\frac{H}{k_B T} = -h \sum \sigma_i - k \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

$$Z \sim \exp\left(S - \frac{\langle E \rangle}{T} \right), \quad U = \langle E \rangle = \sum_s \mathbb{P}(s) H(s)$$

T, S

free energy F : $Z = e^{-F/T}$

$\Rightarrow F = U - TS$



P, V $\qquad \frac{pV}{T} = \text{const}$

μ, N

First principle of thermodyn. : $TdS = dU + PdV$

$$Mdh$$

$$\Rightarrow \boxed{P = -\frac{\partial F}{\partial V}}, \quad S = -\frac{\partial F}{\partial T}$$

$$\boxed{M = \frac{\partial F}{\partial h}}$$

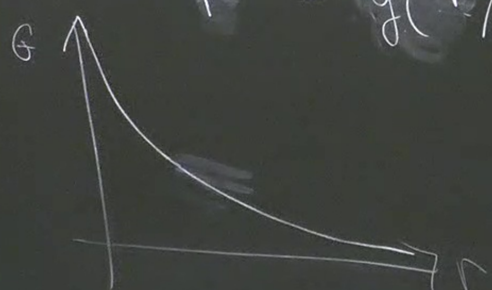
$G(r) = \langle \sigma_r \sigma_0 \rangle - \langle \sigma_r \rangle \langle \sigma_0 \rangle$ - connected 2pt corr. function.
(measures how much two spins are correlated)

$$G(r) = r^{-\frac{d-1}{2}} \, g(r/\xi)$$

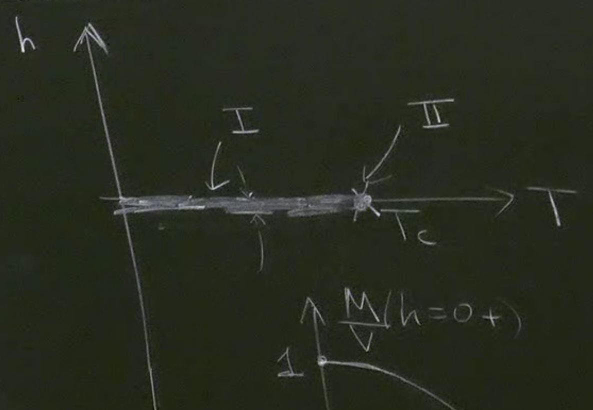
$r \to \infty$

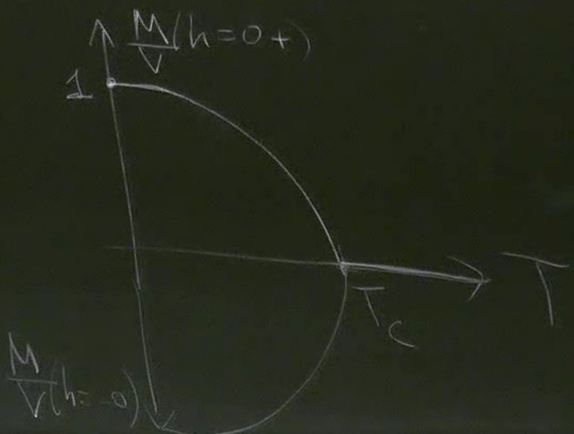
$g(r/\xi) \propto e^{-r/\xi}$, ξ - correlation length

$\xi = \xi(T, h)$



Phase transitions





• Derivatives of F are discontinuous.

→ 1st order: $\frac{\partial F}{\partial h} = M$ — discontinuous.

→ II st order: $\chi = \frac{\partial^2 F}{\partial h^2}$

$C = \frac{\partial^2 F}{\partial T^2}$

$$\chi = \frac{\partial^2 F}{\partial h^2} = \sum_{i,j} G(|i-j|) \propto V \int_0^{+\infty} r^{d-1} G(r)\,dr \sim \xi^{\frac{d+4}{2}}$$

⇒ ξ is diverging at second order p.t.

$t = \frac{T - T_c}{T_c}$

$\xi \propto |t|^{-\nu}, \quad t \to 0, \; h = 0$

$$\boxed{G\left(\begin{smallmatrix} t=0 \\ h=0 \end{smallmatrix}\right) \propto \frac{1}{r^{d-2+\eta}}}$$

η – anomalous scaling dimension

$C|_{h=0} \propto |t|^{-\alpha}$

$M|_{h \to 0-} \propto (-t)^{\beta}, \quad M|_{h \to 0+} \sim 0$

$\chi|_{h=0} \propto |t|^{-\gamma}$

$M|_{t=0} \propto |h|^{1/\delta}$

$\alpha, \beta, \gamma, \delta, \nu, \eta$ – critical exponents.

Universality principle (Landau paradigm): microscopically different systems behave similarly (universally) on larger scales with universality classes beigh labelled by symmetries

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2d CFT allows to find these ones exactly

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Mean field theory

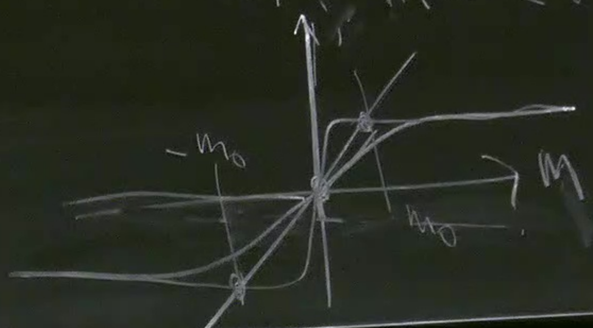
$$\sigma_i = \underset{= \frac{M}{N}}{m} + \underbrace{(\sigma_i - m)}_{\delta\sigma} \qquad H = \ldots + \cancel{\delta\sigma\,\delta\sigma}$$

$$H = -h\sum \sigma_i - k\sum \sigma_i\sigma_j \sim d\cdot k m^2 N - (2dkm + h)\sum_i \sigma_i$$

$$\Rightarrow \quad Z = \left(2\, e^{-dkm^2}\cosh(2dkm + h)\right)^N$$

$$f = \frac{F}{N} \sim dkm^2 - \log\cosh(2dk\underline{m} + \underline{h})$$

Self-consistency: $m = \langle \sigma_i \rangle = -\dfrac{\partial f}{\partial h} = \tanh(2dkm + h)$



$2dk < 1 \Rightarrow m = 0$ $\qquad \boxed{k = \frac{1}{2d}}$

$2dk > 1 \Rightarrow m = 0, \pm m_0(k)$

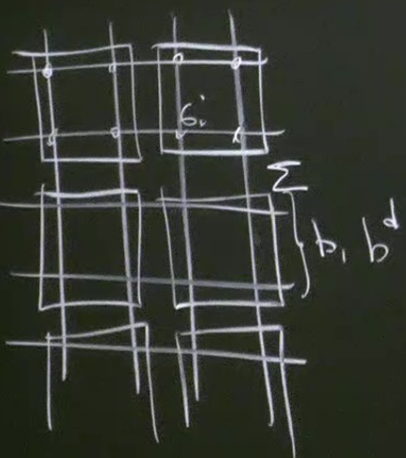
$$f \sim (\nabla m)^2 + a t m^2 + b m^4 \qquad t = \frac{T - T_c}{T_c}$$

$$\alpha = 0, \quad \beta = 1/2, \quad \gamma = 1, \quad \delta = 3$$

$$\nu = 1/2, \quad \boxed{\eta = 0}$$

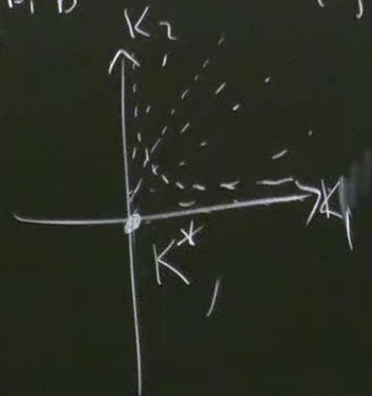
Critical exponents from RG flow (Kadanoff)

RG(K



$$\frac{H}{kT} = -K_0 - K_1\sum_i \sigma_i - K_2 \sum_{i,j}\sigma_i\sigma_j - K_3\sum_{i,j,k}\sigma_i\sigma_j\sigma_k$$

$$Z(K) \sum_{\{\sigma\}} e^{-\frac{H(K)}{T}} = \sum_{\{\Sigma\}} e^{-\frac{H(K')}{T'}} \Rightarrow K' = RG(K)$$



$$RG(K) = RG(K^*) + (K-K')\,RG(K^*) + \dots$$

$$\sim K^* + b^y(K-K^*)$$

$y>0$ - rel.

$y<0$ - irrel

$y=0$ - marg.

y - scaling dimension of operator

$$RG(K)_a - K^*_a \sim \sum_b T_{ab}\,(K_b - K^*_b)$$

$$\sum_a e^i_a T_{ab} = b^{y_i} e^i_a \Rightarrow u_i = \sum e^i_a (K_a - K^i_a)$$

$$\boxed{u_i \mapsto b^{y_i} u_i}$$

$\frac{M}{V}(h=0)$

$$f(K_1, K_2) = g(K_1, K_2) + b^{-d} f_s\left(\underline{RG(K)_1, RG(K)_2}\right)$$

singular specific free energy.

$$f_s(u_1, u_2) = b^{-dn} f_s\left(b^{n y_1} u_1, b^{n y_2} u_2\right) \ominus$$

$u_{2,0}$ $\qquad b^{n y_2} u_2 = u_{2,0}$.

$$\boxed{f_s \ominus \left(u_2/u_{2,0}\right)^{d/y_2} f_s\left(u_1\left(\frac{u_2}{u_{2,0}}\right)^{-y_1/y_2}, u_{2,0}\right)}$$

$$\alpha = 2 - \frac{d}{y_t}$$

$$\beta = \frac{d - y_h}{y_t}$$

$$\gamma = \frac{2y_h - d}{y_t} \Rightarrow$$

$$\delta = \frac{y_h}{d - y_t}$$

$$\nu = \frac{1}{y_t}$$

$$\eta = d + 2 - 2y_h$$

$$\alpha + 2\beta + \gamma = 2$$

$$\alpha + \beta(1+\delta) = 2$$

$$\mathrm{CFT}_{2d}: \quad \boxed{y_t = 1 \qquad y_h = \frac{15}{8}}$$

$$RG(K)_a - K^*_a \sim \sum_b T_{ab}(K_b - K^*_b)$$

$$\sum_a e^i_a T_{ab} = b^{y_i} e^i_a \Rightarrow u_i = \sum e^i_a (K_a - K^i_a)$$

$$\boxed{u_i \mapsto b^{y_i} u_i}$$

$$\boxed{\begin{aligned} y_t &= 1.62 \\ y_h &= 2.12 \end{aligned}}$$