Title: Lecture - Quantum Field Theory III - PHYS 777

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Subject: Quantum Fields and Strings

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Anomalies:

- Classical system with a symmetry G $\begin{cases} \text{- spacetime - symmetry (conformal)} \\ \text{- internal symmetries (Flavor symmetry)} \\ \text{- } \boxed{\text{continous}} \\ \text{- discrete } (P, T, \mathbb{Z}_2) \end{cases}$

- "Obstruction" to G surviving quantization:

n (conformal)

(Flavor symmetry)

$T, \mathbb{Z}_2$)

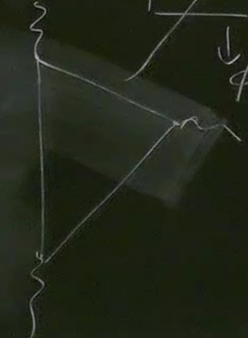
- Explicit breaking / (ABJ anomaly).

$T^{\mu}_{\mu} = 0$ classically

1) $T^{\mu}_{\mu} \neq 0 = \beta(g)\, \mathrm{tr}\, F_{\mu\nu}^{2}$

2) QED $\boxed{U(1)_A} \times U(1)_V$ $\qquad \partial^{\mu} j^{A}_{\mu} = 0$

$\downarrow \phi$



$\partial^{\mu} j^{A}_{\mu} = F_{\mu\nu}^{2}$

QCD $U(1)_A \longrightarrow \mathbb{Z}_{2n}$

$g \quad g^{2n} = 1$

... (conformal)

, (Flavor symmetry)

, T, $\mathbb{Z}_2$).

- Explicit breaking / (ABJ anomaly).

$T^\mu_\mu = 0$ classically

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$\downarrow\!\!\!/$

QCD $U(1)_A \longrightarrow \mathbb{Z}_{2n}$

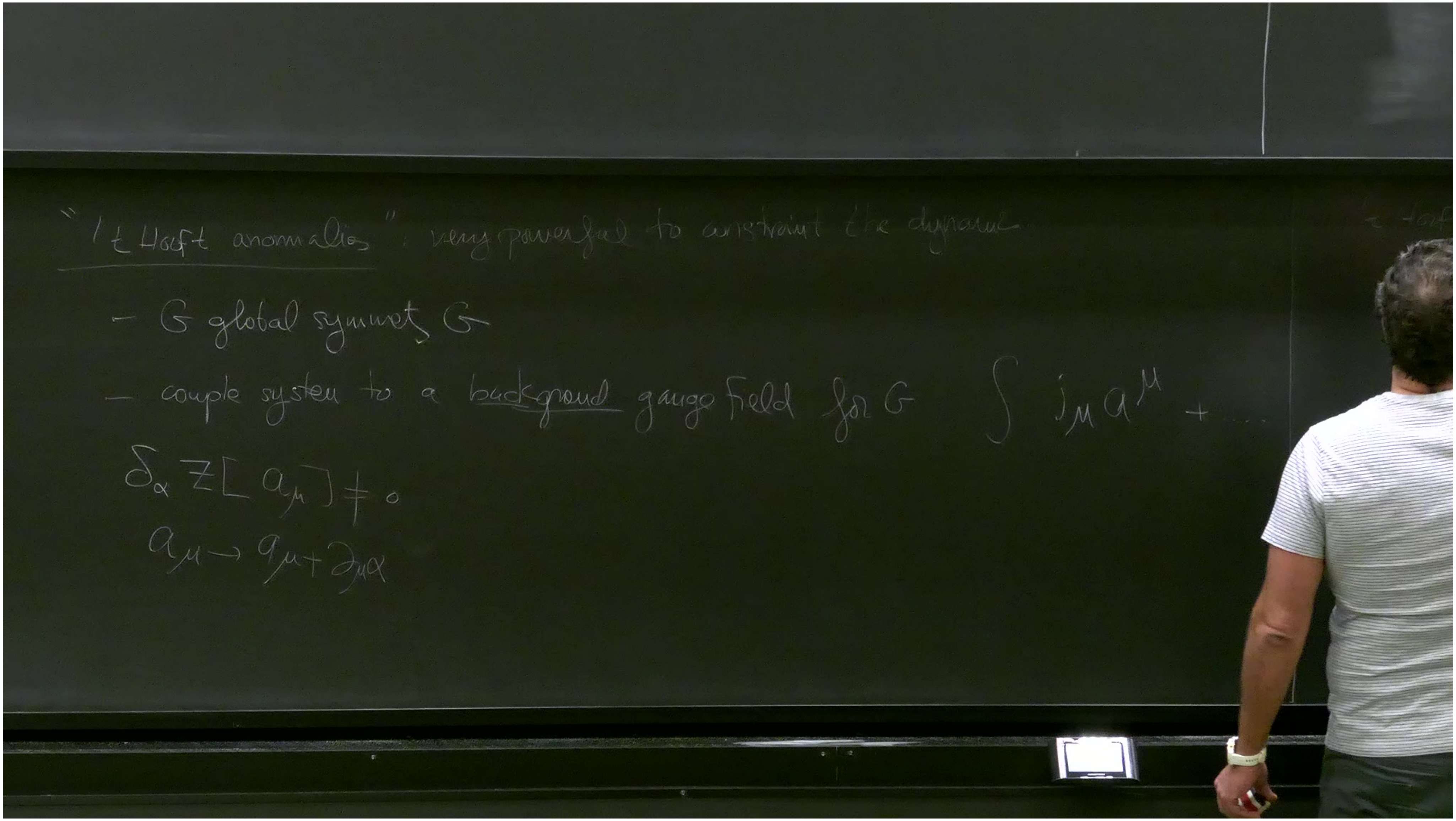
g $g^{2n} = 1$

$\partial^\mu j^A_\mu = 0$

$\partial^\mu j^A_\mu = \boxed{F^2_{\mu\nu}}$

nontrivial operator

$\neq \mathbb{1}$.


"'t Hooft anomalies": very powerful to constraint the dynamics
- G global symmetry G
- couple system to a background gauge field for G $\int j_\mu a^\mu + \ldots$
$\delta_\alpha Z[a_\mu] \neq 0$
$a_\mu \to a_\mu + \partial_\mu \alpha$

$g \quad g^{2n} = 1$

...me.

$\int j_\mu a^\mu + \ldots$

't Hooft anomalies are topological (independent of energy scale)

$\Downarrow$

UV Theory $\quad \mathcal{A}_{UV} \neq 0 \quad$ invariant of RG flow

IR $\boxed{??}$ $\quad$ IR (dynamics cannot be trivial)

$g \quad g^{2n} = 1$

...ue.

$$\int j_\mu a^\mu + \dots$$

't Hooft anomalies are topological (independent of energy scale)

$\Downarrow$

invariant of RG flow

UV Theory $\quad A_{UV} \neq 0$

IR $\quad$ P. Answer, A_{IR} $\quad$ wrong $A_{IR} \neq A_{UV}$

IR (dynamics cannot be trivial)

$a_\mu \to a_\mu + \partial_\mu \alpha$

IR

$$Z[a_\mu] \to \underbrace{e^{i\int A\, d^dx}}_{\uparrow} Z[a_\mu]$$

phase measures the 't Hooft anomaly

d+1 <u>Claim</u>: anomaly is fully charactered by a $\boxed{TQFT_{d+1}}$

d

1+1d

IR $\boxed{\text{P. Answer } A_{IR}}$ IR unless $A_{IR} \neq A_{UV}$

IR (dynamics cannot be trivial)

$$\delta_\alpha \log Z = \int_{M_2} \alpha f \, d^2x$$

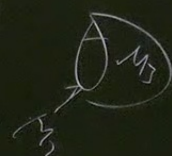
$\boxed{T_{d+1}}$

<u>1+1d</u> U(1) global symmetry

$$\partial^\mu j_\mu = k \overbrace{\epsilon^{\mu\nu} f_{\mu\nu}}^{\parallel f}$$

$$\partial^\mu j_\mu = \mathbb{1} \cdot f(a)$$

Anomaly is captured by 3d topological term



$$S_{TQFT} \int_{S} \frac{a \wedge da}{4\pi}$$

$$\delta_\alpha S_{TQFT} = -\int \alpha f \, d^2x$$

Example

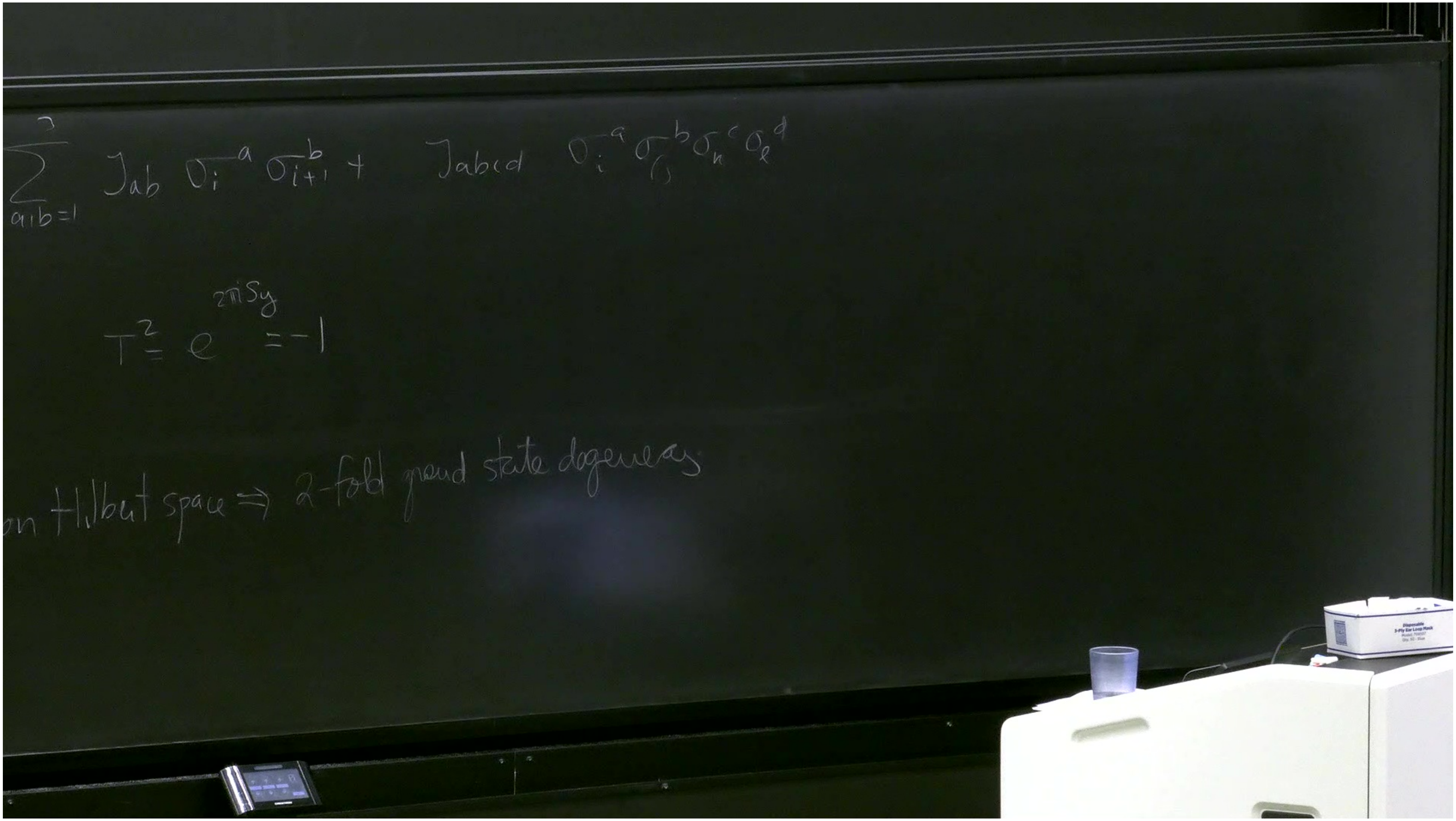
$$H = \sum_{i=1}^{L} J\, \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} + \sum_{a,b=1}^{3} J_{ab}\, \sigma_i^a \sigma_{i+1}^b + J_{abcd}\, \sigma_i^a \ldots$$

- has T-reversal symmet $T^2 = 1$ $\mathbb{Z}_2^T$

$$T \vec{\sigma} T^{-1} = -\vec{\sigma}$$

$$T = e^{i\pi S_y} \cdot K \qquad T^2 = e^{2\pi i S_y} = -1$$

- if L is odd $\mathbb{Z}_2^T$ has a 't Hooft anomaly $\Rightarrow$ $T^2 = -1$ on Hilbert space $\Rightarrow$ 2-fold ground state degen


$\sum_{a,b=1}^{3} J_{ab}\, \sigma_i^a \sigma_{i+1}^b + J_{abcd}\, \sigma_i^a \sigma_j^b \sigma_k^c \sigma_l^d$
$T^2 = e^{2\pi i S_y} = -1$
on Hilbert space ⇒ 2-fold ground state degeneracy

- 't Hooft anomalies are RG invariants
- $\exists$ anomalies (conformal anomalies) they are monotonically decreasing ' $c_{UV} > c_{IR}$

Conformally anomaly in 2d:

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p) \rangle = p_\mu p_\nu p_\rho p_\sigma F_1(p^2) +$$

- 't Hooft anomalies are RG invariants
- $\exists$ anomalies (conformal anomalies) they are monotonically decreasing: $c_{UV} > c_{IR}$

Conformal anomaly in 2d:

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = p_\mu p_\nu p_\rho p_\sigma F_1(p^2) + $$

impose: $p^\mu T_{\mu\nu} = 0$ conservation. $\Rightarrow$ reduces the possible terms in RHS

$$\langle T_{\mu\nu}(p) T_{\rho\sigma}(-p)\rangle = \left(\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\cancel{\Pi_{\nu\rho}}^{0} - \frac{1}{d-1}\Pi_{\mu\nu}\Pi_{\rho\sigma}\right) F(p^2) + \Pi_{\mu\nu}\Pi_{\rho\sigma}\, G(p^2)$$

$$\Pi_{\mu\nu} = p_\mu p_\nu - \eta_{\mu\nu} p^2 \qquad p^\mu \Pi_{\mu\nu} = 0$$

Exercise: prove that in $d=2$

$$\Pi_{\mu\nu} = -\tilde{p}_\mu \tilde{p}_\nu$$

$$\langle T_{\mu\nu}(p) T_{\rho\sigma}(-p)\rangle = \left(\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\rho} - \frac{2}{d-1}\Pi_{\mu\nu}\Pi_{\rho\sigma}\right) f(p) + \Pi_{\mu\nu}\Pi_{\rho\sigma} \ldots$$

$$\Pi_{\mu\nu} = p_\mu p_\nu - \eta_{\mu\nu} p^2 \qquad p^\mu \Pi_{\mu\nu} = 0$$

Exercise: prove that in $d=2$

$$\Pi_{\mu\nu} = -\tilde{p}_\mu \tilde{p}_\nu$$

in 2d

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \frac{c\,\Pi_{\mu\nu}\Pi_{\rho\sigma}}{p^2}$$

– scale invariance

$$\langle T^\mu_\mu(p)\, T_{\rho\sigma}(-p)\rangle = \Pi_{\mu\nu} \Longrightarrow$$

position space

$$\boxed{\langle T^\mu_\mu(x)\, T_{\rho\sigma}(0)\rangle = c\,(\partial_\mu \partial_\nu - \eta_\mu \Box)\,\delta^2(x)}$$

conformal anomaly

Example

$$H = \sum_{i=1}^{L} J\, \vec{\sigma}_i \vec{\sigma}_{i+1} + \sum_{a,b=1}^{3} J_{ab}\, \sigma_i^a \sigma_{i+1}^b + J_{abcd}\, \sigma_i^a \sigma_j^b \sigma_k^c \sigma_l^d$$

- has T-reversal symmetry $T^2 = 1$ $\mathbb{Z}_2^T$

$$T \vec{\sigma} T^{-1} = -\vec{\sigma} \qquad T = e^{i\pi S_y} \cdot K \qquad T^2 = e^{2\pi i S_y} = -1$$

- if L is odd $\mathbb{Z}_2^T$ has a 't Hooft anomaly $\Rightarrow$ $T^2 = -1$ on Hilbert space $\Rightarrow$ 2-fold ground state degeneracy

- $\exists$ anomalies (conformal anomalies) they are ...

Conformal anomaly in 2d:

$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p) \rangle = p_\mu p_\nu p_\rho p_\sigma F_1(p^2) +$ ... $+ (\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}$ ...

$$a_\mu \to a_\mu + \partial_\mu \alpha$$

IR

$$\langle T^\mu_\mu(0)\rangle_g = \langle \int dx \, \langle T^\mu_\mu(0)\, T_{\rho\sigma}(x)\rangle\big|_0 \, \delta g^{\rho\sigma}$$

$$e^{\int \delta g^{\mu\nu} T_{\mu\nu}}$$

$$= c \, \underline{(\partial_\rho \partial_\sigma - \eta_{\rho\sigma}\Box)\,\delta g^{\rho\sigma}}$$

$$\Leftarrow$$

$$\boxed{T^\mu_\mu = cR}$$

IR $\boxed{\text{P. Answer}}$ A_{IR} unity $A_{IR} \neq A$

IR (dynamics cannot be trivial)

D= odd dimensions

$T^\mu_\mu = 0$

$\boxed{D=4}$

$$T^\mu_\mu = a\, E_4 + c\, \mathrm{Weyl}^2 + \cancel{d\, R^2} + \cancel{e\, \Box R}$$

does not obey WZ

$$\delta_\sigma \log Z = \int dx\, \sigma(x)\, A(x)$$

Wess-Zumino consistency condition

$$[\delta_{\sigma_1}, \delta_{\sigma_2}] \log Z = \delta_{[\sigma_1, \sigma_2]} \log Z$$

$$Z_{\text{Jame}} = Z_{\text{you}}\, e^{\int d^4x\, R^2 \Lambda}$$

$$\delta_\sigma \int d^4x\, R^2 = \int \Box R$$

$\boxed{2d}$ / $\boxed{4d}$

$A_{UV} > A_{IR}$

conformal anomaly associated to the Euler density

2d c

4d a