Title: Lecture - Quantum Field Theory III - PHYS 777

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Subject: Quantum Fields and Strings

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Abstract:

$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n)\rangle = \langle \hat{\mathcal{O}}(x_1) \cdots \hat{\mathcal{O}}(x_n)\rangle$ Ward identity for conformal

$$\sum_{i=1}^{n} \langle \mathcal{O}(x_1) \cdots \delta\mathcal{O}(x_i) \cdots \mathcal{O}(x_n)\rangle = 0$$

e.g. scalar operator

$$\langle \mathcal{O}_I(x_1)\, \mathcal{O}_J(x_2)\rangle = \frac{\delta_{IJ}}{x_{12}^{2\Delta_I}}\, \delta_{\Delta_I,\Delta_J}. \qquad x_{ij} = |x_i - x_j|$$

$$\langle \mathcal{O}_I(x_1)\,\mathcal{O}_J(x_2)\,\mathcal{O}_K(x_3)\rangle = \frac{C_{IJK}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3}\; x_{23}^{\Delta_2+\Delta_3-\Delta_1}\; x_{13}^{\Delta_1+\Delta_3-\Delta_2}}$$

$\Rightarrow$ CFT data

$\{\Delta_I, l_I\}$, $\{C_{IJK}\}$

scaling dimension — structure constants

Last lecture.

- studied how SCA is realized on ops.
- come in ∞ dim'al families

- Primaries $[K_{\mu_1}, \mathcal{O}(0)] = 0$
- Descendants $[P_{\mu_1}, [P_{\mu_2}, \dots [P_{\mu_n}, \mathcal{O}]]]$

Ward identities to constraint observables: $x \to \tilde{x}$

$$\langle \mathcal{O}(x_1) \dots \mathcal{O}(x_n) \rangle = \langle \hat{\mathcal{O}}(x_1) \dots \hat{\mathcal{O}}(x_n) \rangle$$ Ward identity for conformal

inf'al $$\sum_{i=1}^{n} \langle \mathcal{O}(x_1) \dots \delta \mathcal{O}(x_i) \dots \mathcal{O}(x_n) \rangle = 0$$

$$x_{ij} = |x_i - x_j|$$

4pt function in d-spacetime

$$x_1 \quad x_2 \quad x_3 \quad x_4$$
$$\downarrow K_\mu$$
$$x_1 \quad x_2 \quad x_3 \quad \infty$$
$$\downarrow P_\mu$$
$$0 \quad x_2 \quad x_3 \quad \infty$$
$$\downarrow D$$
$$0 \quad x_2 \quad |x_3|=1 \quad \infty$$
$$\downarrow M_{12}, M_{13}, \dots M_{1d}$$
$$0 \quad x_2 \quad x_3=(1 0 \dots 0)$$

$$\xrightarrow{M_{23}, \dots M_{2d}} \quad 0 \quad \underset{(a,b,0\dots0)}{x_2} \quad \underset{(10\dots0)}{x_3} \quad \infty$$

$d=3$ used all M's. M_{12}, M_{13}, M_{23}

$d=4$ $M_{12}, M_{13}, M_{14}, M_{23}, M_{24}$ M_{34}

how about higher n-point function

$x_1 \dots x_n$

$$\boxed{dn - \frac{(d+2)(d+1)}{2}} = \text{\# of cross ratios given n-point}$$

4pt function of identical scalar op.

conformal inv. ratios

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta\phi}\, x_{34}^{2\Delta\phi}} = \frac{g(v,u)}{x_{12}^{2\Delta\phi}\, x_{34}^{2\Delta\phi}} \left(\frac{u}{v}\right)^{\Delta\phi}$$

$u = x_{12}^2 x_{34}^2 \dots$

$g(u,v)$ is NOT an independent function

$0 \quad X_2 \quad |X_3| = 1 \quad \infty$

$M_{12}, M_{13}, \ldots M_{1d}$

$0 \quad X_2 \quad X_3 = (1 \, 0 \, \ldots \, 0)$

how about higher n-point function

$x_1 \ldots x_n$

$$\boxed{dn - \frac{(d+2)(d+1)}{2} = \#\text{ of crossratios given n-point}}$$

conformal inv. ratios

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$$

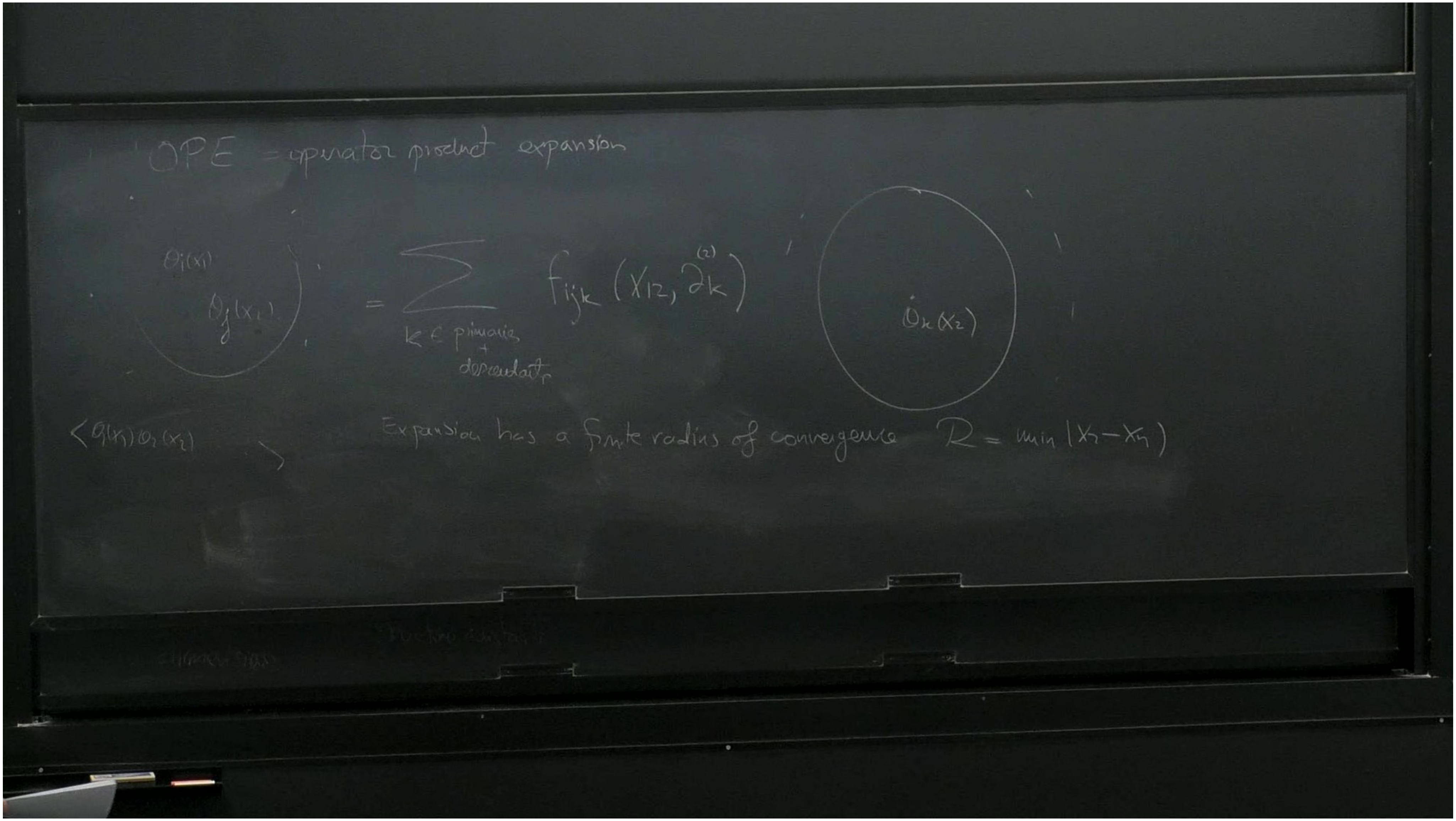
$x_1 \leftrightarrow x_3$

$$v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

4pt function of identical scalar op. $1 \leftrightarrow 3$

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta\phi}\, x_{34}^{2\Delta\phi}} = \frac{g(v,u)}{x_{12}^{2\Delta\phi}\, x_{34}^{2\Delta\phi}} \left(\frac{u}{v}\right)^{\Delta\phi}$$

- $g(u,v)$ is <u>NOT</u> an independent function
- complete determined by $\{\Delta_I\}$, $\{C_{IJK}\}$ + symmetry
- 4pt function constraint the CFT data


OPE = operator product expansion
$O_i(x_1)$ $O_j(x_2)$ = $\sum_{k \in \text{primaries + descendants}} f_{ijk}(x_{12}, \partial_k^{(2)})$ $O_k(x_2)$
$\langle O_1(x_1) O_2(x_2) \rangle$
Expansion has a finite radius of convergence $R = \min|x_2 - x_n|$

$$\langle \mathcal{O}_i(x_1)\, \mathcal{O}_j(x_2)\, \mathcal{O}_k(x_3)\rangle = \sum_{k'} f_{ijk'}(x_{12}, \partial_2)\, \langle \mathcal{O}_{k'}(x_2)\, \mathcal{O}_k(x_3)\rangle$$

$$= \sum_{k'} f_{ijk'}(x_{12}, \partial_2) \left(\frac{\delta_{kk'}}{x_{23}^{2\Delta_k}} \right)$$

expand $\dfrac{|x_{12}|}{|x_{23}|}$

$C_{ijk}\, f(x)$

$$f_{ijk} = C_{ijk}\, x_{12}^{\Delta_k - \Delta_i - \Delta_j} \left(1 + \alpha\, x^\mu \partial_\mu + \beta\, x^\mu x^\nu \partial_\mu \partial_\nu + \cdots \right)$$

Show that $\Delta_i = \Delta_j = \Delta_\phi$

$\Delta_k = \Delta$

$$\alpha = \frac{\Delta + 2}{8(\Delta + 1)}$$

$$v = \frac{x_{14}^2\, x_{23}^2}{x_{13}^2\, x_{24}^2}$$

- 4pt function constraint the CFT data.

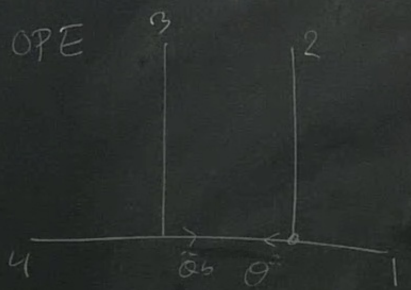
4pt function

$$\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\rangle = \sum_{\mathcal{O},\mathcal{O}'} C_{\phi\phi\mathcal{O}}\, C_{\phi\phi\mathcal{O}'}\, C_a(x_{12},\partial^{(2)})\, C_b(x_{34},\partial^{(4)})\, \langle \mathcal{O}^a(x_2)\, \mathcal{O}'^b(x_4)\rangle$$

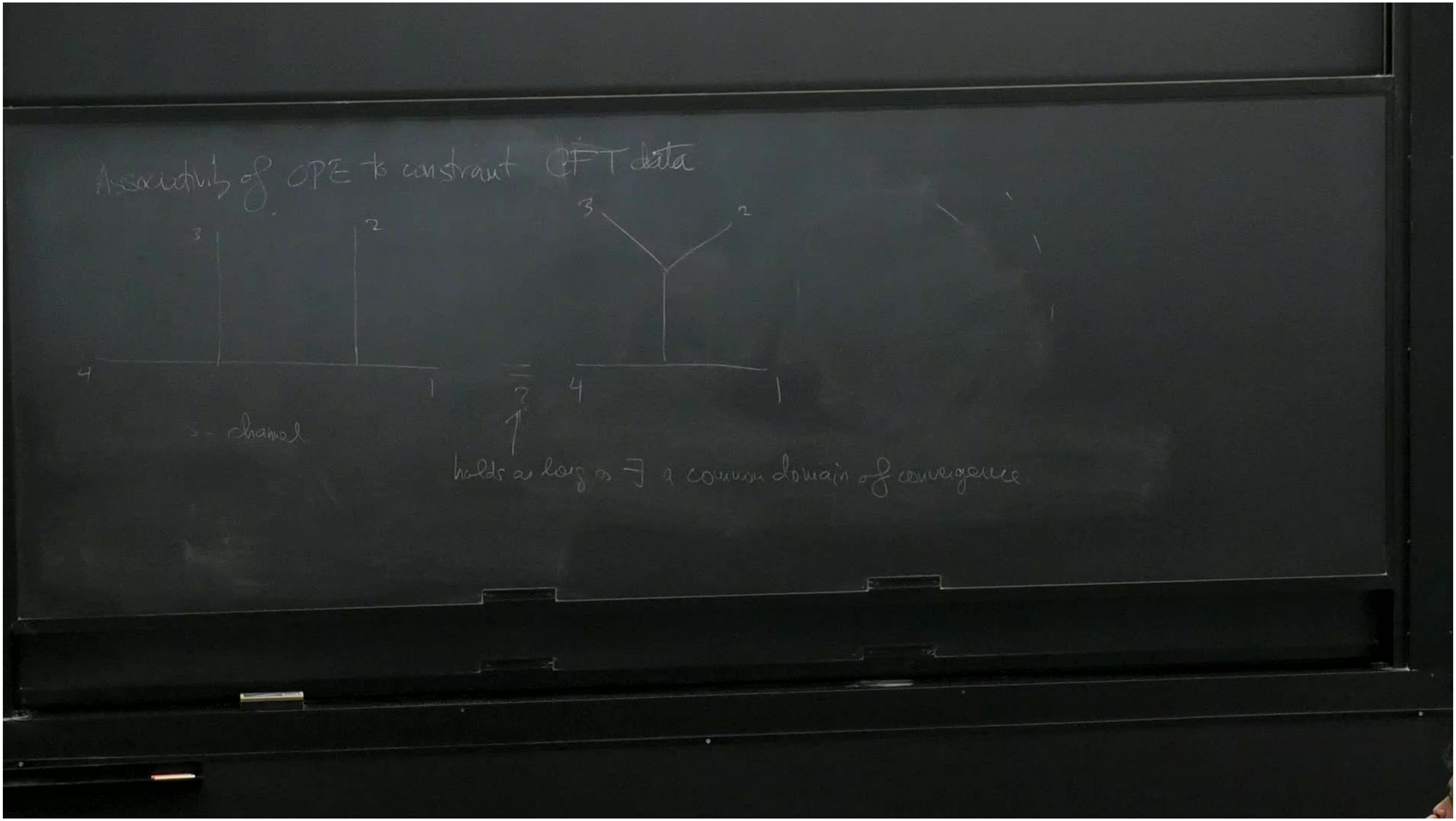
$$\langle \mathcal{O}^a(x_2)\, \mathcal{O}'^b(x_4)\rangle = \delta_{\mathcal{O}\mathcal{O}'}\, \frac{I^{ab}(x_2-x_4)}{x_{24}^{2\Delta_\mathcal{O}}}$$

$$= \frac{1}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}} \sum_{\mathcal{O}} C^2_{\phi\phi\mathcal{O}}\, g_{\Delta_\mathcal{O},l_\mathcal{O}}(u,v)$$

OPE



$$g_{\Delta_\mathcal{O},l_\mathcal{O}}(u,v) = x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}\, C_a(x_{12},\partial^{(2)})\, C_b(x_{34},\partial^{(4)}) \cdot \frac{I^{ab}(x_{24})}{x_{24}^{2\Delta_\mathcal{O}}}$$


Associativity of OPE to constraint CFT data
3
2
4
1
s - channel
=
?
3
2
4
1
holds as long as ∃ a common domain of convergence

s-channel t-channel

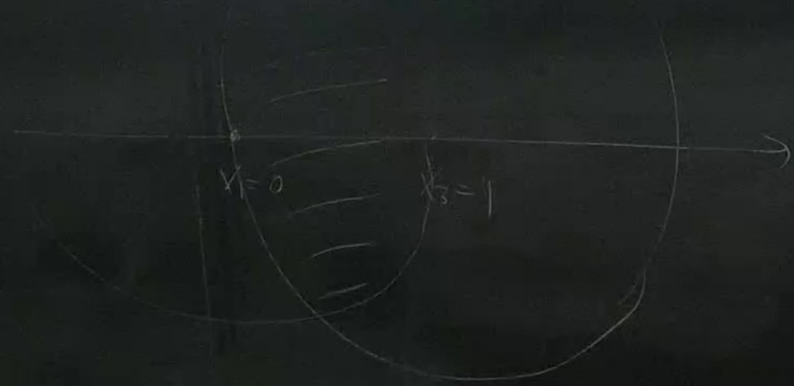
holds as long as $\exists$ a common domain of convergence.

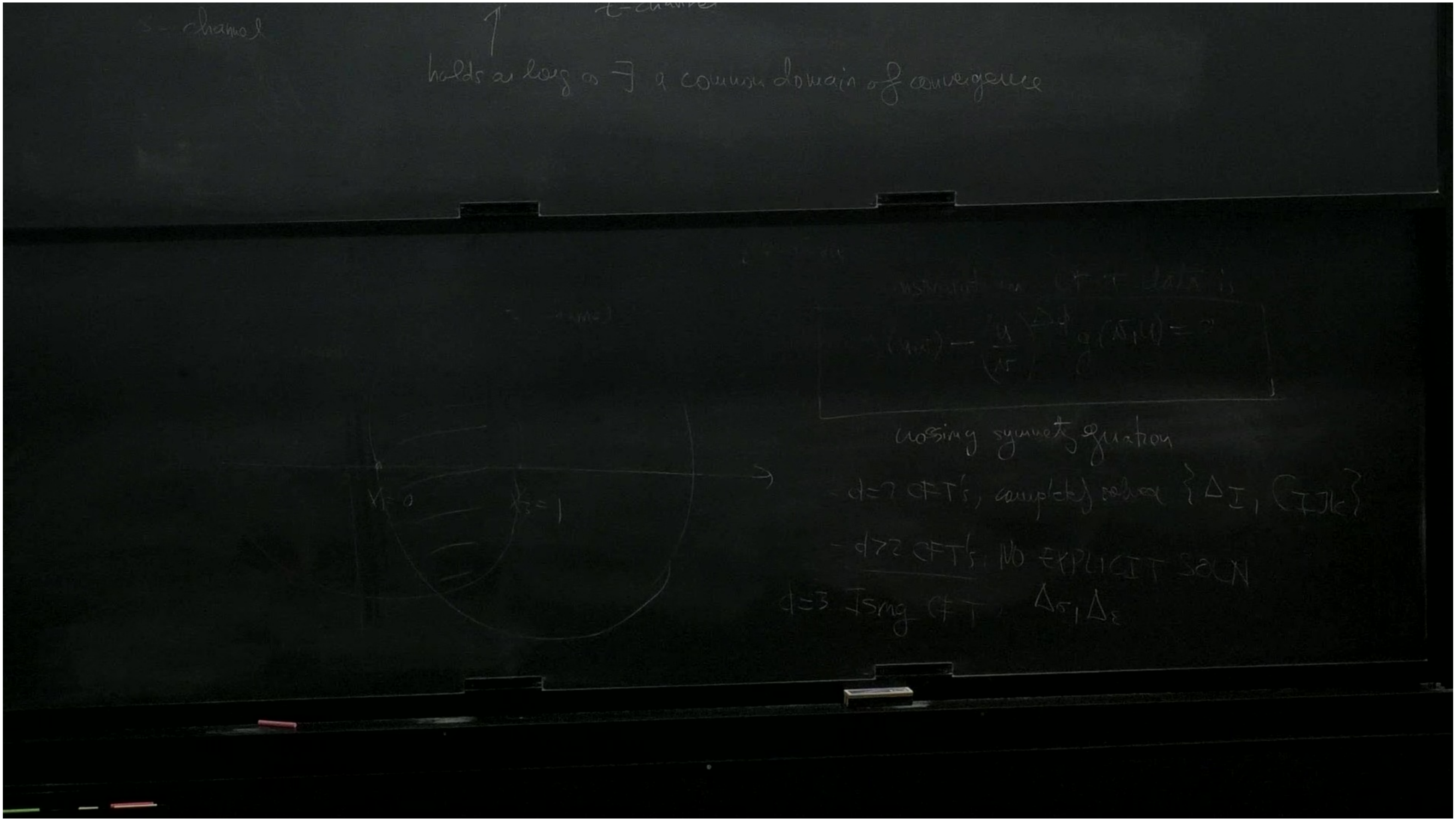
constraint on CFT data is

$$\boxed{g(u,v) - \left(\frac{u}{v}\right)^{\Delta_\phi} g(v,u) = 0}$$

crossing symmetry equation

→ $d=?$ CFT's, completely solved $\{\Delta_I, C_{IJK}\}$.




holds as long as ∃ a common domain of convergence
crossing symmetry equation
d=2 CFT's, completely solved {Δ_I, C_IJK}
d>2 CFT's, NO EXPLICIT SOLN
d=3 Ising CFT Δ_σ, Δ_ε