Title: Lecture - AdS/CFT, PHYS 777

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Collection/Series: AdS/CFT (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Quantum Fields and Strings, Quantum Gravity

Date: March 31, 2025 - 9:00 AM

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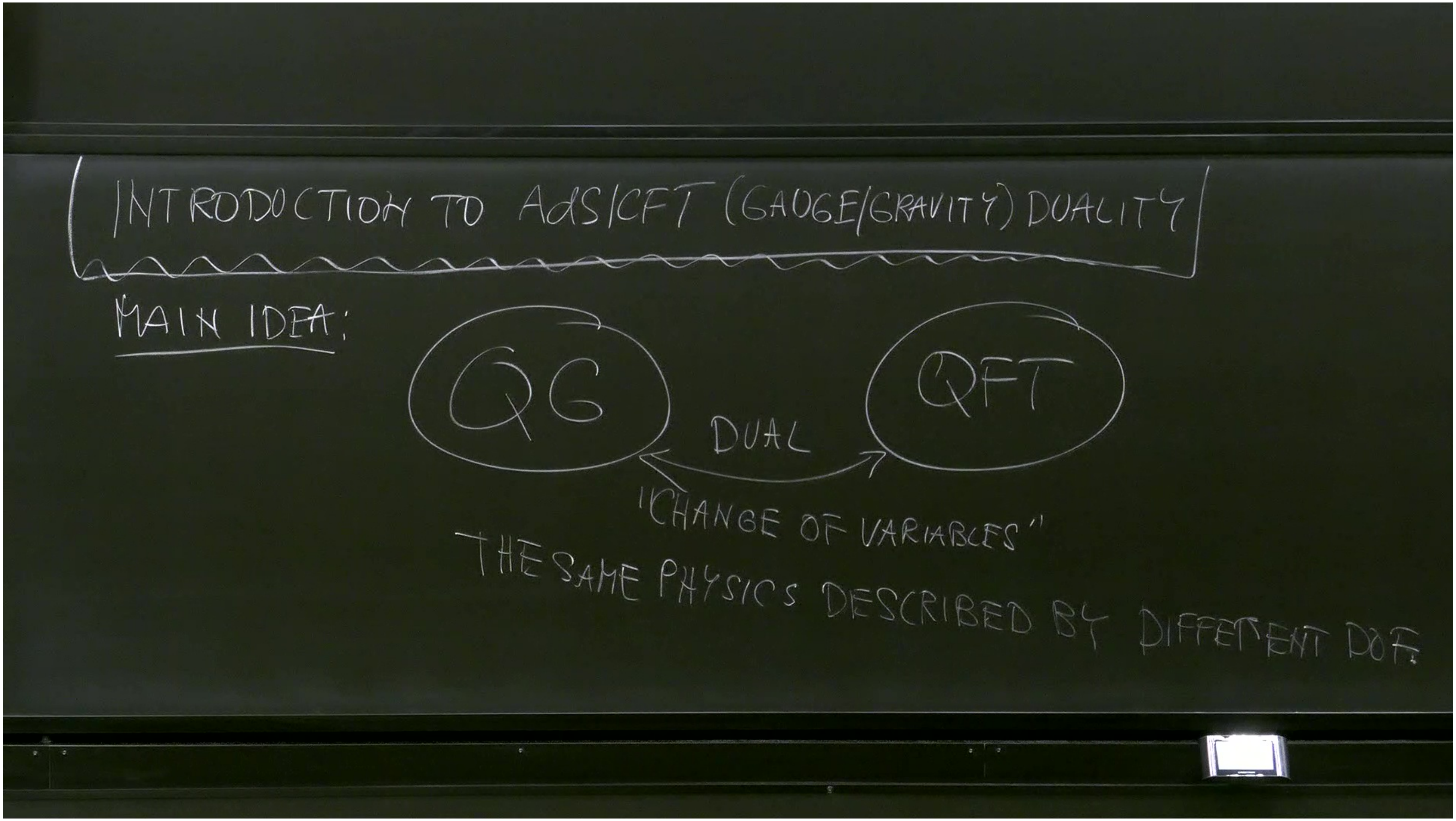
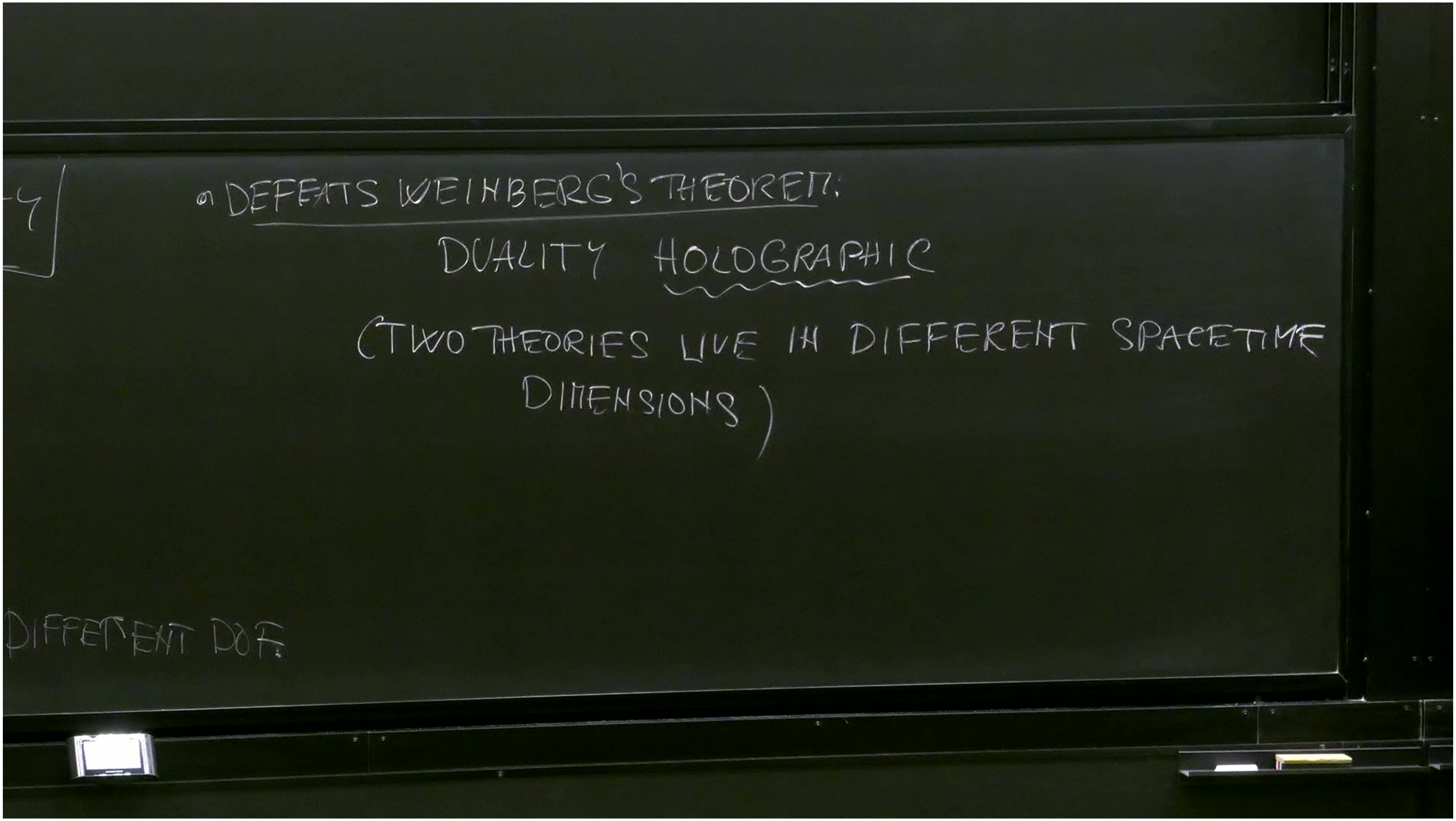
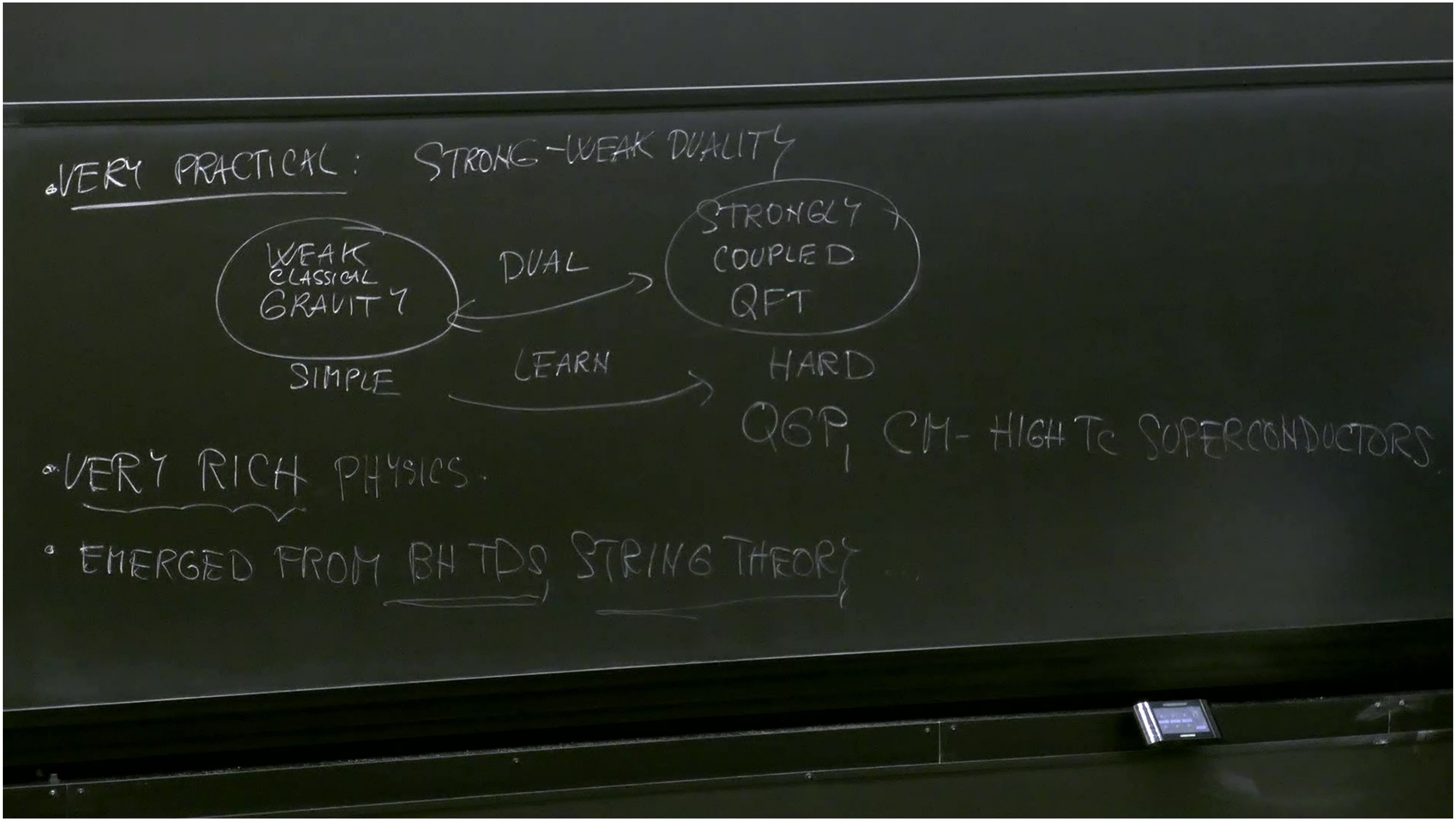

INTRODUCTION TO AdS/CFT (GAUGE/GRAVITY) DUALITY
MAIN IDEA:
QG
QFT
DUAL
"CHANGE OF VARIABLES"
THE SAME PHYSICS DESCRIBED BY DIFFERENT DOF.


• DEFEATS WEINBERG'S THEOREM:
DUALITY HOLOGRAPHIC
(TWO THEORIES LIVE IN DIFFERENT SPACETIME
DIMENSIONS)
DIFFERENT DOF.


• VERY PRACTICAL: STRONG-WEAK DUALITY
WEAK CLASSICAL GRAVITY
DUAL
STRONGLY COUPLED QFT
SIMPLE
LEARN
HARD
QGP, CM- HIGH Tc SUPERCONDUCTORS
• VERY RICH PHYSICS.
• EMERGED FROM BH TDS, STRING THEORY ...

Plan

I. Black holes as thermodynamic objects
   I. From BH mechanics to BH TDs
   II. Black hole evaporation

II. Euclidean magic
   I. Temperature from regularity
   II. Entropy from gravitational action

III. Examples
   I. Schwarzschild BH
   II. Rindler
   III. De Sitter

IV. Summary

Black holes as thermodynamic objects

If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations-then so much the worse for Maxwell's equations. If it is found to be contradicted by observation-well these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.

Sir Arthur Stanley Eddington

Gifford Lectures (1927), *The Nature of the Physical World* (1928), 74.

Black holes and their characteristics

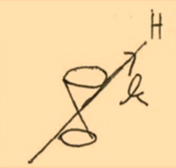
Schwarzschild black hole:

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2 \,, \qquad f = 1 - \frac{2m}{r}$$

- asymptotic mass (total energy)

$$M = -\frac{1}{8\pi}\int_{S^2_\infty} * dk = m \qquad k = \partial_t$$

- black hole horizon: (radius $r_+ = 2M$)



surface gravity $(k^b \nabla_b k^a)_{|H} = \kappa k^a_{|H}$

$$\Rightarrow \quad \kappa = \frac{f'(r_+)}{2} = \frac{M}{r_+^2} = \frac{M}{(2M)^2} = \frac{1}{4M} = \frac{1}{2r_+}$$

horizon area: $A = \int \sqrt{\det \gamma}\, d\theta d\varphi = \int r_+^2 \sin\theta\, d\theta d\varphi = 4\pi r_+^2$

Schwarzschild characteristics: summary

Horizon: $r_+ = 2M$

Mass: M

Surface gravity: $\kappa = \frac{f'(r_+)}{2} = \frac{1}{2r_+}$

Horizon area: $A = \int \sqrt{\det \gamma}\, d\theta d\varphi = 4\pi r_+^2 \,.$

Good idea:

$$dM = \frac{dr_+}{2}\,, \qquad dA = 8\pi r_+ dr_+$$

1st law of black hole mechanics:

$$dM = \frac{\kappa}{2\pi}\frac{dA}{4}$$

Laws of black hole mechanics

- Bardeen, Carter, Hawking (1973)

- **Zeroth law:** The surface gravity κ is constant on the black hole horizon.
- **First law:**

$$\boxed{dM = \frac{\kappa}{2\pi}\frac{dA}{4} + \underbrace{\Omega dJ + \Phi dQ}_{\text{work terms}}\,.} \tag{5.8}$$

  Here, Ω is the angular velocity of the black hole horizon, and Φ is its 'electrostatic potential'.

- **Second law:** Classically, the area of the horizon never decreases (provided the null energy condition holds).

$$\boxed{dA \geq 0\,.} \tag{5.9}$$

- **Third law:** It is impossible to reduce κ to zero in a finite number of steps.

- Essentially equivalent to **gravitational dynamics**
- Despite the resemblance with laws of TDs, **classical** BHs **are black**

Black hole thermodynamics?

$$dM = \frac{\kappa}{2\pi}\frac{dA}{4}$$

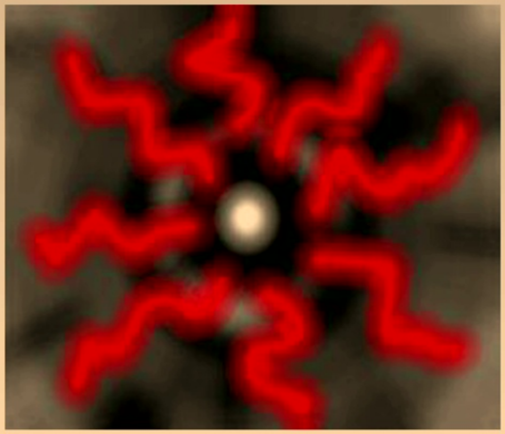
Bekenstein? ⟺

$$dE = TdS$$

Hawking (1974):

$$T = \frac{\kappa}{2\pi} \Rightarrow S = \frac{A}{4}$$

derived using QFT in
curved spacetime



Other approaches: Euclidean path integral approach (Gibbons & Hawking-1977), tunnelling, LQG, string theory,…

Black hole entropy

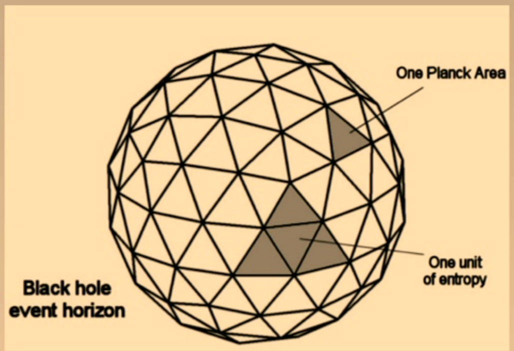
relativity

Stat. mech

$$S = \frac{A}{4}\frac{c^3 k_B}{\hbar G_N}.$$

gravity

QM



- Is huge: $$S = \frac{k_B}{4}\frac{A}{l_P^2}, \quad l_P = \sqrt{\frac{G\hbar}{c^3}}$$

- Is **holographic**: $$S \propto A$$

- Bekenstein's (universal) bound: $$S \leq \frac{A}{4}$$

Black hole evaporation

- Hawking temperature for Schwarzschild

$$T = \frac{\hbar c^3}{8\pi k_B G M} \propto \frac{1}{M} \qquad \sim 6 \times 10^{-8} \frac{M_\odot}{M} K$$

- Effective Stefan-Boltzmann law:

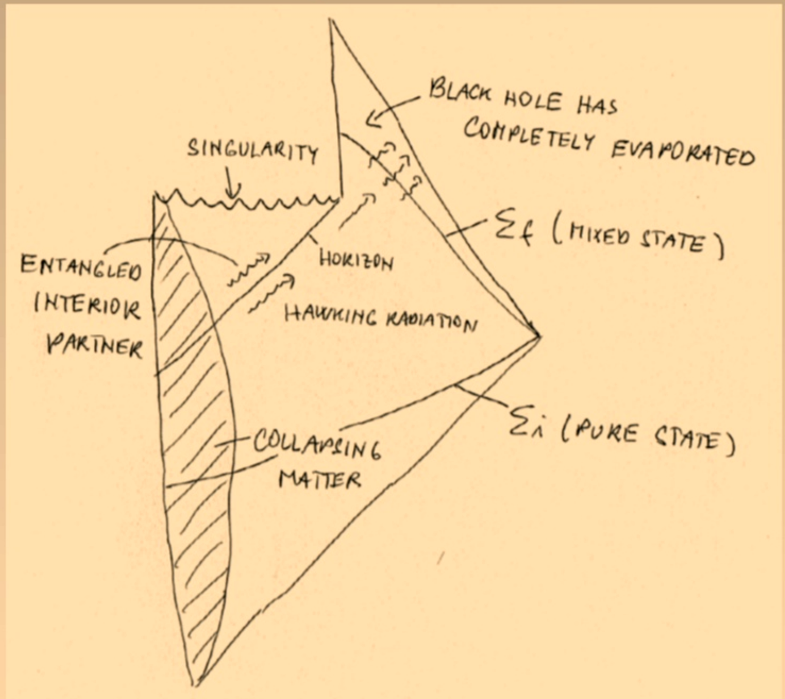
$$\frac{dM}{dt} \propto -\sigma T^4 A \propto -\frac{1}{M^2}$$

- BH completely evaporates $t_{\text{evap}} \approx \left(\frac{M}{M_\odot}\right)^3 \times 10^{71}\, s$

- Note also that:
  i) negative **specific heat** $C = T\frac{\partial S}{\partial T} = -\frac{1}{8\pi T^2}$

  ii) **Generalized 2nd law:** $dS_{\text{tot}} = d(S_{\text{BH}} + S_{\text{outside}}) \geq 0$

Black hole info paradox (Hawking 1976)



- Thermal Hawking radiation leads to black hole evaporation.
- If BH completely evaporates, we violated unitary evolution of QM (evolved from the pure state in the beginning to a mixed state at the end) – **info loss** (see later in the course).
- Contradicts the intuition from **AdS/CFT correspondence**

$$Z = \int Dg e^{-S_E[g]} \approx e^{-S_E(g_c)}$$

II. Euclidean magic

- G. Gibbons, S. Hawking, *Action integrals and partitions functions in quantum gravity*, Phys. Rev D 15, 2752, 1977.
- G. Gibbons and S. Hawking, *Cosmological event horizons, thermodynamics, and particle creation*, Phys. Rev D 15, 2738, 1977.

Euclidean trick (Gibbons & Hawking 1977)

- **Thermal Green functions** have periodicity in **Euclidean** time

$$\tau = it$$

$$G(\tau) = G(\tau + \beta)\,, \quad \beta = 1/T\,.$$

(Conversely, periodicity of G defines a thermal state. A thermometer interacting with the given field for a long time will register this temperature.)

- **Quantum fields** in the vicinity of black holes have this property (as seen by distant static observers).

Euclidean trick (Gibbons & Hawking 1977)

• **Thermal Green functions** have periodicity in **Euclidean** time

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• **Quantum fields** in the vicinity of black holes have this property (as seen by distant static observers).

• What about the **gravitational field** itself? Consider **Euclideanized Schwarzschild**:

$$ds^2 = f\,d\tau^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

Euclidean trick (Gibbons & Hawking 1977)

- **Near horizon** expand:

$$f = \underbrace{f(r_+)}_{0} + \underbrace{(r - r_+)}_{\Delta r}\underbrace{f'(r_+)}_{2\kappa} + \cdots = 2\kappa\Delta r$$

$$ds^2 = 2\kappa\Delta r d\tau^2 + \frac{dr^2}{2\kappa\Delta r} + r_+^2 d\Omega^2$$

- Change variables:

$$d\rho^2 = \frac{dr^2}{2\kappa\Delta r} \quad \Leftrightarrow \quad d\rho = \frac{dr}{\sqrt{2\kappa\Delta r}} \quad \Leftrightarrow \quad \Delta r = \frac{\kappa}{2}\rho^2$$

$$ds^2 = \kappa^2\rho^2 d\tau^2 + d\rho^2 + r_+^2 d\Omega^2 = \rho^2 d\varphi^2 + d\rho^2 + \dots .$$

$\varphi = \kappa\tau$ … looks like **flat space** in polar provided:

φ has a period 2π.

(otherwise conical singularity exists at ρ=0)

Euclidean trick (Gibbons & Hawking 1977)

- **Original manifold** non-singular:

$$\varphi \sim \varphi + 2\pi \quad \Leftrightarrow \quad \tau \sim \tau + \underbrace{2\pi/\kappa}_{\beta} \quad \Leftrightarrow \quad \boxed{T = \frac{\kappa}{2\pi}\,,}$$

… which is the **Hawking's temperature**.

Gravitational partition function

$$\boxed{Z = \int Dg e^{-S_E[g]} \approx e^{-S_E(g_c)}}$$

(using WKB approximation)

- **Free energy**:

$$F = -\frac{1}{\beta}\log Z \approx \frac{S_E}{\beta} \Rightarrow S = -\frac{\partial F}{\partial T}$$

Euclidean trick (Gibbons & Hawking 1977)

- **Gravitational action**:

$$S_E = \int_\Omega \frac{d^4x\sqrt{g}R}{16\pi G} + \int_{\partial\Omega} \frac{d^3x\epsilon\sqrt{h}\mathcal{K}}{8\pi G} + \text{counter terms}$$

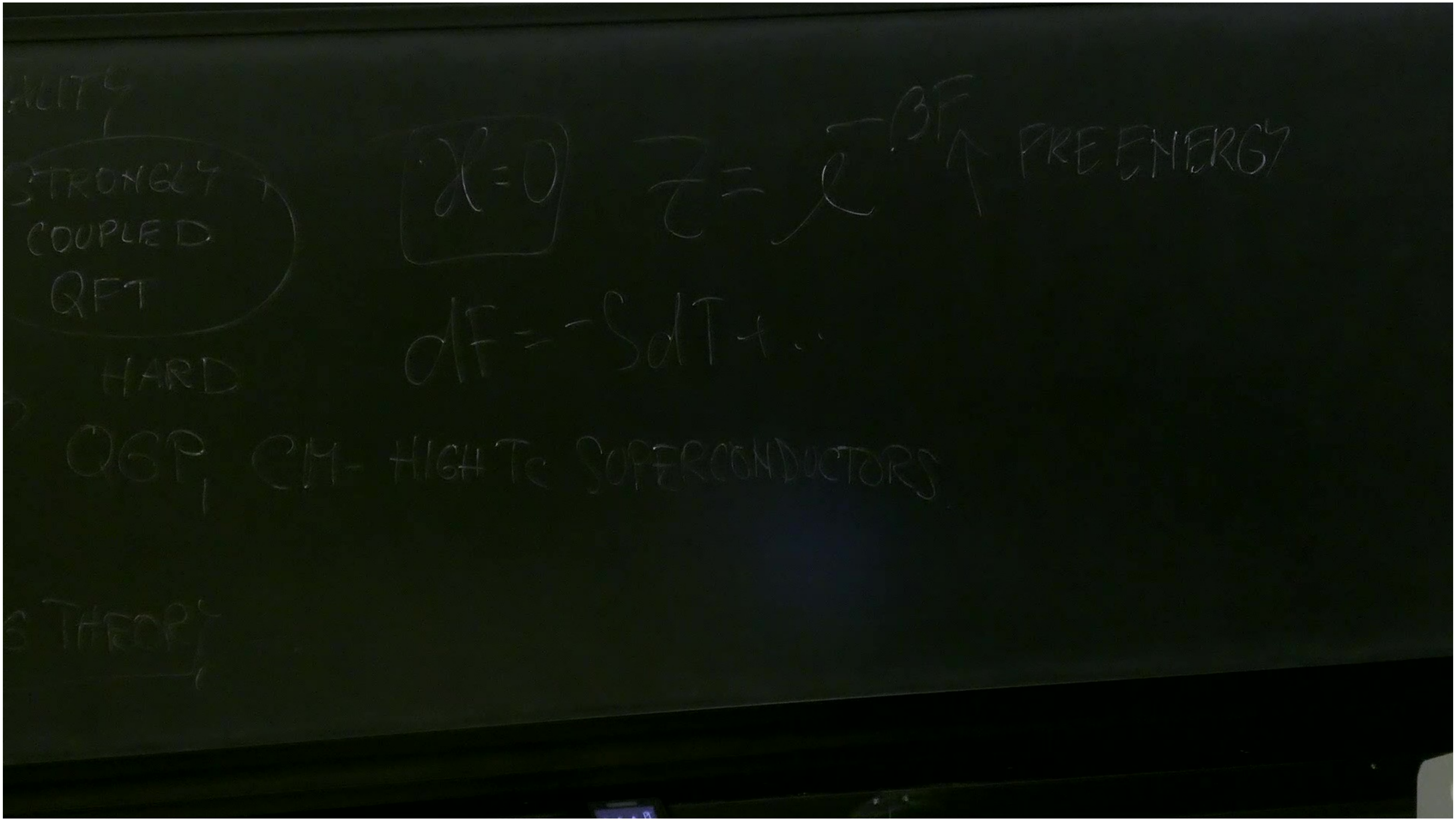
Einstein-Hilbert action
(gives Einstein equations)

York-Gibbons-Hawking term
(yields well posed variational principle with Dirichlet BCs)

Counter terms: "renormalize" the value of the action
(In AdS given covariantly by *holographic renormalization*. In flat space no covariant prescription exists!)

- The prescription confirms **Bekenstein's area law!**

$$S = -\frac{\partial F}{\partial T} = \frac{A}{4}$$


STRONGLY
COUPLED
QFT
HARD
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QGP, CM - HIGH Tc SUPERCONDUCTORS