Title: Lecture - Cosmology, PHYS 621

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Collection/Series: Cosmology (Elective), PHYS 621, March 31 - May 2, 2025

Subject: Cosmology

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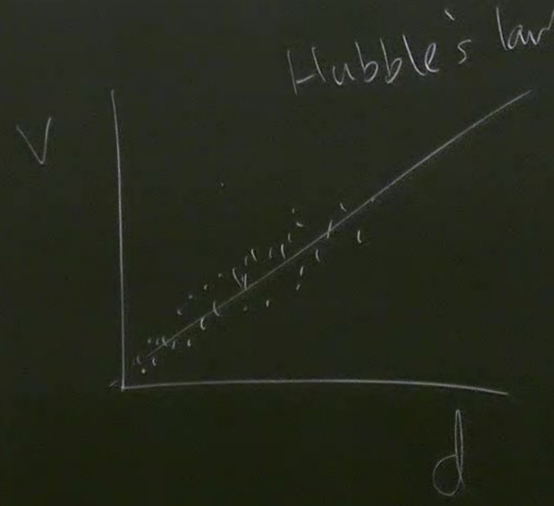
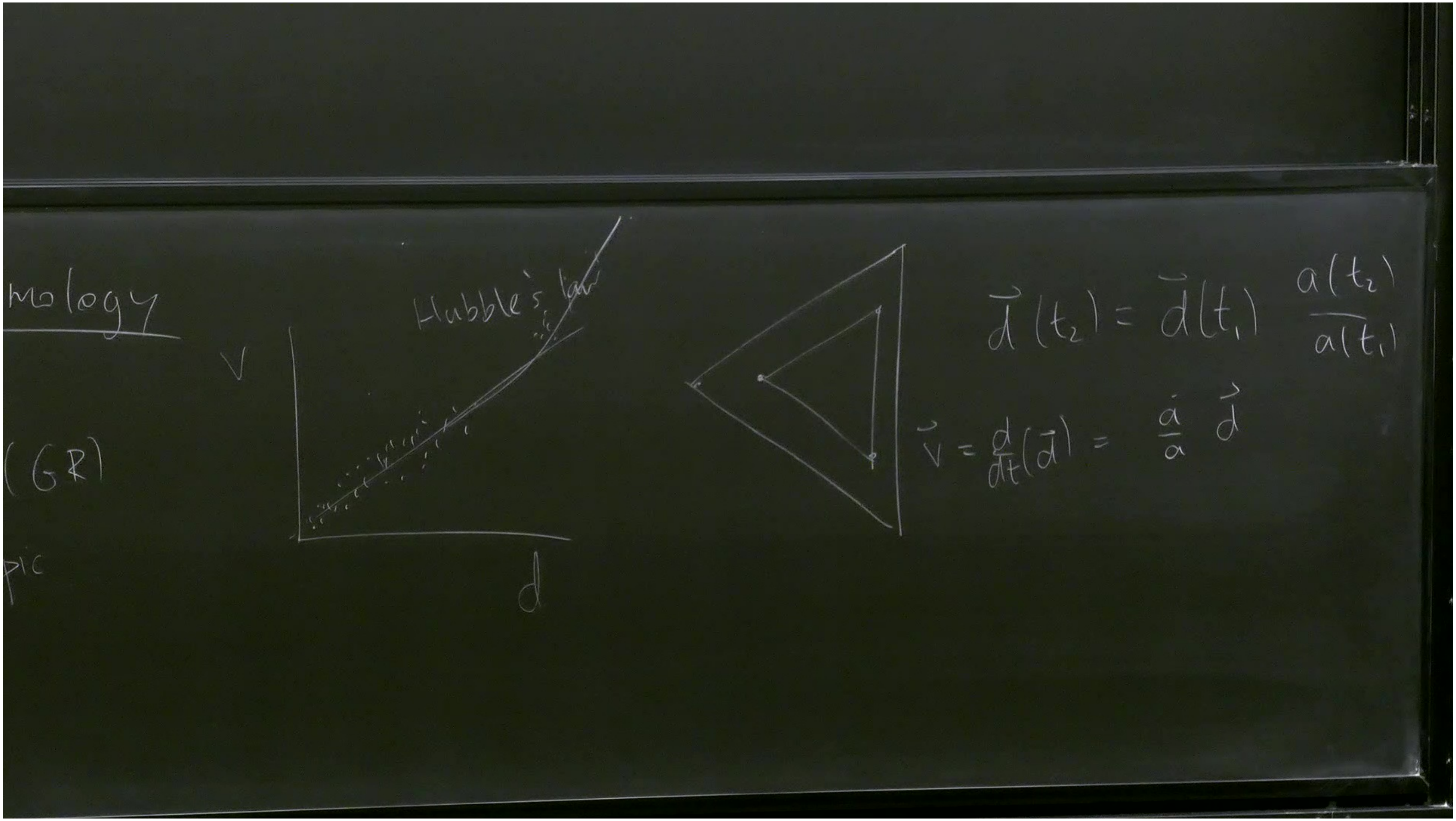
Review of Classical Cosmology

Facts :

1. gravity dominant (GR)
2. homogenous & isotropic
3. universe expanding




mology
v
(GR)
pic
Hubble's law
d
$\vec{d}(t_2) = \vec{d}(t_1) \frac{a(t_2)}{a(t_1)}$
$\vec{v} = \frac{d}{dt}(\vec{d}) = \frac{\dot{a}}{a} \vec{d}$

A $\longrightarrow$ B

$$dv = H\,dx = \frac{\dot{a}}{a}\,dx$$

$$\vec{p},\ E \Rightarrow v = p/E$$

$$dt = \frac{dx}{v} = \frac{E\,dx}{p}$$

$$dp = -E\,dv = -E H\,dx$$

$$\dot{p} = \frac{dp}{dt} = -Hp = -\frac{\dot{a}}{a}\,p$$

$$\frac{dp}{p} = -\frac{da}{a} \Rightarrow p \propto a^{-1}$$

$E\,dv = -EH\,dx$

$Hp = -\frac{\dot{a}}{a}\,p$

$\Rightarrow p \propto a^{-1}$

1. peculiar vel's decay

2. cosmological redshifts

$$p_{obs} = p_{emit}\,\frac{a_{emit}}{a_{obs}}$$

$$\nu_{obs} = \nu_{emit}\,\frac{a_e}{a_0}$$

$a = 1$ today

$1+z = \frac{1}{a}$

$$\lambda_{obs} = \lambda_{emit}\left(1+z_{emit}\right)$$

P α

FRW spacetime

4 dim: t, r, θ, φ

$$ds^2 = -dt^2 + a^2(t)\, d\mathbf{x}^2$$

$$d\mathbf{x}^2 = \gamma_{ij}\, dx^i dx^j$$

$$= -\frac{da}{a} \Rightarrow p \propto a^{-1}$$

$$dx^2 = dr^2 + D^2(r)\, d\Omega^2$$

$$D(r) = \begin{cases} R\sin(r/R) & \text{positive} \\ r & \text{flat} \\ R\sinh(r/R) & \text{negative} \end{cases}$$

$$= R\, k^{-1/2} \sin\left(k^{1/2} \frac{r}{R}\right) \qquad k = \begin{matrix} +1 \\ 0 \\ -1 \end{matrix}$$

$$-\frac{da}{a} \Rightarrow p \propto a^{-1}$$

$$dx^2 = dr^2 + D^2(r)\, d\Omega^2 \qquad r = R \sin^{-1}\left(\frac{D}{R}\right)$$

$$D(r) = \begin{cases} R \sin(r/R) & \text{positive} \\ r & \text{flat} \\ R \sinh(r/R) & \text{negative} \end{cases} \qquad dr = \frac{dD}{\sqrt{1 - \frac{D^2}{R^2}}}$$

$$= R\, k^{-1/2} \sin\left(k^{1/2} \frac{r}{R}\right) \qquad k = \begin{matrix} +1 \\ 0 \\ -1 \end{matrix}$$

$$ds^2 = -dt^2 + a^2\left[dr^2 + D^2\, d\Omega^2\right]$$

$$= -dt^2 + a^2\left[\frac{dD^2}{1-k\left(\frac{D}{R}\right)^2} + D^2\, d\Omega^2\right]$$

$$ds^2 = -dt^2 + a^2\left[dr^2 + D^2 d\Omega^2\right]$$

$$= -dt^2 + a^2\left[\frac{dD^2}{1-k\left(\frac{D}{R}\right)^2} + D^2 d\Omega^2\right]$$

- t, D, θ, ϕ
- $t, D, \theta = 0, \phi$

$$ds^2 = -dt^2 + a^2\left[dr^2 + D^2 d\Omega^2\right]$$

$$= -dt^2 + a^2\left[\frac{dD^2}{1-k\left(\frac{D}{R}\right)^2} + D^2 d\Omega^2\right]$$

t_e t_{obs}

$$dt = a(t)\,dr$$

$$r = \int dr = \int \frac{dt}{a(t)}$$

$$\frac{dp^\mu}{dt} = \frac{\partial p^\mu}{\partial t} + \Gamma^\mu{}_{0\alpha}\, p^\alpha$$

$$\Gamma^\mu{}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu}\left(\partial_\beta g_{\nu\alpha} + \partial_\alpha g_{\nu\beta} - \partial_\nu g_{\alpha\beta}\right)$$

$$\Gamma^r{}_{0\alpha} = \frac{1}{2} \underbrace{g^{rr}}_{=\frac{1}{a^2}}\left(\partial_\alpha g_{r0} + \underbrace{\partial_0 g_{r\alpha}}_{=2a\dot{a}\,\delta_{r\alpha}} - \partial_r g_{0\alpha}\right)$$

$g_{00} = -1$

$g_{rr} = a^2(t)$

$g_{\theta\theta} = a^2 D^2$

$g_{\phi\phi} = a^2 D^2 \sin^2\theta$

$$\frac{\partial p^r}{\partial t} + \frac{\dot{a}}{a}\delta_{r\alpha}p^\alpha = \frac{\partial p^r}{\partial t} + \frac{\dot{a}}{a}p^r = 0$$

$$\frac{\dot{p}^r}{p^r} = -\frac{\dot{a}}{a} \Rightarrow p^r \propto \frac{1}{a}$$

$in^2\theta$

$\frac{1}{a^2}$ $\quad 2a\dot{a}\,\delta_{ra}$

Einstein eqns: $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ $\qquad \mu = 0$

density & flux of energy & momentum

perfect fluid: $T^{\mu\nu} = (\rho + P)\, u^\mu u^\nu + P g^{\mu\nu}$

work in frame where $u^\mu = (1, 0, 0, 0)$

$\nabla_\nu T^{\mu\nu} = 0$

$$T^\mu{}_\nu = \begin{pmatrix} -\rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$$

$\mu = 0$: continuity eqn

$\mu = i$: Euler eqns

$$\frac{d\rho}{dt} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

suppose $P = w\rho$

$$\frac{d\rho}{dt} + 3\frac{\dot{a}}{a}(1+w)\rho = 0 \Rightarrow \frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}$$

$$\Rightarrow \rho \propto a^{-3(1+w)}$$

const w