Title: Generic uniqueness, marginal entanglement, and entanglement transitivity

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Collection/Series: Quantum Foundations

Subject: Quantum Foundations

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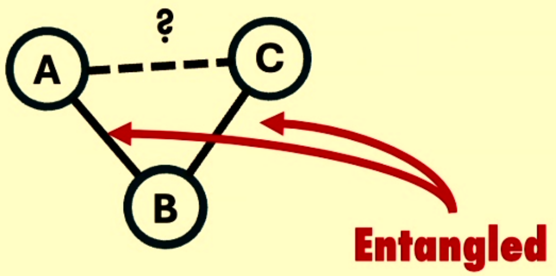
Abstract:

The quantum marginal problem concerns the compatibility of given reduced states. In contrast, the entanglement transitivity problem takes compatible entangled marginals as input and ask if one can infer therefrom the entanglement of some other marginals. When this is possible, the input marginals are said to exhibit entanglement transitivity. Previous studies [Npj Quantum Inf 8, 98 (2022)] have demonstrated that certain families of states show entanglement transitivity. In this talk, we will show that when specific dimension constraints are satisfied, entanglement transitivity is possible and even generic among the marginals of pure state. To this end, we use the fact that given these constraints, the marginals of generic pure states (1) uniquely determine the global state and (2) are entangled. For the latter, our results generalize that of Aubrun et al. [Comm. Pure. Appl. Math. 67, 129 (2013)], which allows us to conclude further that sufficiently large parts of a generic multipartite pure state are entangled for any bipartition.

Generic uniqueness, marginal entanglement, and entanglement transitivity



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Quantum Foundations Seminar, Perimeter Institute for Theoretical Physics, 2025.02.27

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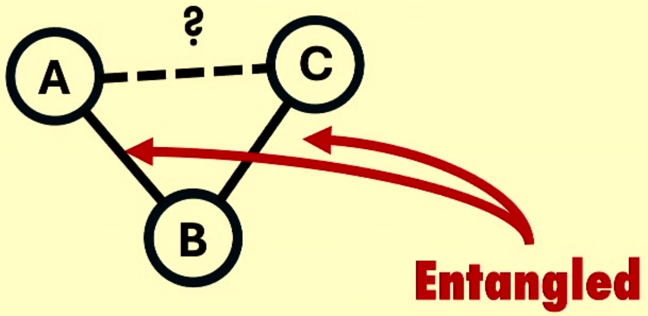
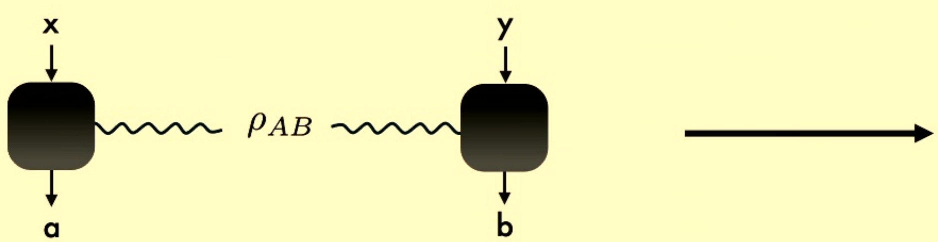
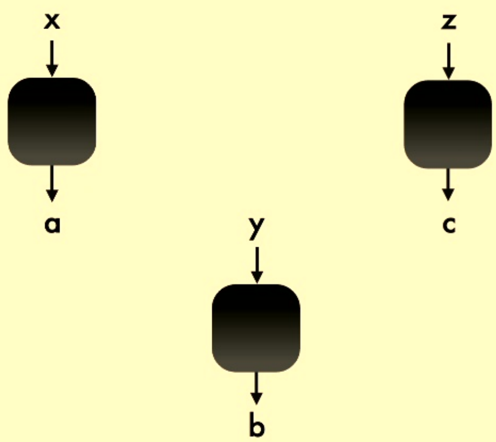
Nonlocality
Transitivity





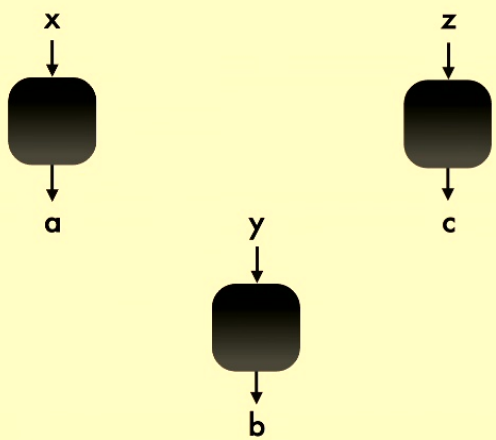
Correlation

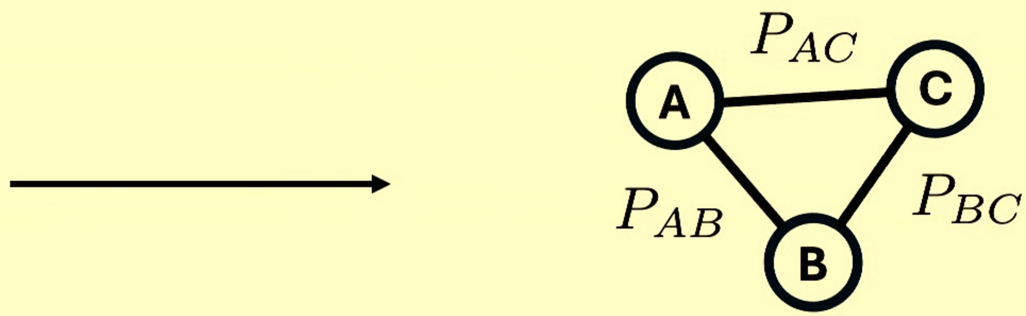
$$P_{AB} = \{p(a,b|x,y)\}$$



$$P_{ABC} = \{p(a,b,c|x,y,z)\}$$

Marginal $$P_{AB} = \{p(a,b|x,y)\}$$



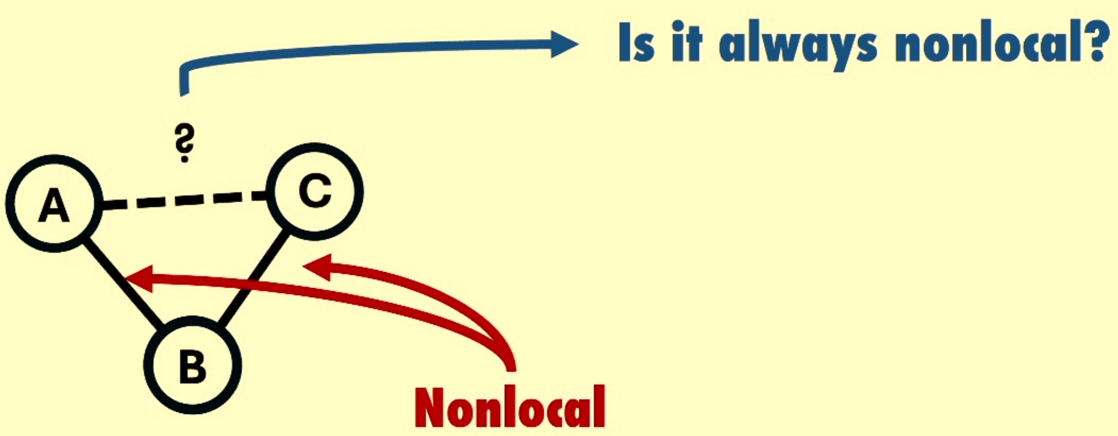


$$P_{ABC} = \{p(a, b, c|x, y, z)\}$$

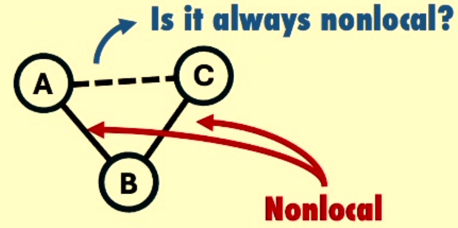
Nonlocality Transitivity

P_{AB}, P_{BC} exhibit nonlocality transitivity if and only if

Compatible nonlocal $P_{AB}, P_{BC} \longrightarrow P_{AC}$ must be nonlocal



Nonlocality Transitivity

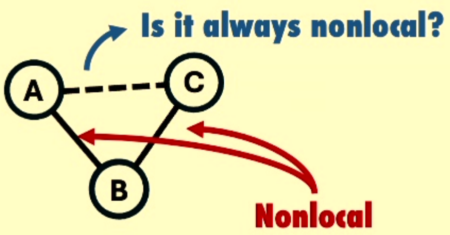


Post-quantum example in (3,4,2) Phys. Rev. Lett. 107, 100402 (2011)

Quantum examples are yet to be found!

- Nonlocality transitivity as a feature that separates quantum theory from other foil theories?

Nonlocality Transitivity



Post-quantum example in (3,4,2) Phys. Rev. Lett. 107, 100402 (2011)

Quantum examples are yet to be found!

- What are the necessary conditions for nonlocality transitivity?

Entanglement Transitivity !

Nonlocality Transitivity

Compatible
nonlocal P_{AB}, P_{BC}

↓

P_{AC} **must be nonlocal**

Entanglement Transitivity

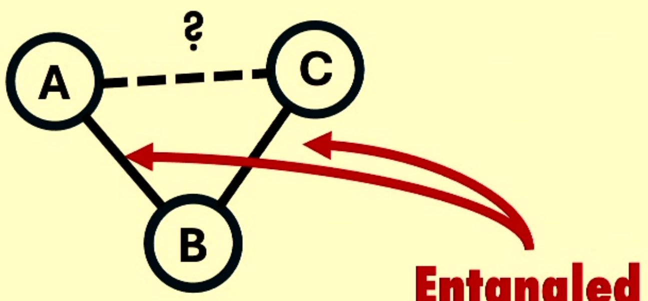
Npj Quantum Inf
8, 98 (2022)

Compatible
entangled ρ_{AB}, ρ_{BC}

↓

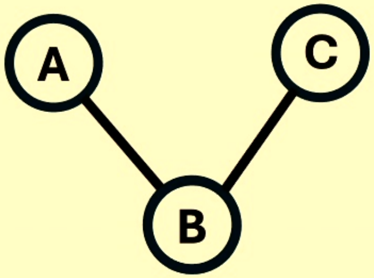
ρ_{AC} **must be entangled**

Entanglement Transitivity



Entanglement Transitivity: Examples

Entangled ρ_{AB}, ρ_{BC} that are compatible



Entanglement monogamy: $\rho_{AB}, \rho_{BC} \neq |\Phi^+\rangle\langle\Phi^+|$

Entanglement Transitivity: Examples

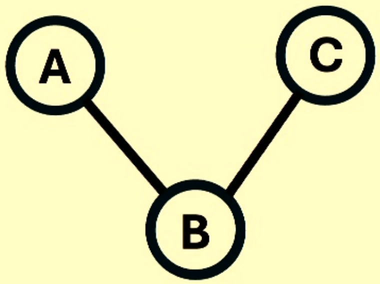
Entangled ρ_{AB}, ρ_{BC} that are compatible

$$|\Psi\rangle_{ABC} = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$

$$\downarrow$$

$$\rho_{AB} = \rho_{BC} = \frac{2}{3}\,|\Psi^+\rangle\langle\Psi^+| + \frac{1}{3}\,|00\rangle\langle 00|$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$



Entanglement Transitivity: Examples

Should all compatible AC marginals be entangled?

$$\rho_{AC} = \frac{2}{3} \left|\Psi^+\right\rangle\!\left\langle\Psi^+\right| + \frac{1}{3} |00\rangle\!\langle 00|$$

This is the only compatible state!

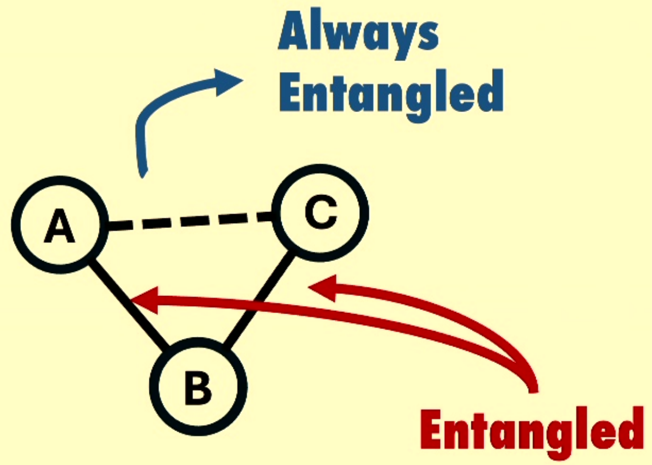
ρ_{AB}, ρ_{BC} uniquely determine the AC marginal.

Entanglement Transitivity: Examples

$$|\Psi\rangle_{ABC} = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$

$$\downarrow$$

$$\rho_{AB} = \rho_{BC} = \frac{2}{3}|\Psi^+\rangle\langle\Psi^+| + \frac{1}{3}|00\rangle\langle 00|$$



AB and BC are said to exhibit entanglement transitivity.

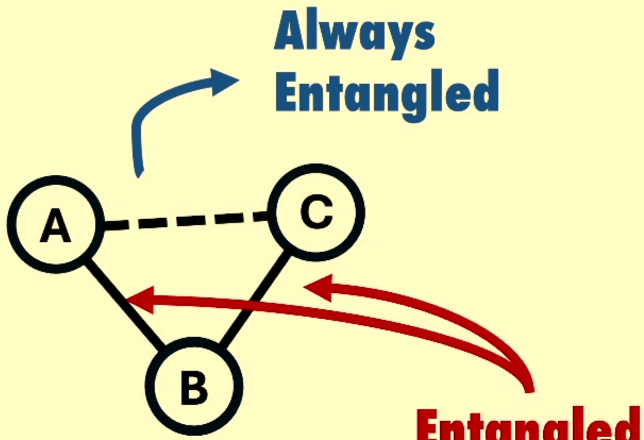
Entanglement Transitivity Problems
npj Quantum Inf 8, 98 (2022)

Entanglement Transitivity: Confusion

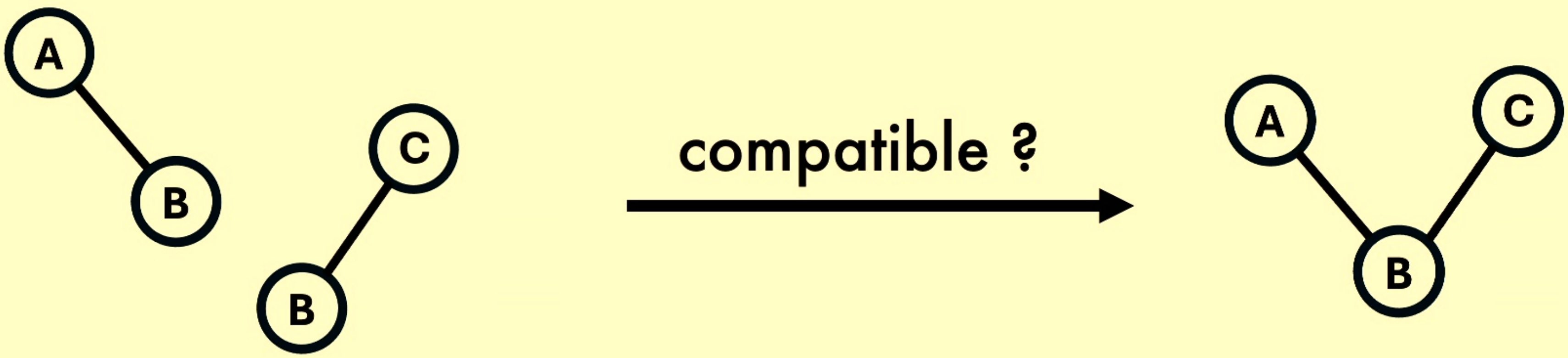
$\rho_{ABC} \rightarrow \rho_{AB}, \rho_{AC}, \rho_{BC}$ are entangled

Do AB and BC exhibit entanglement transitivity ?

Not really!



Quantum Marginal Problems with overlaps



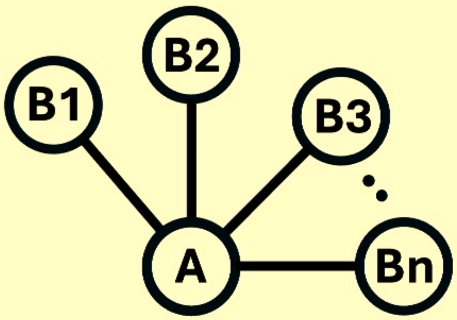
Entanglement monogamy: $\rho_{AB}, \rho_{BC} \neq |\Phi^+\rangle\langle\Phi^+|$

Consistent overlap does not guarantee compatibility.

Quantum Marginal Problems with overlaps

Is AB n-shareable ?

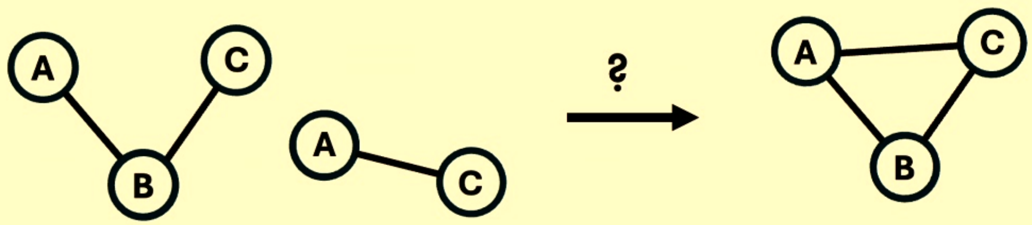
$$\rho_{AB} = \rho_{AB_1} = \rho_{AB_2} = \cdots = \rho_{AB_n}$$



AB is n-shareable for any n $\longleftrightarrow$ AB is separable

Phys. Rev. A 69, 022308 (2004)

Quantum Marginal Problems: Uniqueness



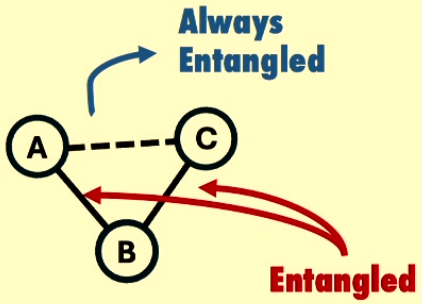
Existence Does there exist an AC that is compatible with AB and BC?

Uniqueness Is AC compatible with AB and BC unique?

Phys. Rev. Lett. 89, 277906 (2002)

Is entanglement transitivity generic ?

Numerical results suggests that it is for three-body pure states with $d_A = d_B = d_C$



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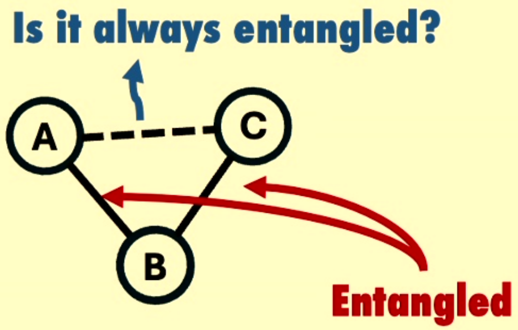
Generic Entanglement Transitivity: Proof

1. A generic pure state ABC is uniquely determined by AB and BC

2. The reduced state AC of a generic pure state is almost surely entangled



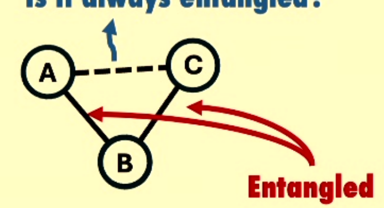
Generic Entanglement Transitivity: Proof

1 A generic pure state ABC is uniquely determined by AB and BC

- There is only one compatible AC marginal.

$$|\psi\rangle_{ABC} \longrightarrow \rho_{AB}, \rho_{BC}, \rho_{AC}$$

$$\sigma_{ABC} \longrightarrow \rho_{AB}, \rho_{BC}, \sigma_{AC}$$

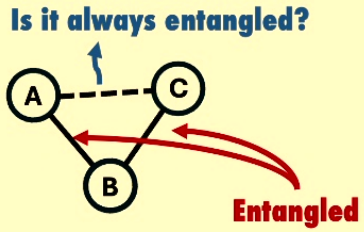


ρ_{AB}, ρ_{BC} uniquely determine $|\psi\rangle_{ABC} \rightarrow \rho_{AC} = \sigma_{AC}$

Generic Entanglement Transitivity: Proof

1 A generic pure state ABC is uniquely determined by AB and BC

- There is only one compatible AC marginal.
- Sufficient conditions:



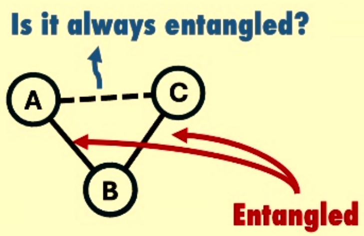
$$d_B \geq \min\{d_A, d_C\}$$ Phys. Rev. A 88, 012109 (2013)

$$d_B \geq 2, d_A = d_C$$ Sci. China Phys. Mech. Astron. 61, 1 (2018)

Generic Entanglement Transitivity: Proof

2 The reduced state AC is almost surely entangled

- Sufficient condition: $d_B \leq (d_A - 1)(d_C - 1)$
- The reduced state AC of a generic pure state is supported on a completely entangled subspace.



J. Phys. A: Math. Theor. 41 375305 (2007)
Commun. Pure Appl. Math. 67, 129 (2014)

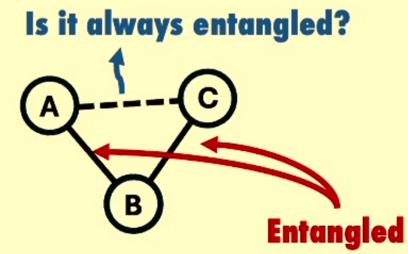
Generic Entanglement Transitivity: Result

1. A generic pure state ABC is uniquely determined by AB and BC if... $d_B \geq \min\{d_A, d_C\}$ or $d_B \geq 2, d_A = d_C$

2. The reduced state AC of a generic pure state is almost surely entangled if... $d_B \leq (d_A - 1)(d_C - 1)$

→ AB and BC almost surely exhibit entanglement transitivity

Entanglement transitivity is generic in any tripartite closed system with equal finite local dimensions.



Multipartite Entanglement Transitivity

$\lfloor \frac{N}{2} \rfloor$ of the m-body reduced states $m \geq \lceil \frac{N}{2} \rceil + 1$

of a generic N-body pure state with equal finite local dimensions

exhibit entanglement transitivity

4 qudits:

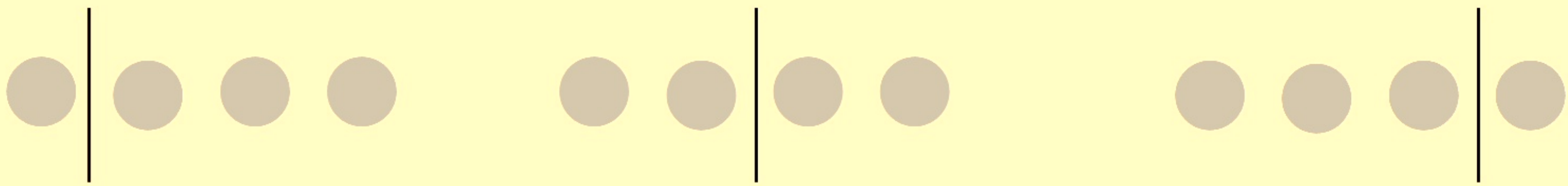
ABC, BCD entangled ⟶ ABD, ACD, ...
must be entangled

Generic multipartite entanglement

Reduced states ρ_{AC} of Haar-random pure states are entangled if they are large enough. $d_B \leq (d_A - 1)(d_C - 1)$

$$d_A = k \qquad d_C = m - k$$

m-body reduced states of a generic N-body pure state are entangled with respect to every bipartition if $m \geq \lceil \frac{N}{2} \rceil + 1$



Generic Entanglement

High-dimensional Haar-random pure states $|\psi\rangle_{AB} \in \mathbb{C}^d \otimes \mathbb{C}^d$ are close to being maximally entangled.

Aspects of Generic Entanglement, Commun. Math. Phys. 265, 95 (2006)

$$\Pr\{S(\rho_A) \leq \log_2 d - K\} < 4^{-d}$$

$$\rho_A = \operatorname{tr}_B |\psi\rangle\langle\psi|_{AB}$$

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Generic Entanglement

Reduced states ρ_{AC} of Haar-random pure states $|\psi\rangle_{ABC}$ are

- close to being separable if they are small enough

  Commun. Pure Appl. Math. 67, 129 (2014)

$$\Pr\{\rho_{AC} \text{ is separable}\} \geq 1 - 2\exp(-cd_B) \quad \text{if} \quad d_B \geq f(d)$$

- entangled if they are large enough

  J. Phys. A: Math. Theor. 41, 375305 (2008)

$$d_B \leq (d_A - 1)(d_C - 1)$$

Large parts are generically entangled

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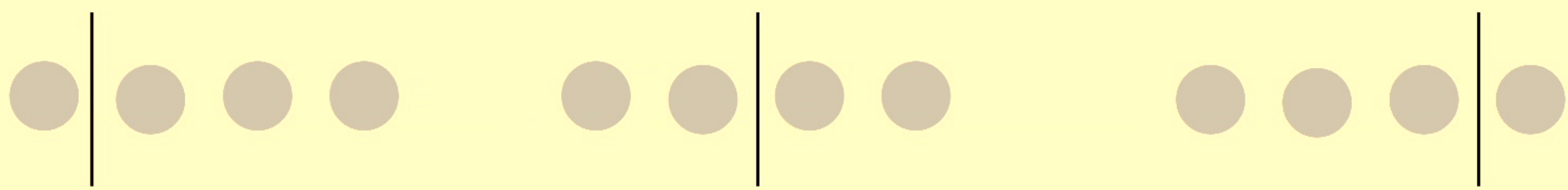
Generic multipartite entanglement

Reduced states ρ_{AC} of Haar-random pure states are entangled if they are large enough. $d_B \leq (d_A - 1)(d_C - 1)$

$d_A = k \qquad d_C = m - k$

m-body reduced states of a generic N-body pure state are entangled with respect to every bipartition if $m \geq \lceil \frac{N}{2} \rceil + 1$



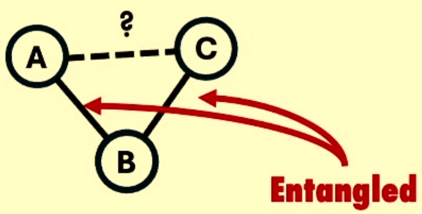
Take-home Messages

Entanglement can be transitive, and the phenomenon is even generic.

Entanglement transitivity is generic in any tripartite closed system with equal finite local dimension !

Large parts are generically entangled with respect to any bipartition.

Thank You



Entanglement Transitivity Problems
Npj Quantum Inf 8, 98 (2022)

Resource Marginal Problems
Quantum 8, 1353 (2024)

Nonlocality of Quantum States can be Transitive
arXiv:2412.10505

Large Parts are Generically Entangled
arXiv:25??.ABCDE

Multipartite Entanglement Transitivity

$\lfloor \frac{N}{2} \rfloor$ of the m-body reduced states $m \geq \lceil \frac{N}{2} \rceil + 1$

of a generic N-body pure state with equal finite local dimensions exhibit entanglement transitivity

4 qudits:

ABC, BCD entangled ⟶ ABD, ACD, ... must be entangled

Entanglement Transitivity: Examples

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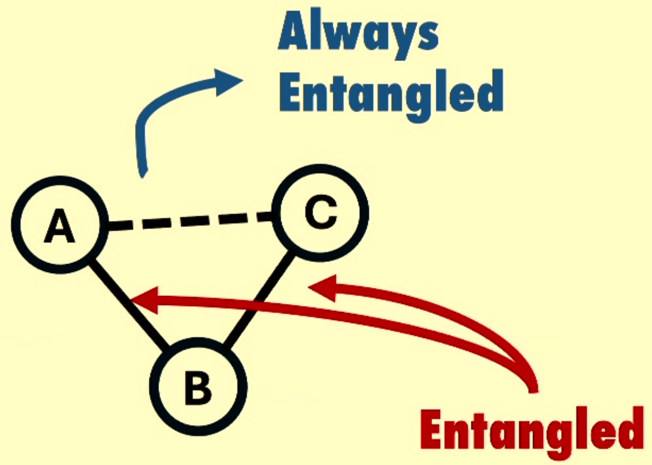

AB and BC are said to exhibit entanglement transitivity.

Entanglement Transitivity Problems
npj Quantum Inf 8, 98 (2022)