Title: Abelian Instantons

Speakers: John Terning

Collection/Series: Particle Physics

Subject: Particle Physics

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Abstract:

It is usually assumed that 4D instantons can only arise in non-Abelian theories. One can, however, explicitly construct instantons for QED in the background of a Dirac monopole. This is the low-energy effective field theory for fermions interacting with a 't Hooft-Polyakov monopole. This theory posesses both a topological instanton number and 't Hooft zero modes. I will show how such instantons provide the underlying mechanism for the Callan-Rubakov process: monopole-catalyzed baryon decay with a cross section that saturates the unitarity bound.

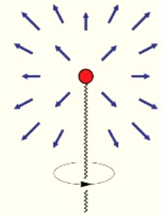
Abelian Instantons

John Terning
UC Davis

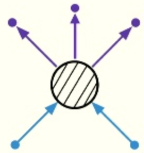
Csaba Csáki, Rotem Ovadia, Ofri Telem,
JT, Shimon Yankielowicz, hep-th/2406.13738

Outline

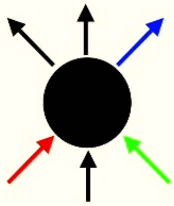
Monopole Review



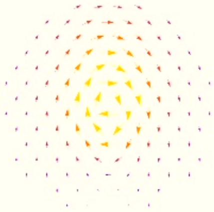
Wigner's Monopole Friend



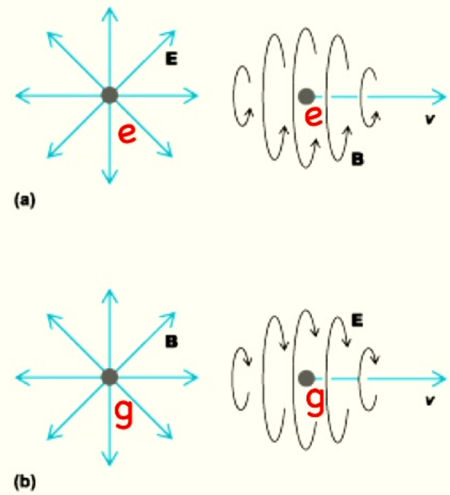
Rubakov Callan



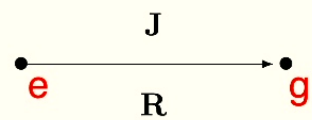
Instantons



J.J. Thomson

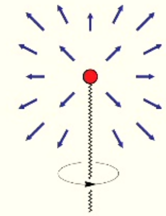


$J = e\ g$

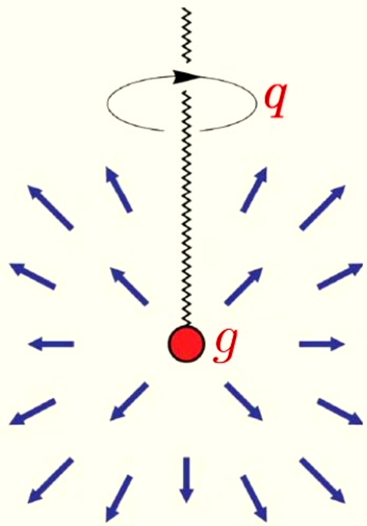


Philos. Mag. 8 (1904) 331

Dirac Charge Quantization



$$\vec{A}(\vec{r}) = \frac{g}{r}\,\frac{\vec{r}\times\vec{n}}{r - \vec{r}\cdot\vec{n}}$$



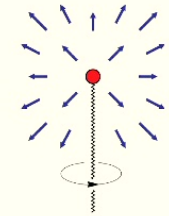
charge quantization

$$q\,g = \frac{n}{2}$$

otherwise the string is physical
and the monopole is confined

Proc. Roy. Soc. Lond. A133 (1931) 60

Monopole QM



$\vec{L} = \vec{r} \times \vec{p}$ does not satisfy $[L_i, L_j] = i\epsilon_{ijk} L_k$

$\vec{L} = \vec{r} \times \vec{p} + eg\hat{r}$ does

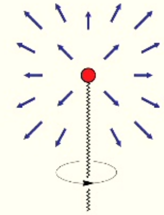
Dirac quantization

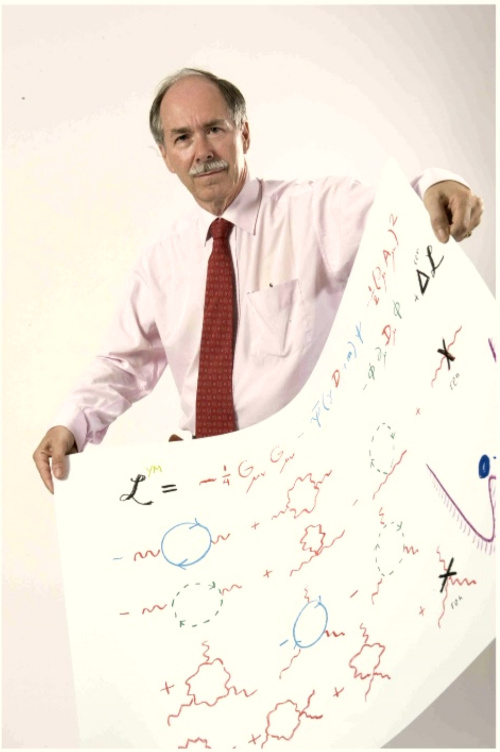
$$e_1\, g_2 - e_2\, g_1 = \frac{n}{2}$$

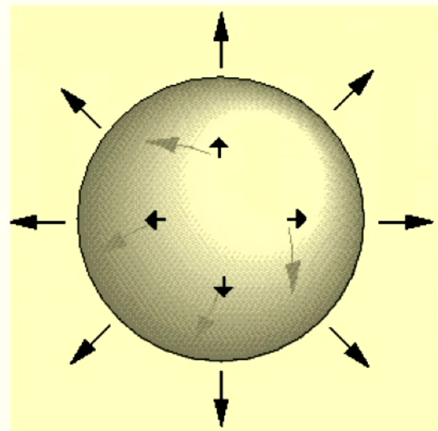
is angular momentum quantization

Lipkin, Weisberger, Peshkin Annals Phys. 53 (1969) 203
Schwinger Science 165 (1969) 757
Zwanziger Phys. Rev. 176 (1968) 1489

‘t Hooft-Polyakov





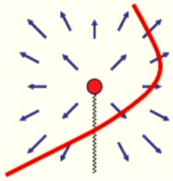


topological monopoles
every GUT predicts monopoles

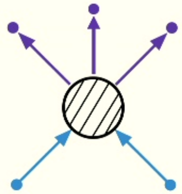
Nucl. Phys., B79 1974, 276
JETP Lett., 20 1974, 194

Three Problems with Monopoles

Weinberg Paradox: Lorentz violating poles

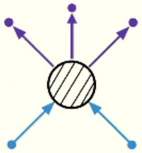


Multiparticle States are not tensor products of Wigner's 1-particle states



Unitarity Puzzle: Callan's "half-particles" or gauge charge violation

Wigner's Little Group



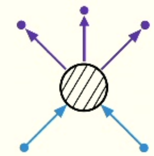
massive: labeled by spin
rest frame invariant
under 3D rotations

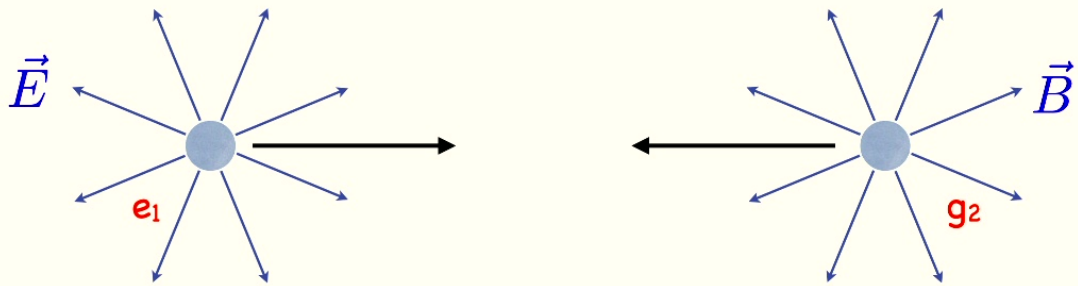
massless: labeled by helicity
momentum invariant
under rotation around
momentum axis

boosted states can transform by a
rotation that leaves the momentum fixed

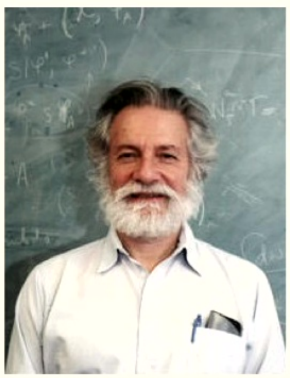
Annals Math **40** (1939) 149

Wigner's Monopole Friend





rotations about COM axis leave system invariant



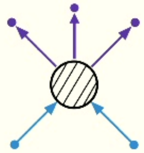
when $h_{12} = e_1\, g_2 - e_2\, g_1 \neq 0$

there is an extra phase from J in field

Zwanziger Phys. Rev. D6 (1972) 458

Pairwise Little Group phase
hep-th/2010.13794

$\flat$ Momenta



for each pairwise helicity:

$$\left(k_{ij}^{\flat\pm}\right)_\mu = p_c\,(1,0,0,\pm 1) \qquad p_c = \sqrt{\frac{(p_1\cdot p_2)^2 - m_1^2 m_2^2}{s}}$$

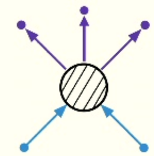
$$p_{ij}^{\flat\pm} = L_p\, k_{ij}^{\flat\pm}$$

$$p_i = \frac{1}{2p_c}\left[(E_i^c + p_c)\, p_{ij}^{\flat+} + (E_i^c - p_c)\, p_{ij}^{\flat-}\right]$$

$$p_j = \frac{1}{2p_c}\left[\left(E_j^c + p_c\right) p_{ij}^{\flat-} + \left(E_j^c - p_c\right) p_{ij}^{\flat+}\right]$$

Kosower hep-th/0406175

Pairwise Spinors



$$\left|k_{ij}^{\flat+}\right\rangle_\alpha = \sqrt{2\,p_c}\begin{pmatrix}1\\0\end{pmatrix} \quad , \quad \left|k_{ij}^{\flat-}\right\rangle_\alpha = \sqrt{2\,p_c}\begin{pmatrix}0\\1\end{pmatrix}$$

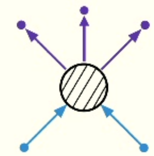
$$\left|p_{ij}^{\flat\pm}\right\rangle_\alpha = (L_p)_\alpha{}^\beta \left|k_{ij}^{\flat\pm}\right\rangle_\beta \quad , \quad \left[p_{ij}^{\flat\pm}\right|_{\dot\alpha} = \left[k_{ij}^{\flat\pm}\right|_{\dot\beta}\left(\tilde{L}_p\right)^{\dot\beta}{}_{\dot\alpha}$$

$$p_{ij}^{\flat\pm}\cdot\sigma_{\alpha\dot\alpha} = \left|p_{ij}^{\flat\pm}\right\rangle_\alpha \left[p_{ij}^{\flat\pm}\right|_{\dot\alpha}$$

$$\Lambda_\alpha{}^\beta \left|p_{ij}^{\flat\pm}\right\rangle_\beta = e^{\pm\frac{i}{2}\phi(p_i,p_j,\Lambda)} \left|\Lambda p_{ij}^{\flat\pm}\right\rangle_\alpha \quad , \quad \left[p_{ij}^{\flat\pm}\right|_{\dot\beta} \tilde{\Lambda}^{\dot\beta}{}_{\dot\alpha} = e^{\mp\frac{i}{2}\phi(p_i,p_j,\Lambda)} \left[\Lambda p_{ij}^{\flat\pm}\right|_{\dot\alpha}$$

spinors transforming covariantly under pairwise LG, with opposite weights

Massless Limit



$m_i \to 0$

$$\left| p_{ij}^{\flat +} \right\rangle_\alpha \;\to\; |\, i \,\rangle_\alpha \qquad , \qquad \left[p_{ij}^{\flat +} \right|_{\dot\alpha} \;\to\; [\, i \,|_{\dot\alpha}$$

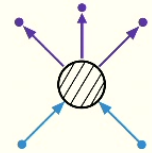
$$\left| p_{ij}^{\flat -} \right\rangle_\alpha \;\to\; \sqrt{2p_c}\; |\hat\eta_i\rangle_\alpha \qquad , \qquad \left[p_{ij}^{\flat -} \right|_{\dot\alpha} \;\to\; \sqrt{2p_c}\; [\hat\eta_i|_{\dot\alpha}$$

Parity flipped

$$\left[p_{ij}^{\flat +}\, i \right] \;=\; \left\langle i\, p_{ij}^{\flat +} \right\rangle \;=\; \left[\hat\eta_i\, p_{ij}^{\flat -} \right] \;=\; \left\langle p_{ij}^{\flat -}\, \hat\eta_i \right\rangle \;=\; 0$$

$$\left[p_{ij}^{\flat -}\, i \right] \;=\; \left\langle i\, p_{ij}^{\flat -} \right\rangle \;=\; \left[\hat\eta_i\, p_{ij}^{\flat +} \right] \;=\; \left\langle p_{ij}^{\flat +}\, \hat\eta_i \right\rangle \;=\; 2p_c$$

origin of mandatory helicity-flip in the
lowest partial wave for charge-monopole scattering

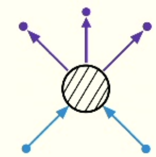
All 3-pt EM Amplitudes 

write most general Lorentz invariant expressions
consistent with Little Group and Pairwise Little Group

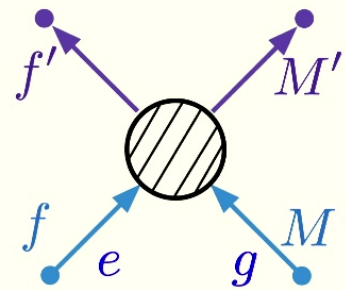
$\text{exponents} \geq 0 \Longrightarrow$ selection rule:

the selection rules are more restrictive than the
h_{ij}=0 case in Arkani-Hamed et al.

Check: Lowest Partial Wave



selection rule: $J \geq |h_{12}| - \frac{1}{2} \quad m \to 0$



$$J = 0 \Rightarrow \quad |\uparrow_{\rm elec.}\rangle\,|\downarrow_{\rm field}\rangle \quad \text{or} \quad |\downarrow_{\rm elec.}\rangle\,|\uparrow_{\rm field}\rangle$$

$h_{12} < 0$ only RH fermion to LH fermion

$h_{12} > 0$ only LH fermion to RH fermion

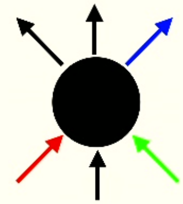
helicity flip vanishes for higher partial waves

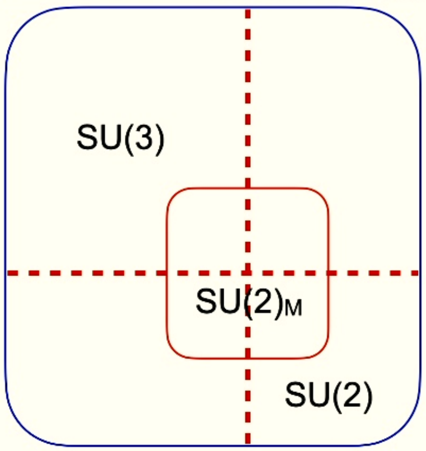
Pairwise Spinor Helicity hep-th/2009.14213

exactly reproduces

Kazama, Yang, Goldhaber Phys Rev D15 (1976) 2287

Georgi-Glashow: SU(5) GUT

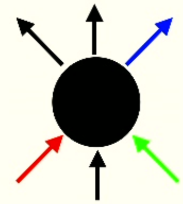




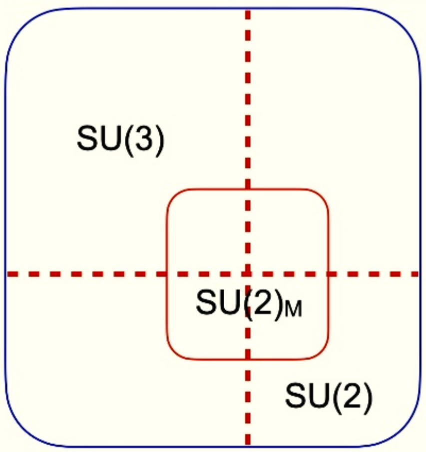
$$\overline{5} = (\overline{d}^1, \overline{d}^2, \overline{d}^3, e^-, \nu_e)$$

$$10 = \begin{pmatrix} 0 & \overline{u}^3 & -\overline{u}^2 & u^1 & d^1 \\ -\overline{u}^3 & 0 & \overline{u}^1 & u^2 & d^2 \\ \overline{u}^2 & -\overline{u}^1 & 0 & u^3 & d^3 \\ -u^1 & -u^2 & -u^3 & 0 & \overline{e} \\ -d^1 & -d^2 & -d^3 & -\overline{e} & 0 \end{pmatrix}$$

Georgi-Glashow: SU(5) GUT



$$\bar{5} = (\bar{d}^1, \bar{d}^2, \bar{d}^3, e^-, \nu_e)$$



$$10 = \begin{pmatrix} 0 & \bar{u}^3 & -\bar{u}^2 & u^1 & d^1 \\ -\bar{u}^3 & 0 & \bar{u}^1 & u^2 & d^2 \\ \bar{u}^2 & -\bar{u}^1 & 0 & u^3 & d^3 \\ -u^1 & -u^2 & -u^3 & 0 & \bar{e} \\ -d^1 & -d^2 & -d^3 & -\bar{e} & 0 \end{pmatrix}$$

monopole in $SU(2)_M$

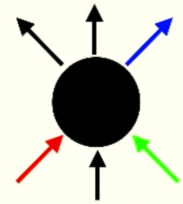
four doublets

outgoing

incoming

$$\begin{pmatrix} e \\ -\bar{d}^3 \end{pmatrix}, \quad \begin{pmatrix} \bar{u}^1 \\ u^2 \end{pmatrix}, \quad \begin{pmatrix} -\bar{u}^2 \\ u^1 \end{pmatrix}, \quad \begin{pmatrix} d^3 \\ \bar{e} \end{pmatrix}$$

Rubakov-Callan



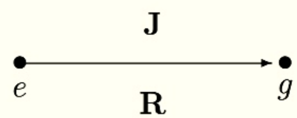
s-wave $\quad u^1 + u^2 + M \to \quad ?$

initial state

$$|\uparrow_{u^1}\rangle\,|\downarrow_{\text{field}}\rangle \times |\uparrow_{u^2}\rangle\,|\downarrow_{\text{field}}\rangle$$

$$\to \left[u^1\, p^{\flat-}_{u^1,M}\right]\left[u^2\, p^{\flat-}_{u^2,M}\right]$$

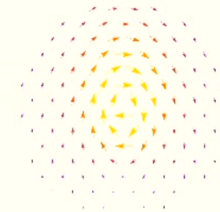
forward scattering not allowed since $\;|\downarrow_{\text{field}}\rangle \to |\uparrow_{\text{field}}\rangle$



only possibility: $\;|\downarrow_{e^\dagger}\rangle\,|\uparrow_{\text{field}}\rangle \times |\downarrow_{\text{d}^3\dagger}\rangle\,|\uparrow_{\text{field}}\rangle$

$$\to \left[\bar{e}^\dagger\, p^{\flat-}_{\bar{e}^\dagger,M}\right]\left[\bar{d}^{3\dagger}\, p^{\flat-}_{\bar{d}^{3\dagger},M}\right]$$

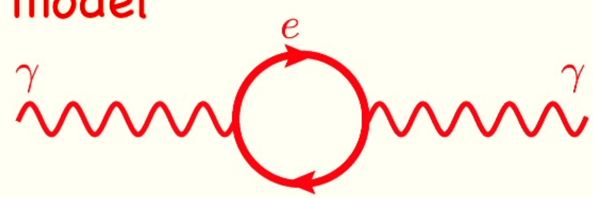
But Why Does it Work?



Rubakov and friends showed

$\int d^4x\, \vec{E} \cdot \vec{B}$ is quantized

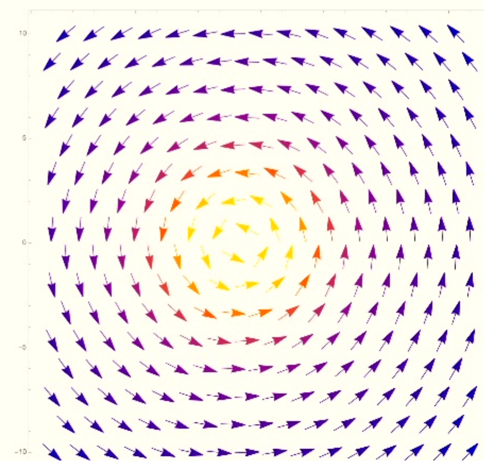
cf Abelian Instanton in Schwinger model



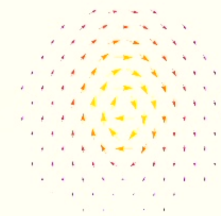
$$m^2 = \frac{2N_f e_{2D}^2}{\pi}$$

$$A_\alpha \to \frac{n}{e_{2D}} \frac{\epsilon_{\alpha\beta} x^\beta}{x^2}$$

$$Ch_1 = \frac{e_{2D}}{2\pi} \int_{S_1, |x| \to \infty} A_\alpha dx^\alpha = n$$



4D Abelian Instanton

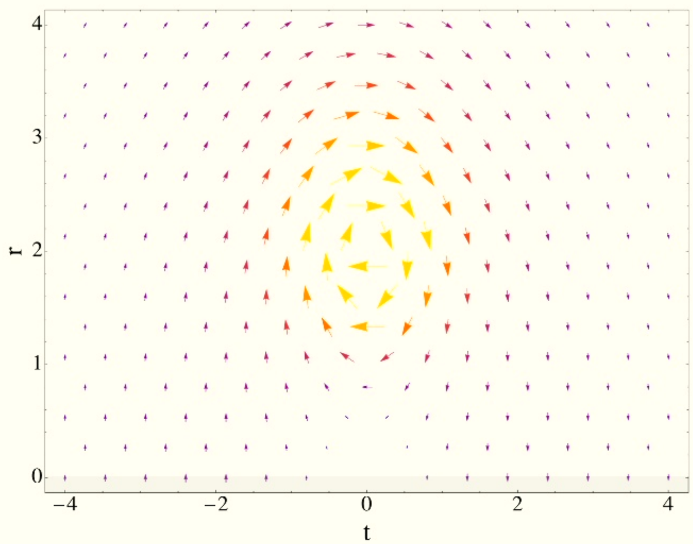


in monopole background

$$Ch_2 = \frac{e^2}{16\pi^2}\int d^4x F_{\mu\nu}\widetilde{F}^{\mu\nu} = \left(\frac{e}{2\pi}\int d\Omega r^2 \vec{B}\right)\cdot\left(\frac{e}{2\pi}\int drdt\vec{E}^{\text{vor}}\right)$$

$$= (2h_{12})(n/2)$$

3D spherical symmetry



Matching

for N_f doublets of SU(2), flavor symmetry SU(N_f)

U(1) EFT: N_f χ_i^+ and χ_i^-
flavor symmetry SU(N_f)xSU(N_f)
broken to SU(N_f) by

$$\text{at } r=0: \; \mathcal{L}_{\text{bound}} = \bar{\psi}_i \tau^+ e^{i\phi} \psi_i + h.c.$$
$$= \bar{\chi}_i^+ e^{i\phi} \chi_i^- + \bar{\chi}_i^- e^{-i\phi} \chi_i^+$$

Lam & Yan Phys. Rev. D**31** (1985) 322

Matching

for N_f doublets of SU(2), flavor symmetry SU(N_f)

U(1) EFT: N_f χ_i^+ and χ_i^-
flavor symmetry SU(N_f)xSU(N_f)
broken to SU(N_f) by

$$\text{at } r=0: \;\; \mathcal{L}_{\text{bound}} = \bar{\psi}_i \tau^+ e^{i\phi} \psi_i + h.c.$$
$$= \bar{\chi}_i^+ e^{i\phi} \chi_i^- + \bar{\chi}_i^- e^{-i\phi} \chi_i^+$$

ϕ has a discrete spectrum: splittings $\propto e^2 M_{GUT}$

low energy EFT: integrate out

Zero Modes

given instanton solution solve Dirac eq.
for zero modes

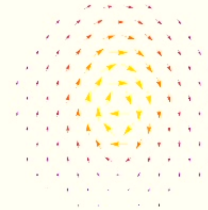
$$\not{D}\psi = 0$$

count using: $$\partial_\mu j^\mu_A(x) = \frac{e^2}{8\pi^2} F^{\mu\nu}\,\widetilde{F}_{\mu\nu}$$

$$\mathcal{L}_{\text{bound}} = \bar{\psi}_i \tau^+ e^{i\phi} \psi_i + h.c.$$

halves number of zero modes

Matching II



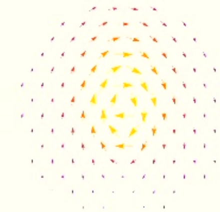
Witten found instantons with
3D spherical symmetry in SU(2)

fun conjecture: 4D Abelian instantons are
EFT remnants of Witten's spherical instantons

evidence: same number of fermion legs

Witten Phys. Rev. Lett. **38** (1977) 121

Path Integral for N_f = 2



helicity flip: $\langle \bar{\psi}_+ \psi_- \rangle$

- only happens in lowest partial wave
- no perturbative contribution
- only instanton sector is relevant

one loop:

$$\left[\frac{\det'(i\not{D})}{\det'(i\not{\partial})}\right]^{N_f} = \exp\{-\int dt\, dr \left[\frac{m^2}{2} A_\alpha \left(\eta^{\alpha\beta} - \frac{\partial^\alpha \partial^\beta}{\partial^2}\right) A_\beta + \ldots\right]\}$$

$$\alpha, \beta = r \text{ or } t \qquad m^2 = \frac{2|h_{12}|e^2}{\pi}$$

"Schwinger mass"

Path Integral for N_f = 2

4D exact: $\psi \to \psi' = e^{-s_h e \gamma_5 \partial^{-2} E_r}\psi$

decouples lowest partial wave:

$$A_t = \; -\partial_r \, \partial^{-2} \, E_r \; , \quad A_r = \partial_t \, \partial^{-2} \, E_r$$

$$\begin{aligned}\overline{\psi}_{j_{\min}} \not{D} \psi_{j_{\min}} &= \overline{\psi}'_{j_{\min}} e^{s_h e \gamma_5 \partial^{-2} E_r} \not{D} e^{s_h e \gamma_5 \partial^{-2} E_r} \psi'_{j_{\min}} \\ &= \overline{\psi}'_{j_{\min}} \gamma^\mu (\partial_\mu - ieA_\mu^{\text{mon}}) \psi'_{j_{\min}}\end{aligned}$$

from axial anomaly:

$$\log \det{}'(i\not{D}) = \int d^4x \, \frac{e^2}{16\pi^2} \, s_h e \, \partial^{-2} E_r \, F_{\mu\nu} \widetilde{F}^{\mu\nu} + \ldots$$

$$= \int d^4x \, \frac{e^2}{4\pi^2} \, E_r \partial^{-2} E_r \, s_h e B_r + \ldots$$

Path Integral for $N_f = 2$

$$\langle \bar{\psi}_+(x_f)\psi_-(x_i)\rangle = \sum_n \int_{Ch_1=n/2} \mathcal{D}A_\alpha \mathcal{D}\overline{\psi}\mathcal{D}\psi\, \bar{\psi}_+(x_f)\psi_-(x_i)e^{-S}$$

$$= \int_{Ch_1=1/2} \mathcal{D}A_\alpha \mathcal{D}\overline{\psi}\mathcal{D}\psi\, \bar{\psi}'_+(x_f)\psi'_-(x_i)e^{-S-S_{\rm rot}}$$

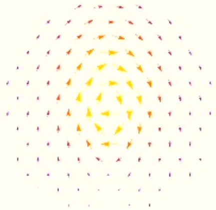
includes anomaly

$$= \bar{\psi}^{(0)}_+(x_f)\psi^{(0)}_-(x_i) \int_{Ch_1=1/2} \mathcal{D}A_\alpha\, e^{-S'-S_{\rm rot}}$$

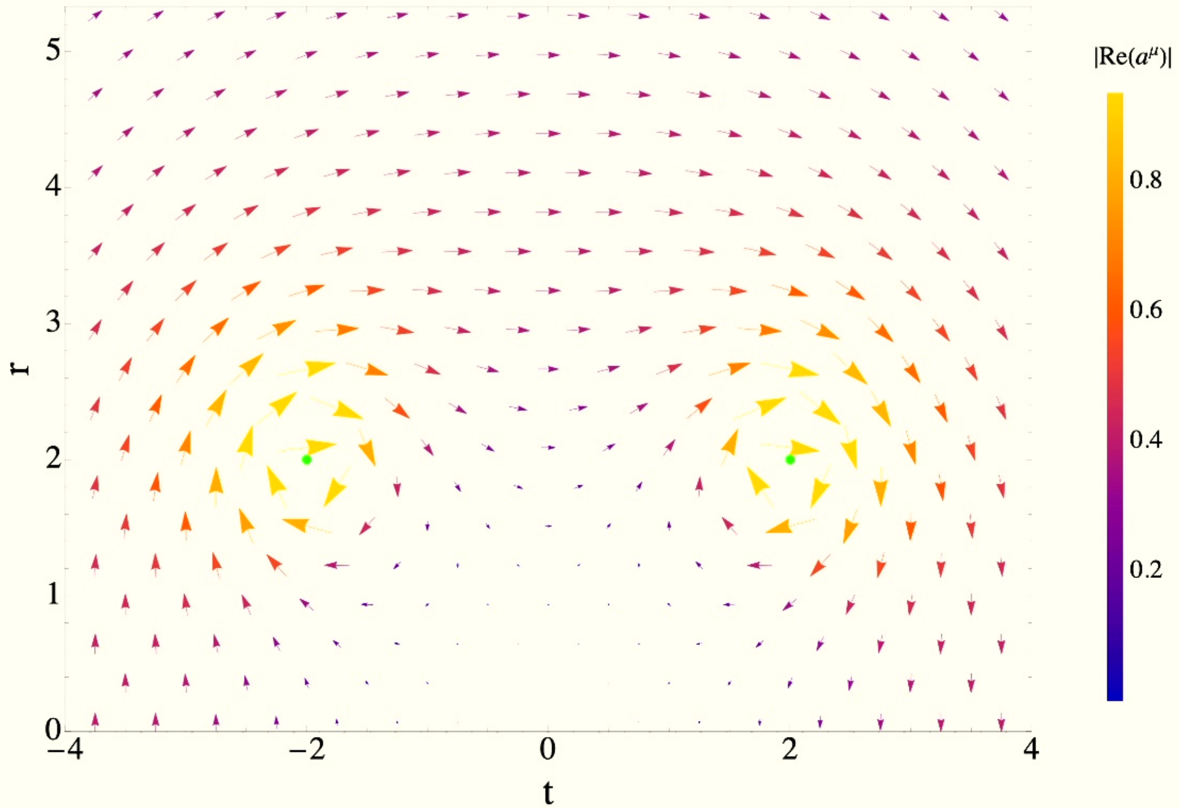
Gaussian

$$= \text{ pairwise spinor helicity result}$$

Path Integral for $N_f = 2$



helicity flip: $\langle \bar{\psi}_+(x_f)\psi_-(x_i)\rangle$



Conclusions

Pairwise Little Group determines monopole partial wave scattering amplitudes

Abelian Instantons are the underlying mechanism for surprising results in lowest-partial waves