Title: Fractional Quantum Hall states Under Decoherence

Speakers: Zijian Wang

Collection/Series: Quantum Matter

Subject: Condensed Matter

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Abstract:

In this talk, I will introduce our recent studies on the effect of decoherence on fractional quantum Hall fluids and their robustness as topological quantum memories. Specifically, we examine the behavior of Laughlin states and Moore-Read states under dephasing noise using the plasma analogy as well as the field theory description. Notably, we identify a critical filling (which is 1/4 for the Renyi-2 case and between 1/4 and 1/8 in the replica limit) separating two regimes of mixed-state phase transition. Below the critical filling, a Berezinskii-Kosterlitz-Thousless (BKT) transition occurs at finite noise strength, separating the topologically ordered phase from a critical phase. This transition is characterized by quantum-information quantities that are non-linear functions of the density matrix. On a torus, the BKT transition marks the critical decoherence above which the quantum memory encoded in the ground state manifold is degraded. In contrast, above the critical filling, the quantum memory is extremely resilient against dephasing noise, with the transition occurring only at infinite noise strength. For the Moore-Read state, we further show that the transition also characterizes whether different fusion outcomes of non-Abelian anyons remain distinguishable under decoherence.

Fractional Quantum Hall States Under Decoherence

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Acknowledgement

Ruihua Fan

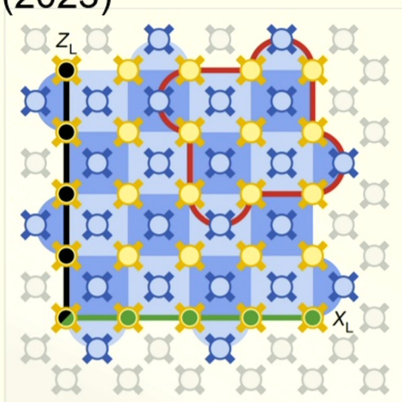
Tianle Wang

Samuel J. Garratt

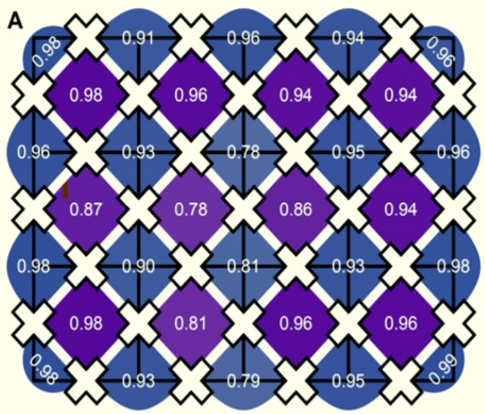
Ehud Altman

Noisy quantum architectures

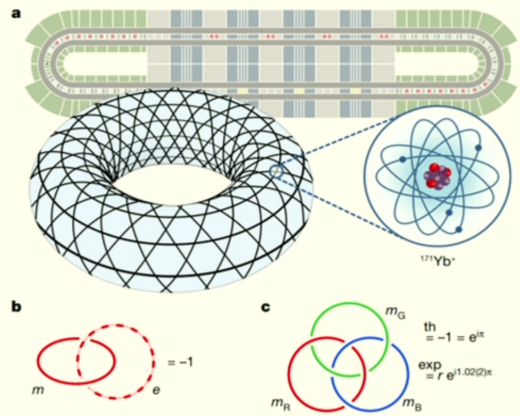
Google quantum AI (2023)



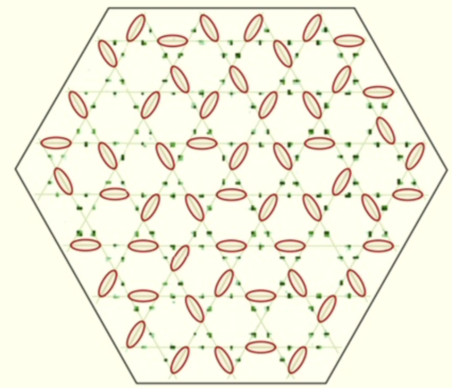
Satzinger et al. (2021)



Iabal et al. (2023)



Semeghini et al. (2021)



Mixed-state phases of matter

Error-correction codes as mixed-state phase of matter

Dennis, Kitaev, A Landahl, Preskill (2002)
Fan, Bao, Altman, Vishwanath (2023)
…

New patterns of long-range entanglement:

Wang, Wu, Wang (2023)
Sohal, Prem (2024)
Ellison, Cheng (2024)
Lessa, Cheng, Wang (2024)
…

New definitions of phases and phase transitions:

Hastings (2011)
Bao, Fan, Vishwanath, Altman (2023)
Lee, Jian, Xu (2023)
Sang, Zou, Hsieh (2023)
Sang, Hsieh (2023)
Chen, Grover (2023)
…

New types symmetries+symmetry-breaking/SPT/SET:

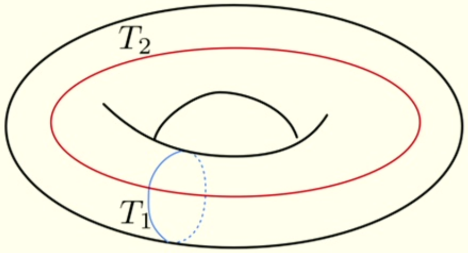
Ma, Wang (2022)
Ma, Zhang, Bi, Cheng, Wang (2023)
Lessa, Ma, Zhang, Bi, Cheng, Wang (2024)
Sala, Gopalakrishnan, Oshikawa, You (2024)
Gu, Wang, Wang (2024)
Zhang, Xu, Zhang, Xu, Bi, Luo (2024)
…

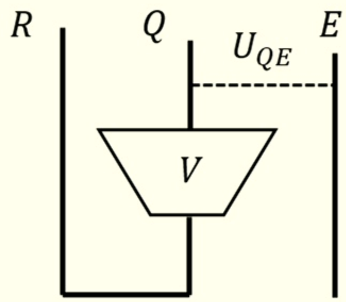
Why FQH states?

Compared to studied cases, FQH states have the following nice features:

- Widely realized in solid-state systems
- Solvable models beyond stabilizer code
- Chiral Topological order
- Enriched by global U(1) symmetry

Q1: Can these features lead to novel mixed-state phases of matter?





Q2: How good are FQH states as quantum codes?

Q3: How resilient are non-Abelian anyons?

A brief Review of Laughlin states

Laughlin wavefunction:

$$\Psi(z_1,\cdots,z_N)=\prod_{i<j}\left(z_i-z_j\right)^m e^{-\sum_i |z_i|^2/4l_B^2}$$

Laughlin (1983)

- It describes a gapped state in lowest landau level at filling $\nu=\frac{1}{m}$
- Ground-state degeneracy on torus=m;
- Quasi-holes/Quasi-electrons are fractional excitations with $\theta=\frac{\pi}{m}, e^*=\pm\frac{e}{m}$.

- The IR theory can be described by a $U(1)_m$ Chern-Simons theory:

$$S=\frac{m}{4\pi}\int \epsilon^{\alpha\beta\gamma}a_\alpha\partial_\beta a_\gamma$$

- Chiral edge states on an open manifold:

$$S_{\text{edge}}=\frac{m}{4\pi}\int \partial_t\varphi\partial_x\varphi-v(\partial_x\varphi)^2$$

All these properties play crucial roles in the decoherence problem!

Noise models

- Density dephasing noise

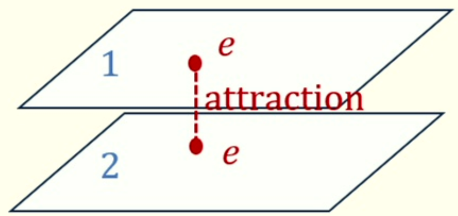
$$\mathcal{L}[\rho] = \gamma \int d^2z\, n(z)\rho n(z) - \frac{1}{2}\{n(z), \rho\},$$

$$\rho = e^{\mathcal{L}t}[\rho_0]$$

Doubled space description:

$$\rho = \sum_{m,n} \rho_{mn} |m\rangle\langle n| \to |\rho\rangle\rangle = \sum_{mn} \rho_{mn}\, |m\rangle \otimes |n^*\rangle$$

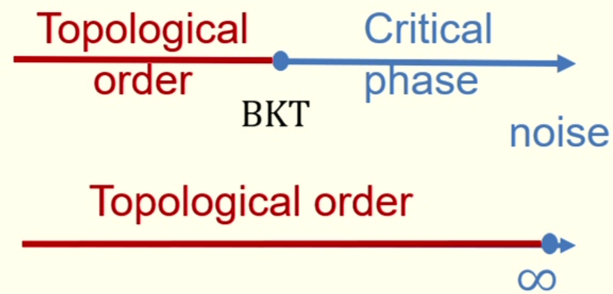
$$\Rightarrow\ |\rho\rangle\rangle = e^{-\gamma t \int d^2z (n_1(z) - n_2(z))^2} |\Psi \otimes \Psi^*\rangle$$



Phase diagram:

$\nu = \frac{1}{m} < \nu_c$

$\nu > \nu_c$



- Low-frequency noise, such as
  a. LLL-projected density dephasing,
  b. Incoherent errors of anyons that preserves U(1) symmetry.

Plasma analogy

- The wavefunction norm of Laughlin states

$$\langle\Psi|\Psi\rangle = \int d^2 z_1 \cdots d^2 z_N e^{2m\sum_{i<j}\log|z_i - z_j| - \sum_i \frac{|z_i|^2}{4l_B^2}}$$

Partition function of 2D plasma with $T = 1, q = \sqrt{2m}$, and neutralizing background charge

For $\mathrm{m} < 140$, the plasma is in the screening phase. Correspondingly, the Laughlin state describes a topological order

- In the doubled space, there are two copies(layers) of Laughlin plasmas.

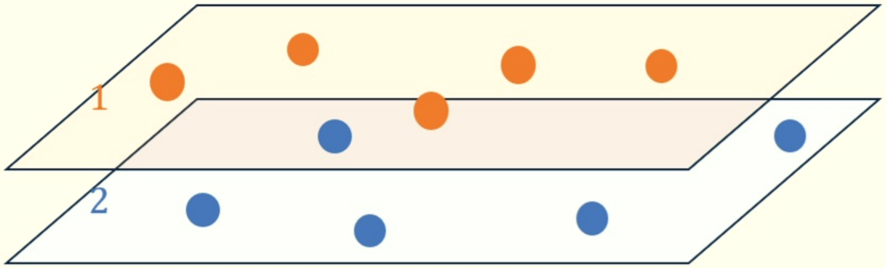
$$tr(\rho^2) = \langle\langle\rho|\rho\rangle\rangle = \prod_{i,j} \int d^2 z_i^1 \int d^2 z_j^2 \, e^{-(\Phi_1 + \Phi_2 + \mu \mathrm{V}_{12})}$$

$$\Phi_{\alpha=1,2} = -2m \sum_{i<j} \log\left|z_i^\alpha - z_j^\alpha\right| + \sum_i \frac{|z_i^\alpha|^2}{4l_B^2}$$

μ: decoherence strength,
V_{12}: short-range attraction between layers

Plasma analogy

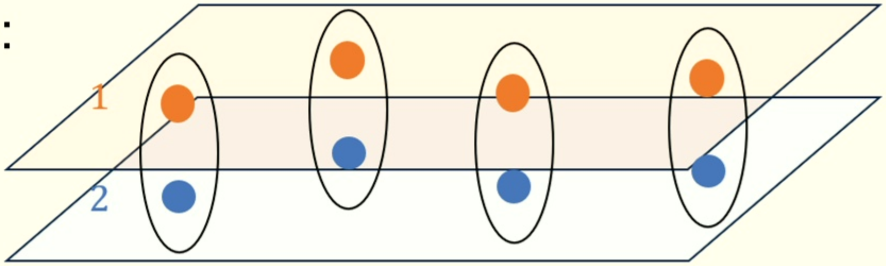
$\mu = 0$:



Fully screening

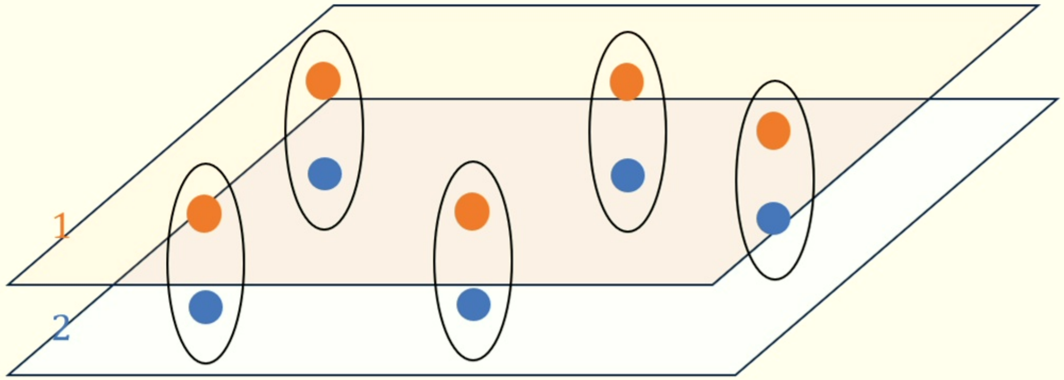
increasing
decoherence

$\mu = \infty$:



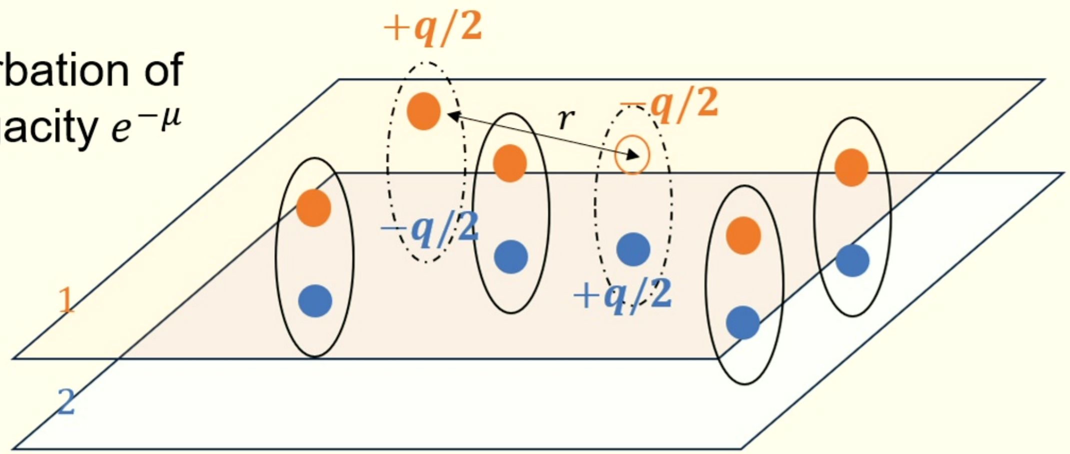
Partially screening

Kosterlitz-Thouless type instability at strong decoherence



Kosterlitz-Thouless type instability at strong decoherence

Consider perturbation of infinetisimal fugacity $e^{-\mu}$



$$\beta\Delta E \approx 2m \log r \xrightarrow{\text{screening}} \beta\Delta E \approx m \log r$$

Kosterlitz-Thouless type instability at strong decoherence

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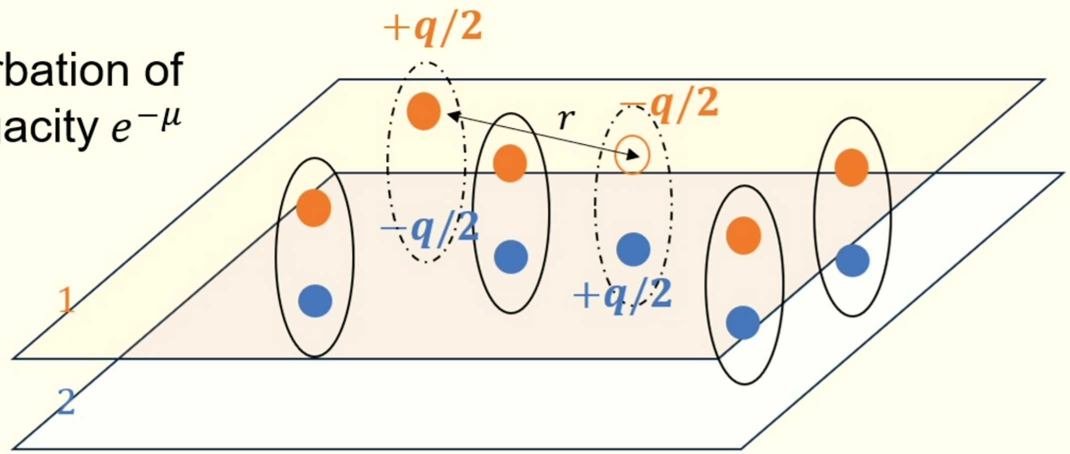


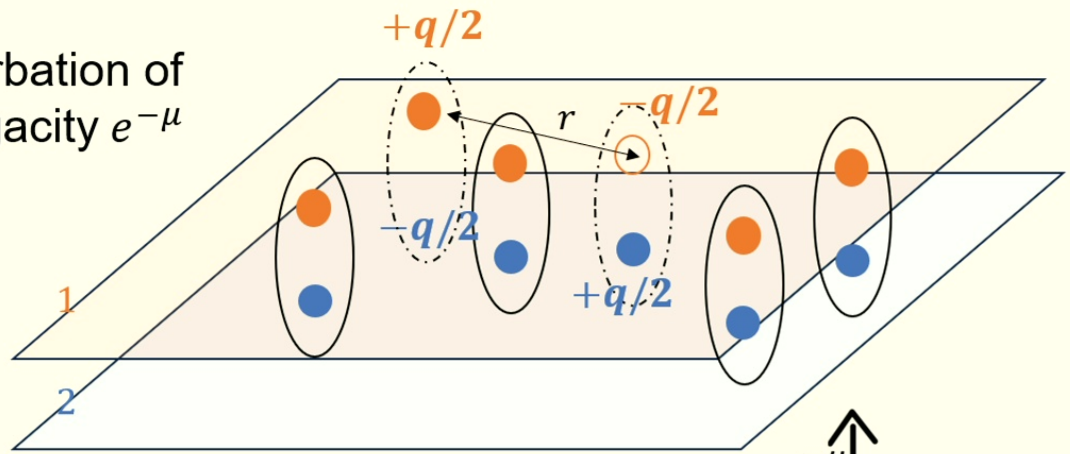
$$\beta\Delta E \approx 2m\log r \xrightarrow{\text{screening}} \beta\Delta E \approx m\log r$$

For $\mathrm{r} \sim O(L), \Delta S \approx \log L^2 r^2 \approx 4\log L$

At $m \leq 4$, charges always unbind for $\mu < \infty$

Kosterlitz-Thouless type instability at strong decoherence

Consider perturbation of infinetisimal fugacity $e^{-\mu}$



screening

$$\beta\Delta E \approx 2m \log r \xrightarrow{\text{screening}} \beta\Delta E \approx m \log r$$

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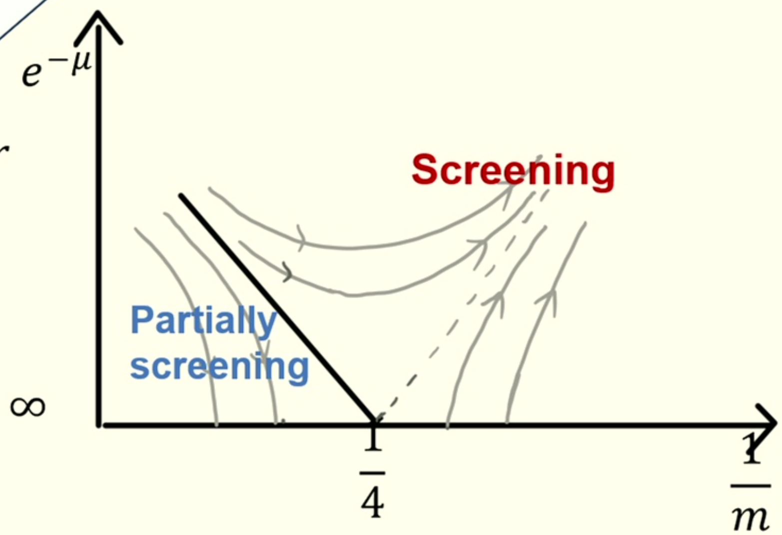

Kosterlitz-Thouless type instability at strong decoherence

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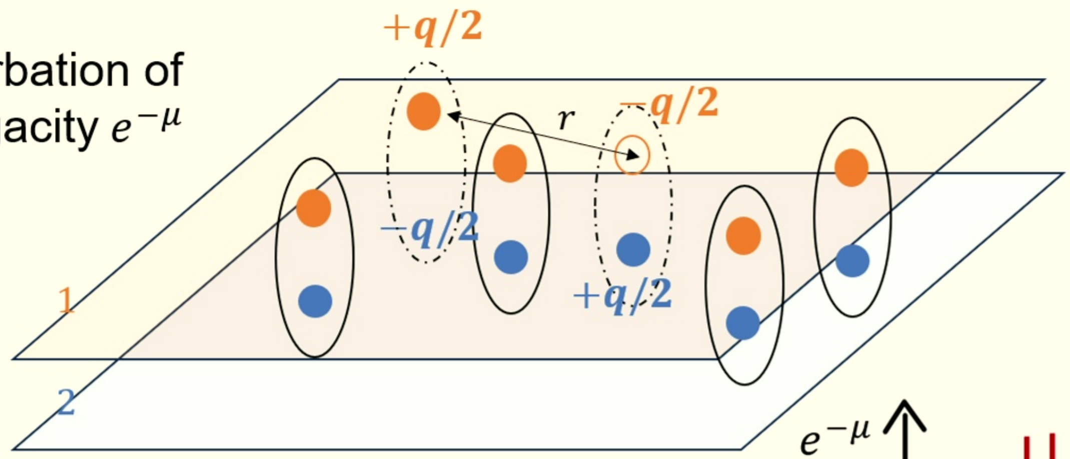
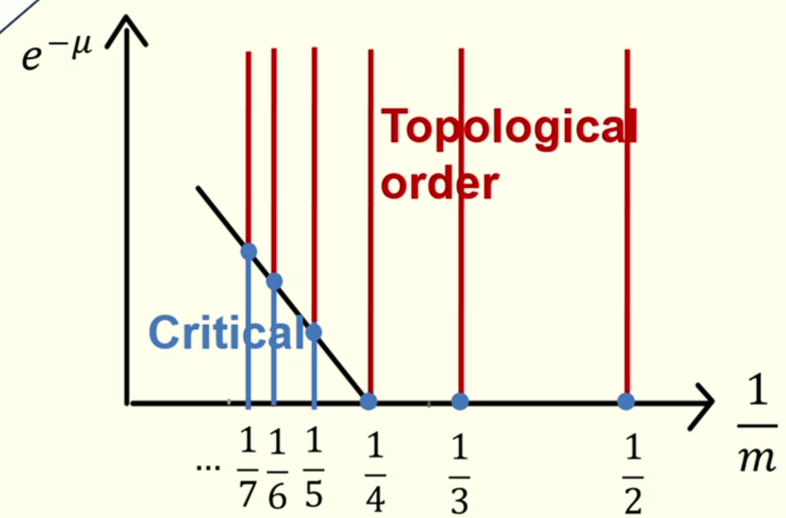


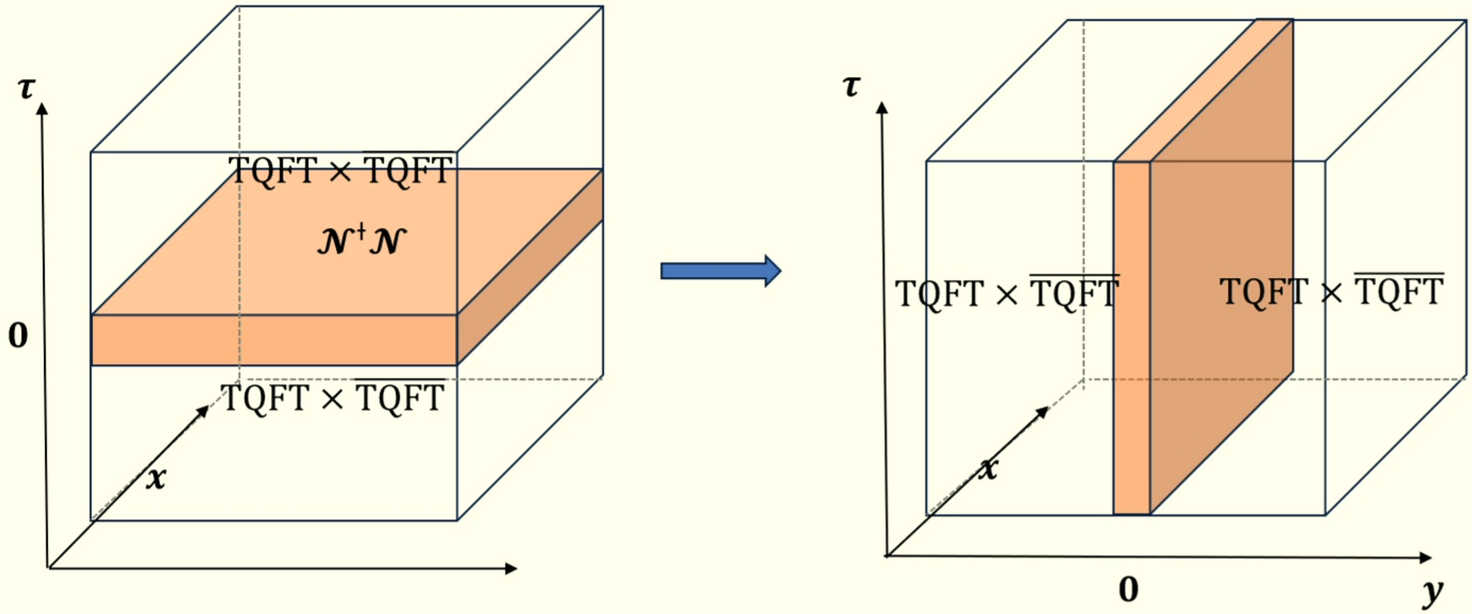
What's the nature of the critical phase?
Why is it critical?

What's the operational meaning of the BKT transition?



Errorfield double formalism

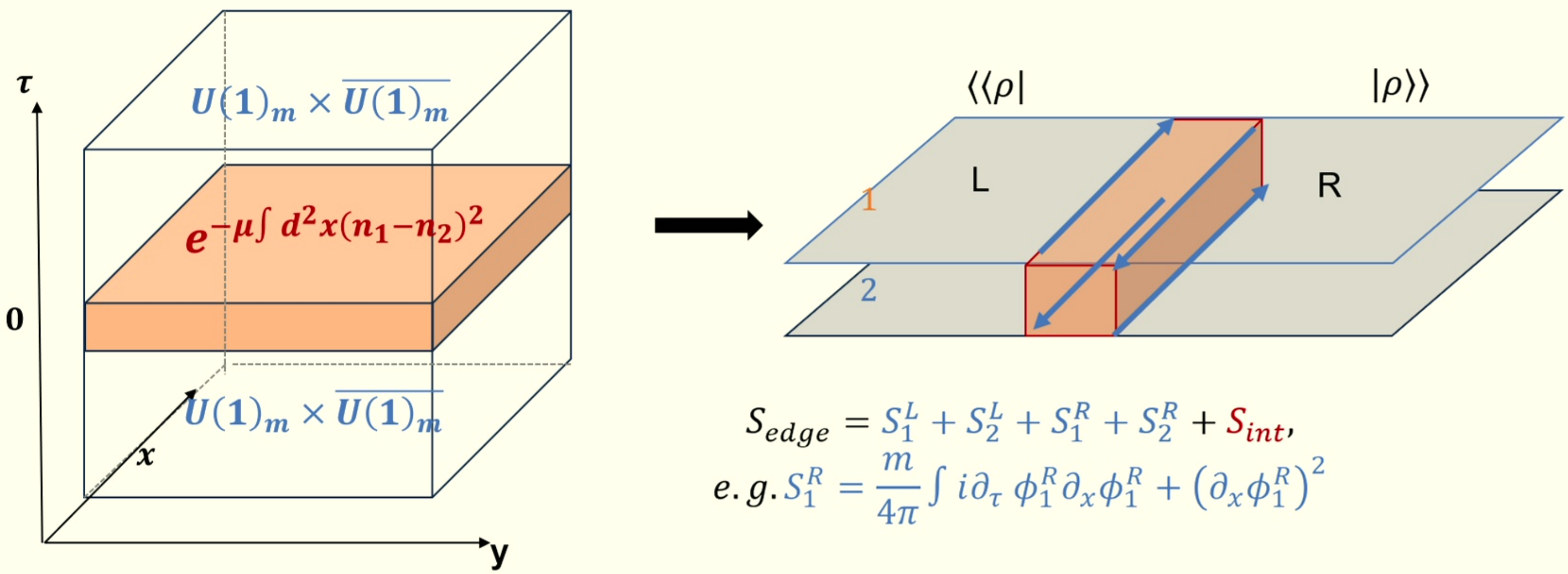
$$|\rho_0\rangle\rangle = \lim_{\beta\to\infty} e^{-\beta(H\otimes I+I\otimes H^*)}|\text{ref}\rangle\rangle, \qquad \langle\langle\rho|\rho\rangle\rangle = \langle\langle\rho_0|\mathcal{N}^\dagger\mathcal{N}|\rho_0\rangle\rangle$$



Bao, Fan, Vishwanath, Altman. 2023

Errorfield double formalism

$U(1)_m$ Chern-Simons: $S_{CS} = \int d^2x \frac{m}{4\pi} \epsilon^{\alpha\beta\gamma} a_\alpha \partial_\beta a_\gamma$



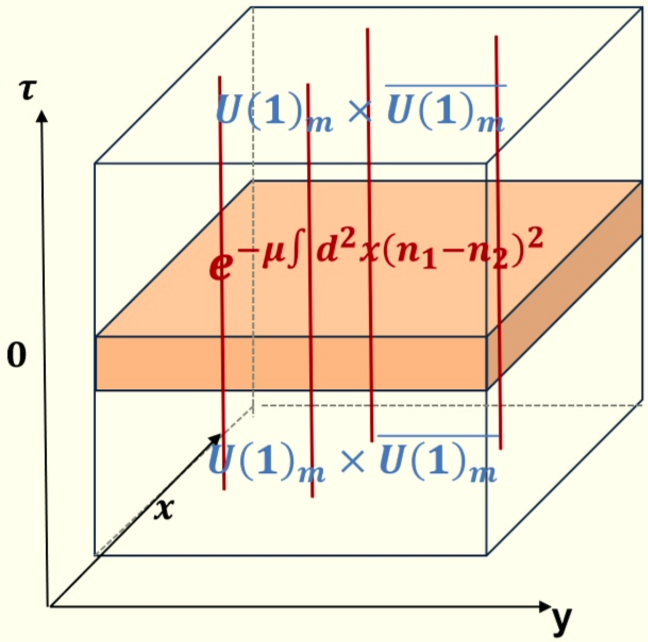
$S_{edge} = S_1^L + S_2^L + S_1^R + S_2^R + S_{int}$,

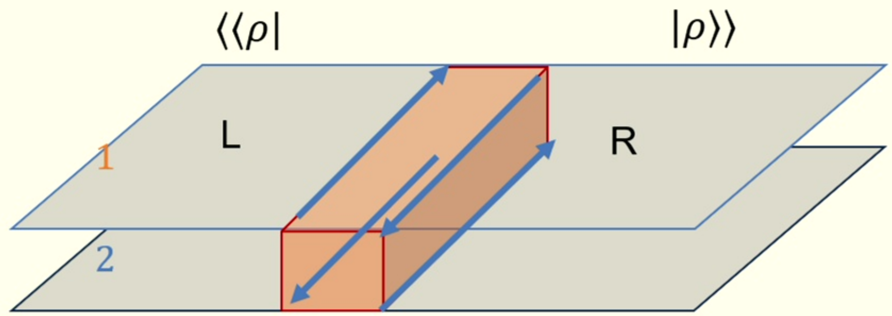
$e.g.\, S_1^R = \frac{m}{4\pi} \int i\partial_\tau \phi_1^R \partial_x \phi_1^R + \left(\partial_x \phi_1^R\right)^2$

Errorfield double formalism

$U(1)_m$ Chern-Simons: $S_{CS} = \int d^2x \frac{m}{4\pi} \epsilon^{\alpha\beta\gamma} a_\alpha \partial_\beta a_\gamma + m a_\mu j_e^\mu$

For simplicity, we consider infinitely massive matter field: $j_e^{x,y} = 0, j_e^0 = n(x,y)$



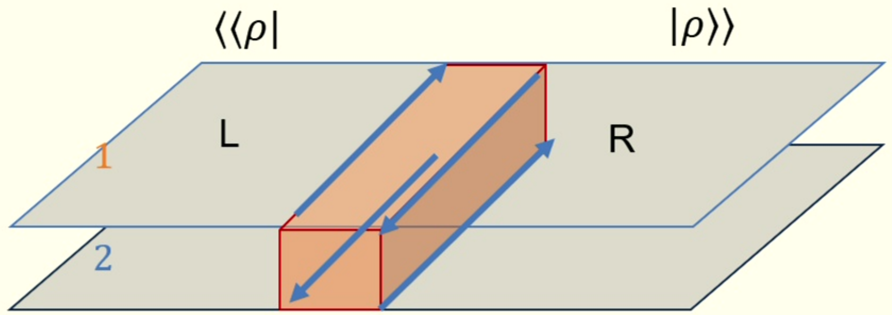


$S_{edge} = S_1^L + S_2^L + S_1^R + S_2^R + S_{int}$,

$e.g.\, S_1^R = \frac{m}{4\pi} \int i\partial_\tau \phi_1^R \partial_x \phi_1^R + \left(\partial_x \phi_1^R\right)^2$

Effective field theory for decohered Laughlin states

$$\mathrm{tr}(\rho^2) = \int D\varphi_{1,2}^{L,R} e^{-S_1^L - S_2^L - S_1^R - S_2^R - S_{int}}$$



$$e^{-S_{int}} = \prod_{x,\tau} \sum_{n_1,n_2} e^{-\mu(n_1-n_2)^2 + im(\varphi_1^L - \varphi_1^R)n_1 - im(\varphi_2^L - \varphi_2^R)n_2}$$

$$\sim \delta\left(\frac{m}{2}\left(\varphi_1^L - \varphi_2^L - \varphi_1^R + \varphi_2^R\right)\right) e^{e^{-\mu}\int dx d\tau \cos\frac{m}{2}(\varphi_1^L + \varphi_2^L - \varphi_1^R - \varphi_2^R)}$$

θ_+: phase conjugate to $n_1 + n_2$. Always pinned

ϕ_-: phase conjugate to $n_1 - n_2$

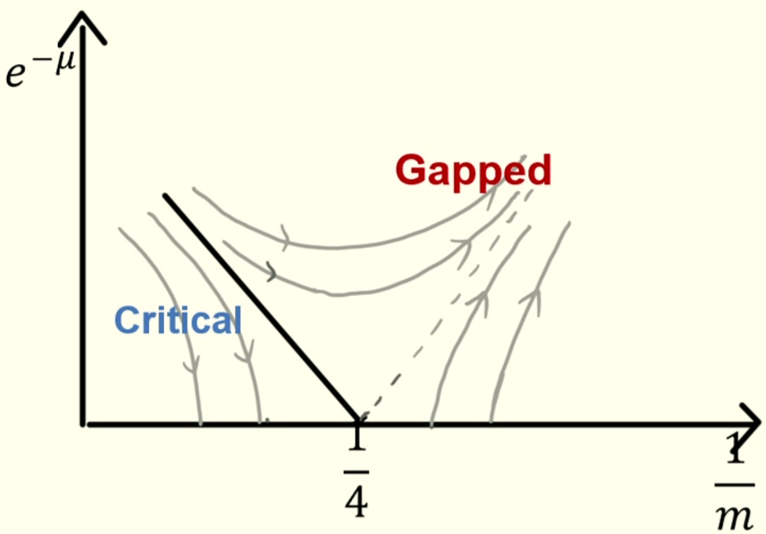
Non-chiral boson field:
$\phi_\pm = (\varphi_1^L + \varphi_2^L) \pm (\varphi_1^R + \varphi_2^R)$

Dual field: $\theta_\pm$

Nature of the critical phase

$$\mathrm{tr}(\rho^2) = \int D\phi_- D\theta_+ e^{-S[\phi_-]}\delta(\theta_+)$$
$$= \int D\phi_- e^{-S[\phi_-]}$$

$$S[\phi_-] = \int dx d\tau \frac{m}{4\pi}(\partial\phi_-)^2 - e^{-\mu}\cos m\phi_-$$



Lessa, Ma, Zhang, Bi, Cheng, Wang (2024)

Lee, Jian, Xu (2023)

Power-law Renyi-2 correlation at $\mu > \mu_c$

$$\frac{\mathrm{tr}\, c(x)c^\dagger(x')\rho c(x')c^\dagger(x')\rho}{\mathrm{tr}\,\rho^2}$$
$$= \langle\langle e^{i\theta_+(x)}e^{-i\theta_+(x')}\rangle\rangle\langle\langle e^{i\theta_-(x)}e^{-i\theta_-(x')}\rangle\rangle$$

$= \langle\langle e^{i\theta_-(x)}e^{-i\theta_-(x')}\rangle\rangle \quad \sim \frac{1}{|x-x'|^m}$ at $\mu = \infty$

For anyons: $= \langle\langle e^{\frac{i\theta_-(x)}{m}} e^{-\frac{i\theta_-(x')}{m}}\rangle\rangle \sim \frac{1}{|x-x'|^{1/m}}$

Anyons cannot truly condense
because they carry global charges

Nature of the critical phase

How does decoherence create power-law correlation?

Related to quasi-long-range order (QLRO) of composite bosons:

$$b(z) = c(z)e^{im\int d^2z' n(z')\,\mathrm{Im}\log(z-z')}$$

Girvin, Macdonald. 1987

$\langle b^\dagger(z)b(z')\rangle \sim \frac{1}{|z-z'|^{m/2}}$ for Laughlin states

In the doubled space: $\langle\langle b_1^\dagger(z)b_2^\dagger(z)b_1(z')b_2(z')\rangle\rangle \sim \frac{1}{|z-z'|^m}$ Hidden QLRO

↓ Flux attachment phase factor cancels under dephasing

Lu, Zhang, Vijay, Hsieh. 2023

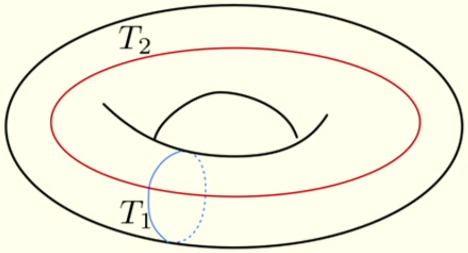
$\langle\langle c_1^\dagger(z)c_2^\dagger(z)c_1(z')c_2(z')\rangle\rangle \sim \frac{1}{|z-z'|^m}$ True QLRO

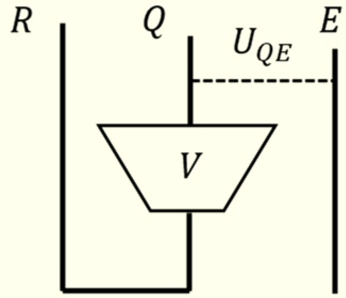
Why FQH states?

Compared to studied cases, FQH states have the following nice features:

- Widely realized in solid-state systems
- Solvable models beyond stabilizer code
- Chiral Topological order
- Enriched by global U(1) symmetry

Q1: Can these features lead to novel mixed-state phases of matter?



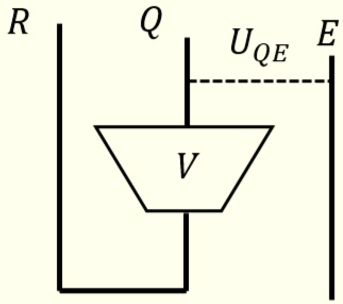


Q2: How good are FQH states as quantum codes?

Q3: How resilient are non-Abelian anyons?

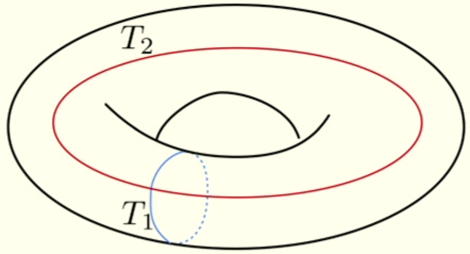
Decodability

An intrinsic measure of quantum memory: coherent information



$$I_c(R\rangle Q) = S_Q - S_{RQ}$$

Fan, Bao, Altman, Vishwanath (2023)



Amount of encoded quantum information that can be recovered after decoherence

Use the effective field theory to calculate the Renyi-2 coherent information:

$$I_c^{(2)} = \log \frac{\mathrm{tr}\,\rho_{RQ}^2}{\mathrm{tr}\,\rho_Q^2} = -\log \frac{1}{m} \sum_{n=0,1,\cdots,m-1} \frac{\widetilde{Z_n}}{Z_n}$$

Decodability

$$S[\phi_-] = \int dx d\tau \frac{m}{4\pi}(\partial\phi_-)^2 - e^{-\mu}\cos m\phi_-$$

$$I_c^{(2)} = -\log\frac{1}{m}\sum_{n=0,1,\cdots,m-1}\frac{\widetilde{Z_n}}{Z_n}$$

$\widetilde{Z_n}$: partition function with twisted BC of ϕ_-: $\phi_-(x+L) = \phi_-(x) + \frac{2\pi n}{m} \bmod 2\pi$

Z_n : partition function with twisted BC of θ_-: $\theta_-(x+L) = \theta_-(x) + 2\pi n \bmod 2\pi m$

$\mu = \infty$: $\widetilde{Z_n} = Z_n$ due to T- duality $\phi_- \leftrightarrow \frac{\theta}{m}$, so $I_c^{(2)} = 0$.

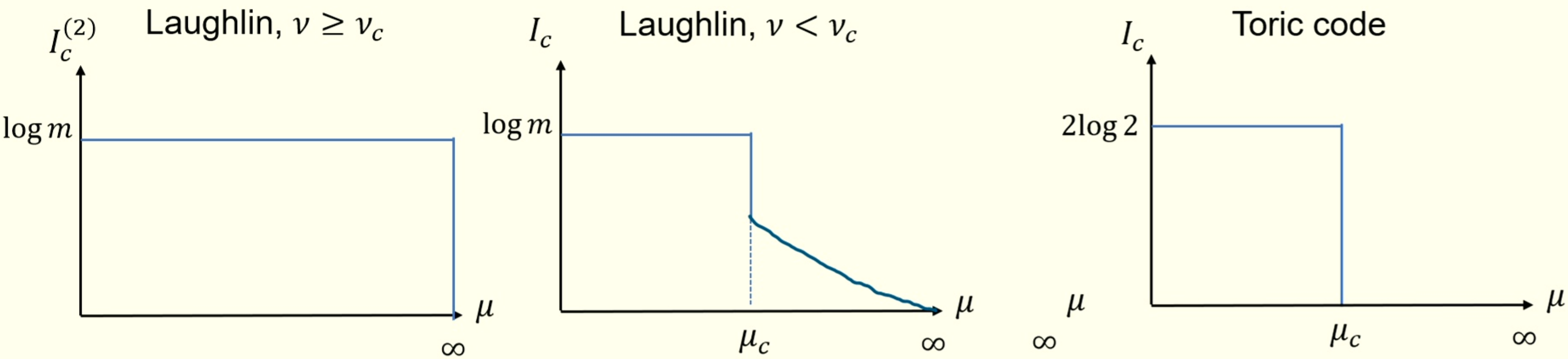
$\mu < \mu_c$: gapped by cosine term, twisted BC of ϕ_- cost $O(L)$ free energy
$\Rightarrow \frac{\widetilde{Z_n}}{Z_n} = \delta_{n0} \Rightarrow I_c^{(2)} = \log m$

$\mu_c \leq \mu < \infty$: cosine term irrelevant, leading to O(1) free energy cost for twisted BC of ϕ_-

$\Rightarrow 0 < I_c^{(2)} < \log m$

Decodability

ν: filling factor



The critical phase has partial quantum memory.

Replica limit $n \to 1$

Both the plasma analogy and errorfield double can be generalized to n-replicas, but the replica limit $n \to 1$ is ill-defined.

Alternatively, we can directly use the von-Neumann entropy or the canonical purification to study the transition.

We show in this case, $\frac{1}{8} \leq \nu_c \leq \frac{1}{4}$

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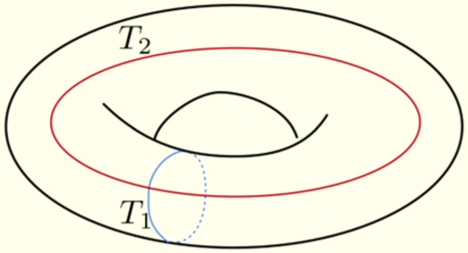
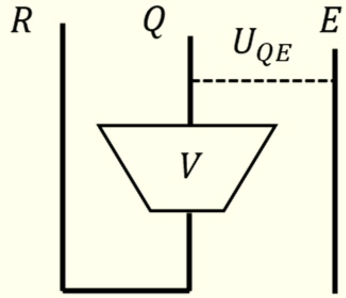




Q2: How good are FQH states as quantum codes?

Q3: How resilient are non-Abelian anyons?

Moore-Read state

$\nu = \frac{1}{m}$ Moore-Read wavefunction

$$\Psi(\{z_i\}) = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^m \, e^{-\sum_i |z_i|^2/4l_B^2}$$

$$= \langle \prod_i \psi(z_i) e^{im\varphi(z_i)} \, e^{-i2\pi l_B^{-2} \int d^2 z \varphi(z)} \rangle$$

Moore, Read (1991)

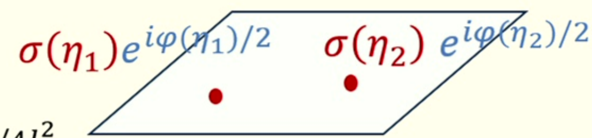
Ising $\times$ $\mathrm{U(1)_m}$ chiral boson CFT $(R = \sqrt{m}\,)$

Electron operator: $\psi_e = \psi \times e^{im\varphi}$

Primary fields:

$$e^{in\varphi}, \sigma e^{i(n+\frac{1}{2})\varphi}, \psi e^{in\varphi}, n = 0,1, \cdots m-1$$

Non-Abelian anyons: $\sigma(\eta_1) e^{i\varphi(\eta_1)/2}$ $\quad e_{qh} = \frac{e}{2m}$



$$\Psi(\eta_1, \eta_2, \{z_i\}) = \mathrm{Pf}\left(\frac{(z_i - \eta_1)(z_j - \eta_2) + i \leftrightarrow j}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^m \, e^{-\sum_i |z_i|^2/4l_B^2}$$

$$= \langle \, \sigma(\eta_1) e^{i\varphi(\eta_1)/2} \sigma(\eta_2) e^{i\varphi(\eta_2)/2} \, \prod_i \psi(z_i) e^{im\varphi(z_i)} \rangle$$

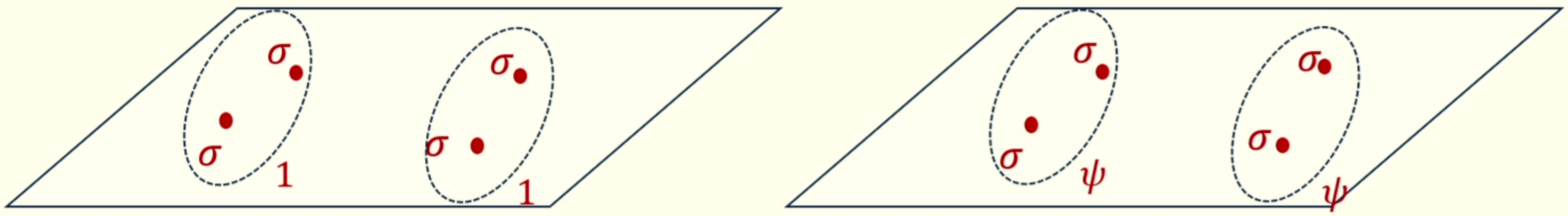
How resilient are non-Abelian anyons?

$$\Psi_{\alpha=1,\psi}(\eta_1,\eta_2,\eta_3,\eta_4,\{z_i\}) = \left\langle \prod_{\mu=1,2,3,4} \sigma(\eta_\mu) e^{\frac{i\varphi(\eta_\mu)}{2}} \prod_i \psi(z_i) e^{im\varphi(z_i)} \right\rangle_\alpha$$

$$\equiv F_\alpha(\{\eta_\mu\},\{z_i\})$$

2 Conformal blocks

$$\sigma \times \sigma = 1 + \psi$$



The fusion channels of non-Abelian anyons can be used to encode quantum information.

Do the two fusion channels remain well distinguishable under decoherence?

How resilient are non-Abelian anyons?

$$|\rho\rangle\rangle = e^{-\frac{\mu}{2}\int d^2z\left(n_1(z)-n_2(z)\right)^2}|\Psi\rangle\otimes|\Psi^*\rangle$$

Fugacity of unpaired operators induced by decoherence

$$\mathrm{tr}\left(\rho^2\right) = \langle\langle\rho|\rho\rangle\rangle = \prod_{1\le i,j\le N}\int d^2z_i^1\int d^2z_j^2\, e^{-\mu V(z_i^1-z_j^2)}\left\langle\prod_i \varepsilon^1\left(z_i^1\right)e^{im\phi^1(z_i^1)}\prod_j \varepsilon^2\left(z_j^2\right)e^{im\phi^2\left(z_j^2\right)}\right\rangle$$

As before, we introduce $\phi_\pm = \frac{\phi_1\pm\phi_2}{2}$

Moreover, we bosonize the Ising^2 CFT

$$\varepsilon_1\varepsilon_2 \to (\partial\Phi)^2,\ \varepsilon_{1,2} \to -2i(\cos\Phi \pm \cos 2\Theta)$$

$$\sigma_1\sigma_2 \to \cos\frac{\Phi}{2},\ \mu_1\mu_2 = \sin\frac{\Phi}{2}$$

How is the theory changed by the insertion of the 4N local operators?

Effectively, it set $\phi_+ = \nabla\Phi = 0$, so Φ becomes a constant field

$$\mathrm{tr}\left(\rho^2\right) \to \int d\Phi \int D[\phi_-(x)]\,\exp\left\{-\int d^2x\,\frac{m}{4\pi}(\nabla\phi_-)^2 + e^{-\mu}\cos\Phi\int d^2x\cos m\phi_-\right\}$$

Moore-Read state

$\nu = \frac{1}{m}$ Moore-Read wavefunction

$$\Psi(\{z_i\}) = \mathrm{Pf}\left(\frac{1}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^m \, e^{-\sum_i |z_i|^2 / 4 l_B^2}$$

$$= \langle \prod_i \psi(z_i) e^{im\varphi(z_i)} \, e^{-i 2\pi l_B^{-2} \int d^2 z \varphi(z)} \rangle$$

Moore, Read (1991)

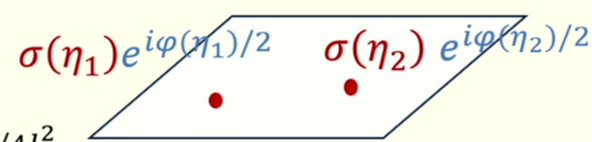
Ising $\times$ $\mathrm{U(1)_m}$ chiral boson CFT $(R = \sqrt{m}\,)$

Electron operator: $\psi_e = \psi \times e^{im\varphi}$

Primary fields:

$$e^{in\varphi}, \sigma e^{i(n+\frac{1}{2})\varphi}, \psi e^{in\varphi}, n = 0,1, \cdots m-1$$

Non-Abelian anyons: $\sigma(\eta_1) e^{i\varphi(\eta_1)/2}$ $\quad e_{qh} = \frac{e}{2m}$



$$\Psi(\eta_1, \eta_2, \{z_i\}) = \mathrm{Pf}\left(\frac{(z_i - \eta_1)(z_j - \eta_2) + i \leftrightarrow j}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^m \, e^{-\sum_i |z_i|^2 / 4 l_B^2}$$

$$= \langle \, \sigma(\eta_1) e^{i\varphi(\eta_1)/2} \sigma(\eta_2) e^{i\varphi(\eta_2)/2} \, \prod_i \psi(z_i) e^{im\varphi(z_i)} \rangle$$

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How resilient are non-Abelian anyons?

$$\mathrm{tr}(\rho^2) \approx \int d\Phi \int D[\phi_-(x)] \exp\left\{-\int d^2x \frac{m}{4\pi}(\nabla\phi_-)^2 + e^{-\mu_{\mathrm{eff}}} \cos\Phi \int d^2x \cos m\phi_-\right\}$$

This field theory is a good effective description in the sense that it gives correct qualitative behavior of correlation of operators of the form $O[\Phi, \phi_{1,2}(x)]$, e.g., $\sigma_1\sigma_2 \sim \cos\frac{\Phi}{2}$, $\varepsilon_1 + \varepsilon_2 \sim 2\cos\Phi$, $e^{im\phi_-}$

This result can be obtained by applying Coulomb-gas formalism to Ising CFT and then using plasma analogy.

Bonderson, Gurarie, Nayak (2011)

How resilient are non-Abelian anyons?

Coherent information for the topological qubit encoded by 4 quasi-holes

$$\rho_Q = e^{\mathcal{L}t}\left[\sum_{\alpha=1,\psi}\left|\Psi_{\alpha,\{\eta_i\}}\right\rangle\left\langle\Psi_{\alpha,\{\eta_i\}}\right|\right]$$

Correspond to conformal block F_α

$$\mathrm{tr}\left(\rho^2\right) \approx \int d\Phi \int D[\phi_-(x)] \exp\left\{-\int d^2x \frac{m}{4\pi}(\nabla\phi_-)^2 + e^{-\mu}\cos\Phi \int d^2x \cos m\phi_-\right\}$$

$$I_c^{(2)} = \log\frac{\mathrm{tr}\rho_{RQ}^2}{\mathrm{tr}\rho_Q^2} = \log\frac{\langle \cos^4\frac{\Phi}{2}\rangle}{\frac{3}{2}\langle\cos^2\frac{\Phi}{2}\sin^2\frac{\Phi}{2}\rangle + \frac{1}{2}\langle\cos^4\frac{\Phi}{2}\rangle}$$

$\mu < \mu_c$ (Topologically ordered phase): $\Phi, \phi_-(x)$ are pinned: $I_c^{(2)} = \log 2$

$\mu > \mu_c$ (critical phase): both $\Phi, \phi_-(x)$ are depinned, $I_c^{(2)} = 0$.

Future directions

- Other noise models
- Decoder for FQH states
- A more comprehensive description of decohered non-Abelian states
- How to experimentally detect the breakdown of non-Abelian anyons?