Title: Anatomy of a massless field: a group-theoretical glance at flat holography

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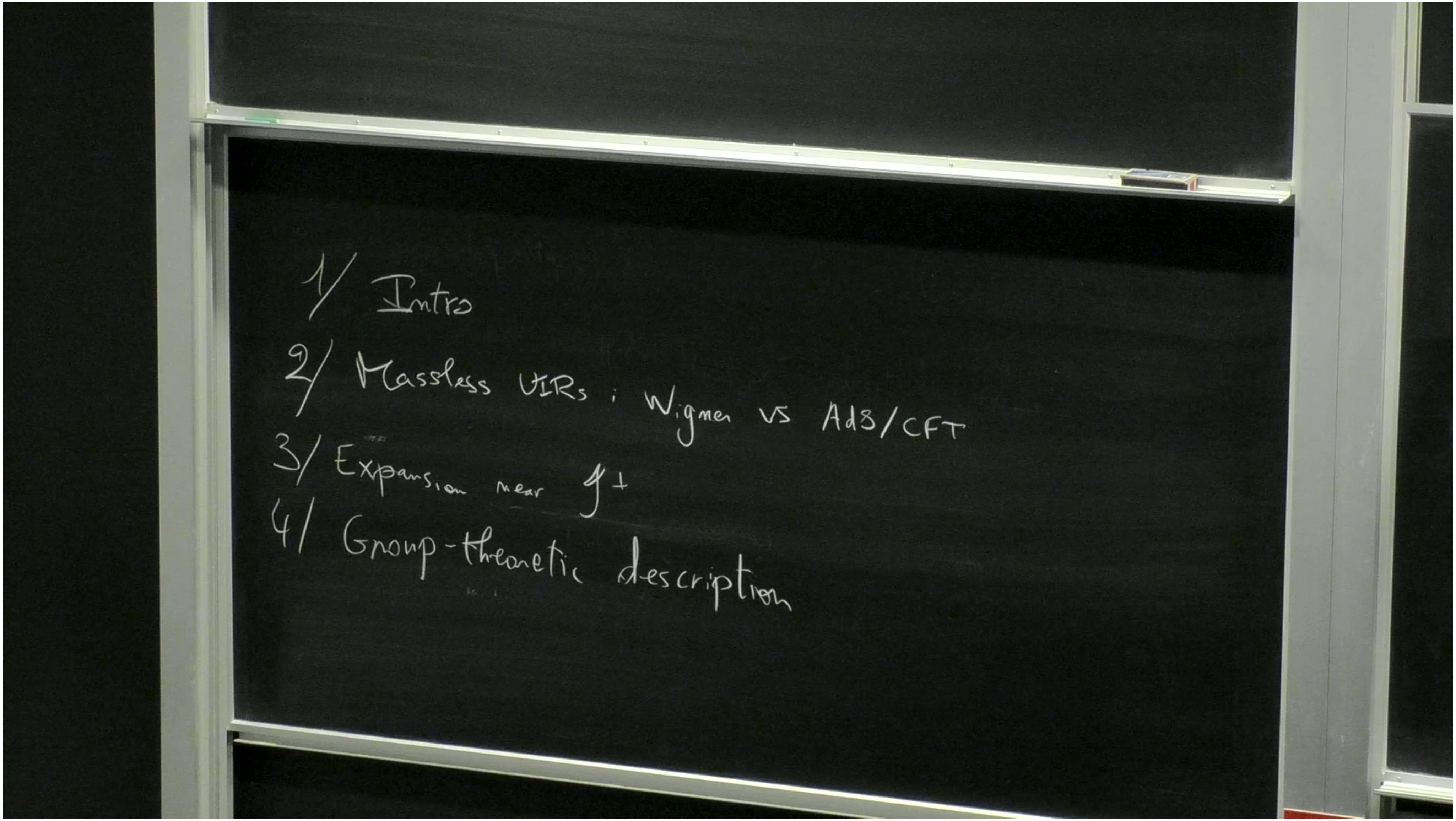
Abstract:

Many aspects of classical AdS/CFT can be understood from the point of view of group theory. In this talk, I will argue that applying the same philosophy can provide us with some powerful insights regarding the construction of its flat counterpart, dubbed flat holography. The basic setup will be a massless gauge field of arbitrary (integer) spin propagating on Minkowski spacetime of any dimensions. An asymptotic analysis will reveal the data composing the solution space, as well as the asymptotic symmetries acting on the latter. I will show how part of this solution space can be understood in terms of unitary irreducible representations of the Lorentz group, seen as the group of conformal transformations of the celestial sphere. This interpretation allows us to capture some universal aspects of field theories in Minkowski space-time within the framework of celestial holography, while we will see that the other parts of the solution space have a more natural Carrollian interpretation.


1/ Intro
2/ Massless UIRs : Wigner vs AdS/CFT
3/ Expansion near $\mathscr{I}^+$
4/ Group-theoretic description

Massless in Minkowski $ISO(D-1,1)$

——— in AdS $SO(D-1,2)$
(gauge)

field in $AdS_D \longleftarrow$ UIR $\longrightarrow$ field in CFT_{D-1}

Any $D \geq 4$, any $s \in \mathbb{N}^*$

Massless in Minkowski $ISO(D-1,1)$

——— in AdS $SO(D-1,2)$
(gauge)

field in AdS_D $\longleftarrow$ UIR $\longrightarrow$ field in CFT_{D-1}

* Any $D \geq 4$, any $s \in \mathbb{N}^*$

* linearised fields over a fixed bckgnd

2) UIRs of Poincaré

* $p^2 = 0 \xrightarrow[\text{little group}]{} ISO(D-2) \xrightarrow[\text{helicity}]{} SO(D-2)$

$$\left(m^2 = 0\,,\ Y_{D-2}\right) \qquad \boxed{\quad s \quad}$$

* $$\Box \varphi_{\mu(s)} - s\,\partial_\mu \partial^\nu \varphi_{\mu(s-1)\nu} + \frac{s(s-1)}{2}\,\partial_\mu \partial_\mu \varphi'_{\mu(s-2)} = 0$$

$$\delta \varphi_{\mu(s)} = s\,\partial_\mu \xi_{\mu(s-1)} \quad \Big| \quad \varphi'' = 0 = \xi'$$

$$\nabla^2_{AdS_D}\varphi_{\mu(s)} - s\nabla_\mu\nabla^\nu\varphi_{\nu\mu(s-1)} + \frac{s(s-1)}{2}\nabla_\mu\nabla_\mu\varphi'_{\mu(s-2)}$$

$$- \frac{1}{R^2}\left[R^2 m^2\varphi_{\mu(s)} + s(s-1)\, g_{\mu\mu}\varphi'_{\mu(s-2)}\right] = 0$$

$$R^2 m^2 = -\Delta\tilde{\Delta} - s \qquad \delta\varphi_{\mu(s)} = s\nabla_\mu\xi_{\mu(s-1)}$$

$$\Delta = D-3+s$$

$$\tilde{\Delta} = D-1-\Delta \leq \Delta$$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = \frac{R^2}{z^2}\left[dz^2 + \eta_{ij}\, dx^i dx^j\right] \qquad x^\mu = (z, x^i)$$

$$\text{GF}: \nabla^\nu \varphi_{\nu\mu(s-1)} = 0 = \varphi'_{\mu(s-2)}$$

$$\varphi_{i(s)} \ (\text{traceless}) \sim z^{\tilde{\Delta}-s}\, \tilde{\phi}_{i(s)} + \cdots + z^{\Delta-s}\, \underline{\phi_{i(s)}} + \cdots$$

$$\text{CFT unitarity} \rightarrow \Delta \geq D-3+s$$

$$\tilde{\Delta} = D-1-\Delta \leq \Delta$$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = \frac{R^2}{z^2}\left[dz^2 + \eta_{ij}\, dx^i dx^j\right] \qquad x^\mu = (z, x^i)$$

GF: $\nabla^\nu \varphi_{\nu\mu(s-1)} = 0 = \varphi'_{\mu(s-2)}$ $\qquad \boxed{\partial^i \phi_{i(s)} = 0}$

$$\varphi_{i(s)} \text{ (traceless)} \sim z^{\tilde{\Delta}-s}\, \cancelto{0}{\tilde{\phi}_{i(s)}} + \cdots + z^{\Delta-s}\, \underline{\phi_{i(s)}} + \cdots$$

CFT unitarity $\to \Delta \geq D-3+s$ $\qquad \boxed{\partial^i \mathcal{O}_{i(s)} = 0}$

3/ Expansion near $\mathscr{I}^+$

$D = d+2$

$$ds^2 = -du^2 - 2du\,dr + r^2 \underbrace{\gamma_{AB}}_{S^d} dx^A dx^B \qquad (A = 1, \dots, d)$$

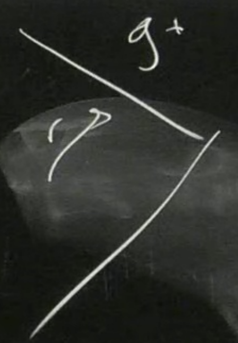
$$\varphi_{r\mu(s-1)} = 0 \,, \quad \varphi_{AB\mu(s-2)}\gamma^{AB} = 0$$

$$\hookrightarrow \varphi_{u(k)A(s-k)}$$

$$* \quad \Box\varphi_{\mu(s)} - s\,\partial_\mu\partial^\nu\varphi_{\mu(s-1)\nu} + \frac{s(s-1)}{2}\partial_\mu\partial_\mu\varphi'_{\mu(s-2)} = 0$$

$$\delta\varphi_{\mu(s)} = s\,\partial_\mu\xi_{\mu(s-1)} \quad \Big| \quad \varphi'' = 0 = \xi'$$

$$
\begin{aligned}
\overset{k=0}{\overline{\varphi}}_{A(s)} = {} & \sum_{t=0}^{s-1} \frac{1}{r^{2-2s+t}} \left[\tilde{\varphi}^{(s,t)}_{A(s)}(\hat{x}) + \cdots \right] \\
& + \frac{1}{r^{d/2-s}} \varphi^{rad}_{A(s)}(u, \hat{x}) + \cdots + \\
& + \sum_{t=0}^{s-1} \frac{1}{r^{d-2-t}} \left[\varphi^{(s,t)}_{A(s)}(x) + \cdots \right] \\
& + \sum_{n \geq 0} \frac{1}{r^{d-1+n}} \left(\chi^{(n)}_{A(s)}(x) + \cdots \right)
\end{aligned}
$$



$$k=0$$

asymptotic symmetries

$$\delta \tilde{\varphi}^{(s,t)}_{A(s)} = \mathcal{D}_{\langle A} \cdots \mathcal{D}_{A\rangle} \xi^{(s,t)}_{A(s-t-1)}$$

$$\varphi_{A(s)} = \sum_{t=0}^{s-1} \frac{1}{r^{d-2s+t}} \left[\tilde{\varphi}^{(s,t)}_{A(s)}(\hat{x}) + \cdots \right]$$

$$+ \frac{1}{r^{d/2-s}} \varphi^{rad}_{A(s)}(u,\hat{x}) + \cdots +$$

$$+ \sum_{t=0}^{s-1} \frac{1}{r^{d-2-t}} \left[\varphi^{(s,t)}_{A(s)}(x) + \cdots \right]$$

$$+ \sum_{n\geq 0} \frac{1}{r^{d-1+n}} \left(\chi^{(n)}_{A(s)}(x) + \cdots \right)$$

$$\varphi_{A(s)} = \sum_{t=0} r^{2-(s+t)} \left[\varphi_{A(s)}^{(t)} \cdots \right] \cdots^{(t+1)}$$

$$+ \frac{1}{r^{d/2-s}} \varphi^{rad}_{A(s)}(u,\hat{x}) + \cdots + \qquad \overbrace{D^A \cdots D^A}^{(t+1)} \varphi^{(s,t)}_{A(s)} = 0$$

$$+ \sum_{t=0}^{s-1} \frac{1}{r^{d-2-t}} \left[\varphi^{(s,t)}_{A(s)}(x) + \cdots \right]$$

$$+ \sum_{n \geq 0} \frac{1}{r^{d-1+n}} \left[\chi^{(n)}_{A(s)}(x) + \cdots \right]$$

$\varphi_{u(k)A(s-k)}$ $\quad k \geq 1 \qquad \varphi_{u(k)A(s-k)} = r^{2s-2-k}(\ldots) + \frac{1}{r^{d-1}}\left[m_{u(k)A(s-k)}(\hat{x}) + \right]$

$\varphi_{i(s)}$ (traceless) $\sim z^{\tilde{\Delta}-s}\,\cancel{\tilde{\varphi}_{i(s)}} + \cdots + z^{\Delta-s}\,\underline{\phi_{i(s)}} +$

CFT unitarity $\to \Delta \geq D-3+s \qquad \boxed{\partial^i O_{i(s)} = 0}$

$j_{i(s)} \sim \varphi^* \overleftrightarrow{\partial \cdots \partial_i}\, \varphi \qquad \Box\varphi = 0$

4/ Holographic interpretation

* $\varphi^{rad}_{A(s)}(u,\hat{x}) \xrightarrow{\text{Fourier}} \varphi^{rad}_{A(s)}(\omega,\hat{x}) \xrightarrow{\text{Mellin}} \varphi^{rad}_{A(s)}(\Delta,\hat{x})$

$\Delta \in \frac{d}{2} + i\mathbb{R}$

$SO(d+1,1)$ (PCS)

* Exceptional series, $\delta \varphi^{(s,t)}_{A(s)} = D_{(A} \cdots D_{A}\, \xi^{(s,t)}_{A(s-t-1))}$

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$\gamma_{AB} + \varphi^{(2,0)}_{AB} + \partial(\varphi^2)$

$$S_{current} = \int \sqrt{\gamma}\, d^d x \; \varphi^{(s,t)}_{A(s)} \; \tilde{\varphi}^{A(s)}_{(s,t)}$$

* Exceptional series $\delta \alpha^{(s)}(\hat{x}) = \xi^{(s,s-1)}(x)$ $\qquad D^A \cdots D^A \xi$

$\delta \alpha^{(1)} = \alpha(\hat{x})$

$\delta \alpha^{(2)} = T(\hat{x})$

$T_i{}^j(x^i)$

$(d+1)$-dim CFT

$\partial_j T_i{}^j = 0 \longrightarrow \begin{cases} \frac{1}{c}\partial_t T_t{}^t + \partial_A T_t{}^A = 0 \\ \frac{1}{c}\partial_t T_t{}^A + \partial_B T_A{}^B = 0 \end{cases}$

$T_i{}^i = 0$

$\downarrow c \to 0$

$m_B(x)$

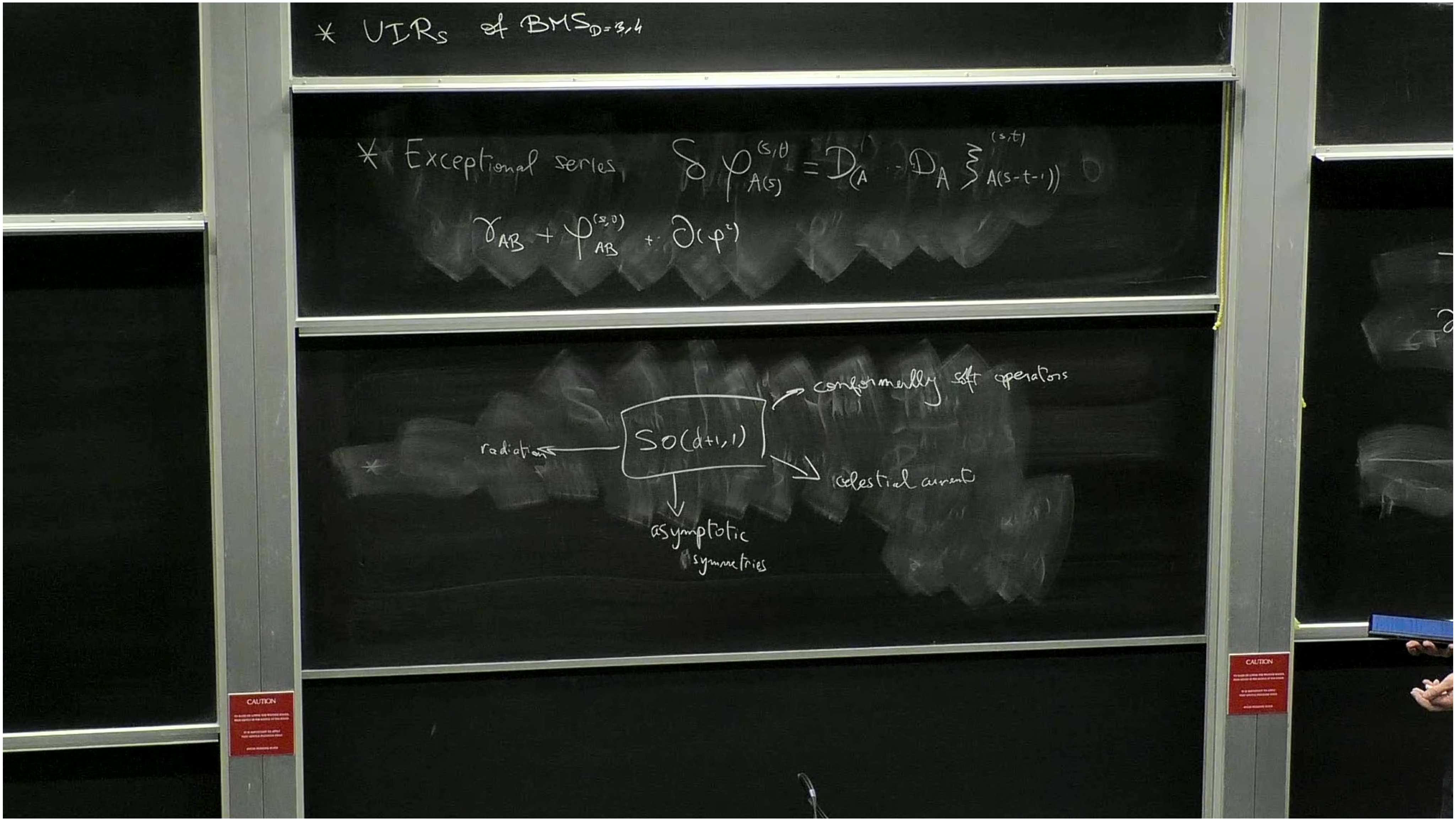
$\begin{cases} \partial_t T_t{}^t = 0 \\ \partial_t T_A{}^t - \frac{1}{d}\partial_A T_t{}^t = 0 \end{cases}$

$N_A(x)$

$\sum_{n \geq 0} t^n T^{(n)}_{\langle AB \rangle}(x)$

$\|$

$T^{(t,x)}_{\langle AB \rangle}$


* UIRs of BMS$_{D=3,4}$
* Exceptional series, $\delta\varphi^{(s,t)}_{A(s)} = D_{(A} \cdots D_{A}\,\xi^{(s,t)}_{A(s-t-1))}$
$\gamma_{AB} + \varphi^{(s,0)}_{AB} + \partial(\varphi^2)$
conformally soft operators
SO(d+1,1)
radiation
celestial current
asymptotic symmetries