

Title: Quantum droplets of dark matter

Speakers: Ian Moss

Collection/Series: Cosmology and Gravitation

Subject: Cosmology

Date: February 25, 2025 - 11:00 AM

URL: <https://pirsa.org/25020045>

Abstract:

Vacuum polarisation can destabilise dark matter models made from ultralight, weakly coupled scalar fields. The dark matter condenses into quantum droplets, like those seen in Bose Einstein condensates, and could contribute to explaining some cosmological conundrums.



Quantum droplets of dark matter

Ian Moss

February 2025



Contents

- Cosmological conundrums
- Fuzzy dark matter droplets
- Structure formation
- BEC analogues

Based on arXiv 2407.13243



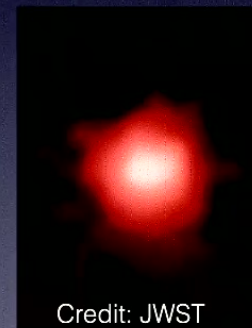
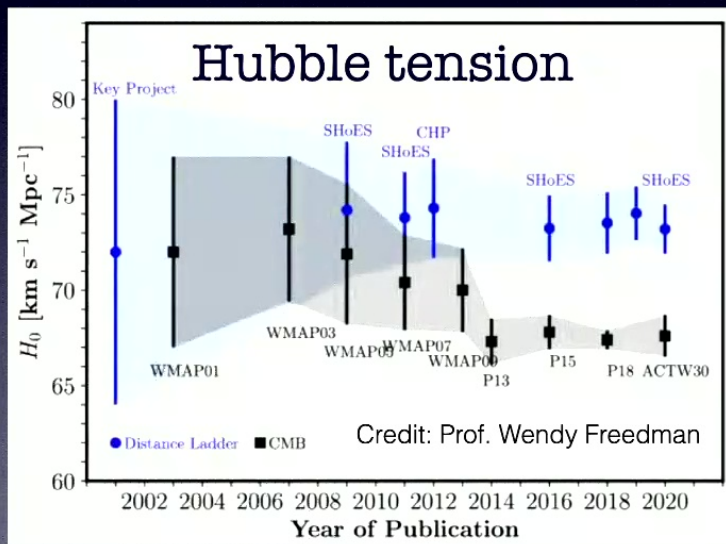
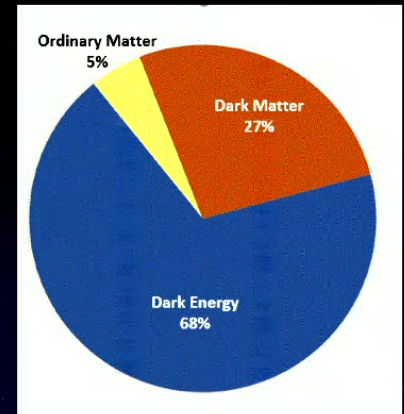
+Tom and Nick



<https://blogs.ncl.ac.uk/cosmology/>

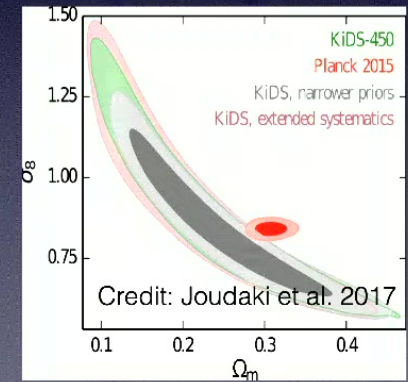
Beyond Lambda-CDM

The standard model of cosmology includes pressure free matter (CDM) and a cosmological constant (Lambda). There are some hints that we may need to go beyond Lambda-CDM:



galaxies at large z

density fluctuations

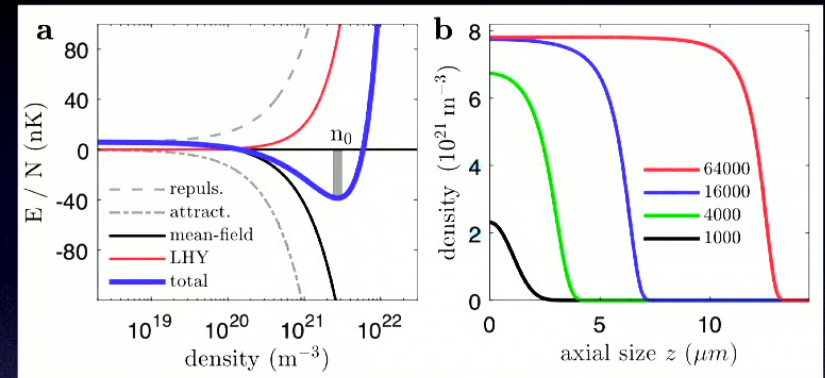


Dark matter quantum droplets

The idea comes from Bose Einstein Condensates

Attractive forces balanced by repulsive quantum forces (LHY)

Ultra-light scalar field as dark matter with a quantum corrected potential



Credit: Bottcher et al. 2020

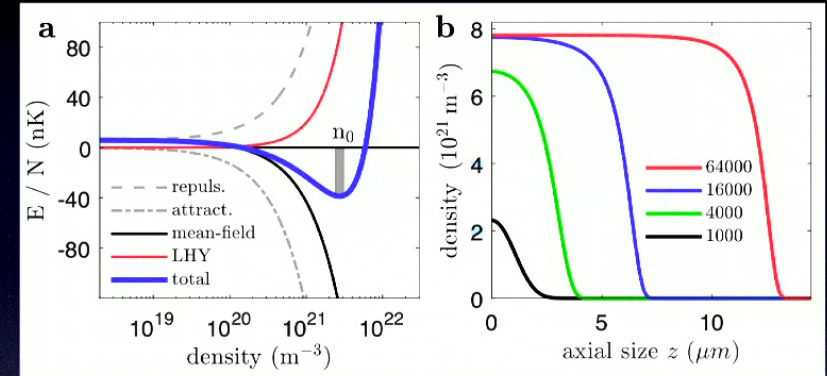


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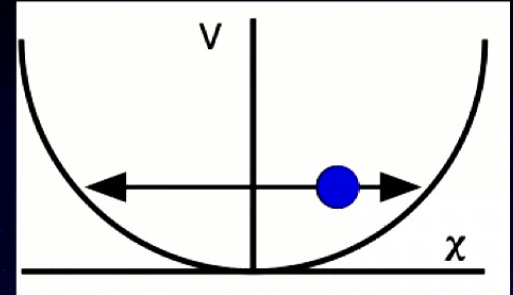
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Fuzzy dark matter with vacuum polarisation

Fuzzy dark matter consists of a **light scalar field** χ

In an expanding universe: $\ddot{\chi} + 3H\dot{\chi} - \frac{1}{a^2}\nabla^2\chi + \frac{\partial V}{\partial\chi^*} = 0$



In the homogeneous WKB limit the field oscillates

$$\chi = \sqrt{n(t)}e^{-i\omega t}$$

$$V = \lambda|\chi|^4 \implies n \propto a^{-4}$$

dark radiation

$$V = m^2|\chi|^2 \implies n \propto a^{-3}$$

dark matter

Droplets form for a Coleman-Weinberg potential

$$V = m^2|\chi|^2 + \alpha^2|\chi|^4 \ln \frac{|\chi|^2}{\mu_R^2}$$

Cosmological Structure formation

Use density perturbation theory: $\delta = \delta n/n$

$$\ddot{\delta} + 2H\dot{\delta} + \left(\frac{k^4}{4m^2 a^4} + \frac{gnk^2}{ma^2} \ln \frac{n}{n_d} - 4\pi Gnm \right) \delta = 0 \quad g = \alpha^2/m^2$$

pressure
droplet
gravity

Jeans length λ_J gravitational instability - power-law growth

Uniform density droplets can form when $n < n_d$

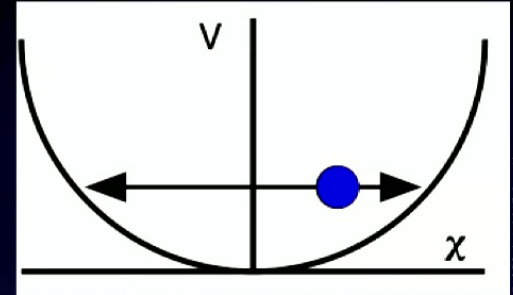
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droplet size λ_d droplet instability - **exponential** growth

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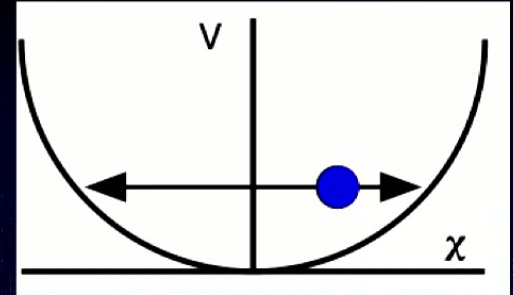
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Structure formation - BEC

Need to consider:

The range of droplet sizes

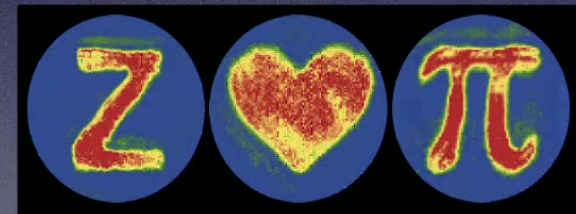
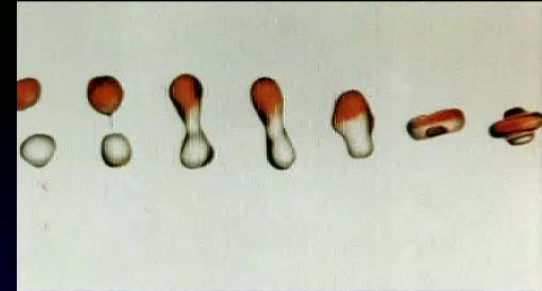
The initial velocity of the droplets

Merging droplets

These could be studied in a laboratory BEC: the quantum potential in 2D has the logarithmic form. The Hubble parameter is replaced by the rate of change of some external parameter.

cosmology: lower n fix n_d rate H

BEC: fix n raise n_d rate v

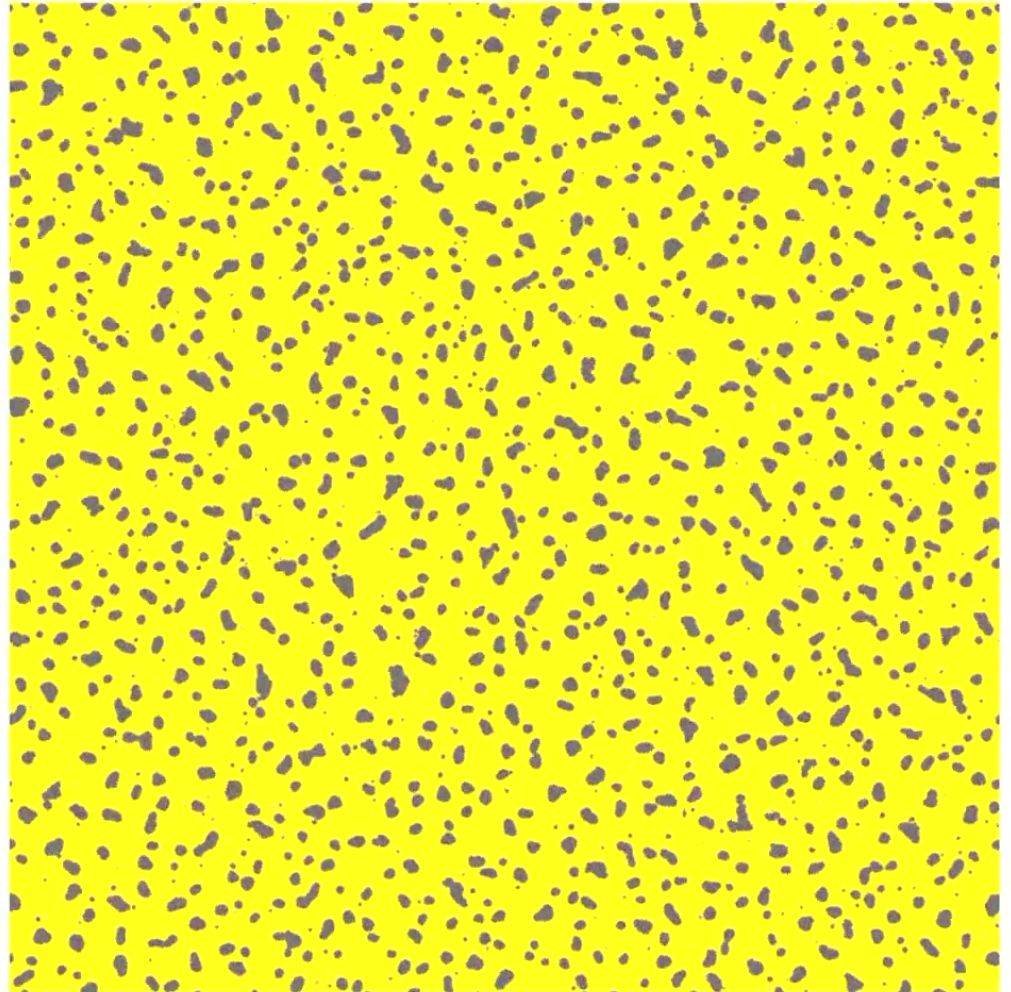


Cambridge 2D optical trap

GPE simulations for K39

GPE solutions with vacuum
fluctuation initial conditions


Raise n_d rather
than lower n



Structure formation - 2D BEC

For a BEC mixture close to instability, the mean condensate field satisfies

$$i\hbar\dot{\psi} = -\frac{\hbar^2}{2m}\nabla^2\psi - \mu\psi + g \ln\left(\frac{|\psi|^2}{n_d}\right) |\psi|^2\psi \quad \text{GPE}$$

LHY quantum term 

Unstable modes when $n < n_d$: Growth rate v_d :

$$\omega^2 = \frac{k^2}{2m} \left[\frac{\hbar^2 k^2}{2m} + 2gn \ln \frac{n}{n_d} \right] \quad v_d = |\omega|_{\max} = \frac{\hbar k_{\max}^2}{2m} = \frac{gn}{\hbar} \left| \ln \frac{n}{n_d} \right|$$

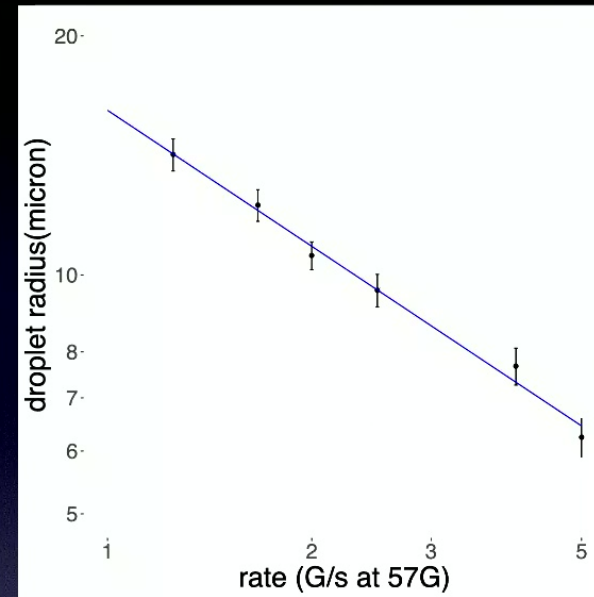
Compare v_d to $v = dn/dt$ $k_{\max} \propto v^{1/2}$

(similar to Kibble-Zurek mechanism)

GPE simulations for K39

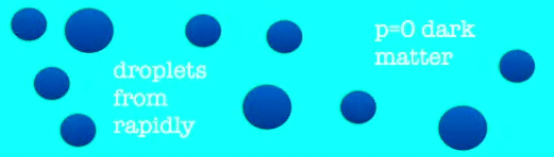
Droplet radius depends on
the rate of parameter
change v during the ramp

$$r \propto v^{-d} \quad d = -0.57$$



Zero T: **not** Kibble-Zureck, but $\zeta \sim |n - n_d|^{-\nu}$ $\tau \sim |n - n_d|^{1-\nu/d} \implies r \propto v^{-d}$

Questions around the validity of the LSY terms: renormalisation? Crossover: 3D to 2D?



CMB observations

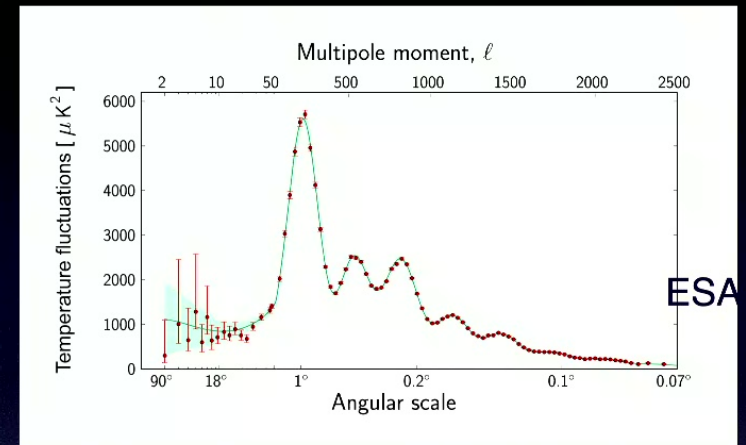
Last scattering surface at redshift z_*

$$\theta_s = \frac{\text{sound horizon at } z_*}{\text{distance to } z_*} = \frac{2c_s/H_*}{D_*}$$

$$h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$D_* \propto 1/h$$

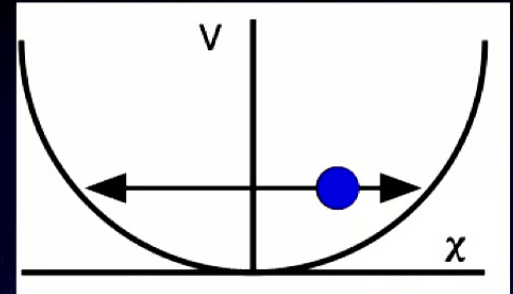
In the droplet model, there is an increase in the energy density compared to CDM



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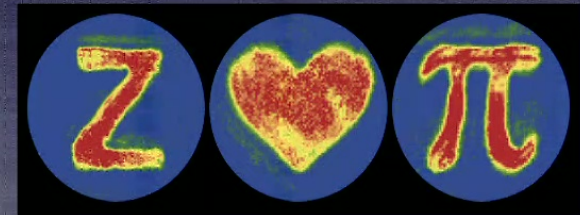
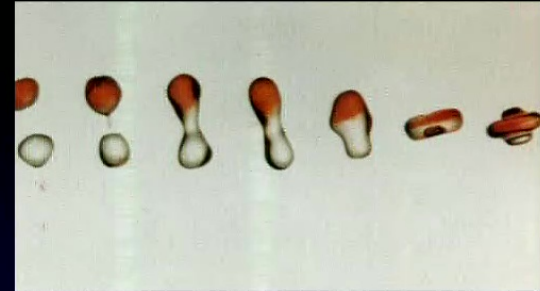
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CMB observations

The droplet model has higher density than Λ CDM :

$$\delta \ln \rho_{\text{DM}^*} = \frac{1}{2} \frac{gn_d}{mc^2} \left(\frac{1+z_*}{1+z_d} \right)^3 \left\{ \ln \left(\frac{1+z_*}{1+z_d} \right)^3 - 1 \right\}$$

The sound horizon is held fixed:

$$\delta \ln \theta_s \approx 0.2 \delta \ln h + 0.14 \delta \ln \omega_m - 0.55 \delta \ln \rho_{\text{DM}^*}$$

The CMB prediction of the expansion rate increases by

$$\delta \ln h \approx 2.5 \delta \ln \rho_{\text{DM}^*}$$

CMB observations

For a more sophisticated analysis, the theory has three new parameters

gn_d/m z_d control the CMB observations

$$\lambda_{CW} = \frac{2\pi\hbar}{mc}$$

controls the droplet size/density

$$\lambda_d = \left(\frac{mc^2}{gn_d} \right)^{1/2} \lambda_{CW}$$

$mc^2(\text{eV})$	z_d	$\lambda_d(\text{m})$	$M_d(\text{kg})$	α	$\delta \ln h$
10^{-20}	200	1.58×10^{16}	8.9×10^{27}	7.24×10^{-41}	5.0×10^{-2}
10^{-16}	200	1.58×10^{12}	8.9×10^{15}	7.24×10^{-33}	5.0×10^{-2}
10^{-10}	200	1.58×10^6	8.9×10^{-3}	7.24×10^{-16}	5.0×10^{-2}
10^{-2}	200	1.58×10^{-2}	8.9×10^{-27}	7.24×10^{-5}	5.0×10^{-2}



From fantasy to reality?

Quantum corrections to the Fuzzy dark matter models can lead to droplets

Structure forms rapidly in droplet models and the cosmological parameters extracted from the CMB are altered.

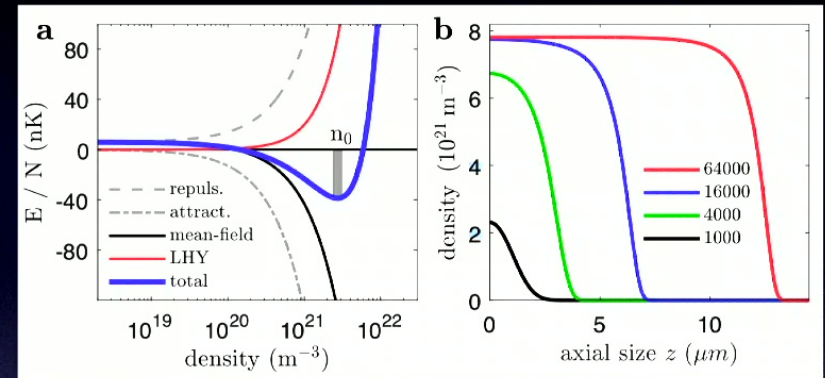
Droplet interactions could be explored in the laboratory.

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