Title: Nonabelian Anyon Condensation in 2+1d topological orders: A String-Net Model Realization

Speakers: Yidun Wan

Collection/Series: Quantum Matter

Subject: Condensed Matter

Date: February 18, 2025 - 3:30 PM

URL: https://pirsa.org/25020042

Abstract:

In this talk, we develop a comprehensive framework for realizing anyon condensation of topological orders within the string-net model by constructing a Hamiltonian that bridges the parent string-net model before and the child string-net model after anyon condensation. Our approach classifies all possible types of bosonic anyon condensation in any parent string-net model and identifies the basic degrees of freedom in the corresponding child models. Compared with the traditional UMTC perspective of topological orders, our method offers a finer categorical description of anyon condensation at the microscopic level. We also explicitly represent relevant UMTC categorical entities characterizing anyon condensation through our model-based physical quantities, providing practical algorithms for calculating these categorical data.

NON-ABELIAN ANYON CONDENSATION IN 2+1D TOPOLOGICAL ORDERS: A STRING-NET MODEL REALIZATION

Yidun Wan

PI Quantum Matter Seminar

Feb 18, 2025

based on arXiv:2409.05852, to appear in JHEP

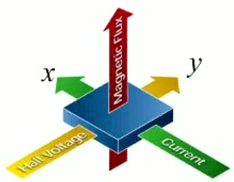
Yu Zhao

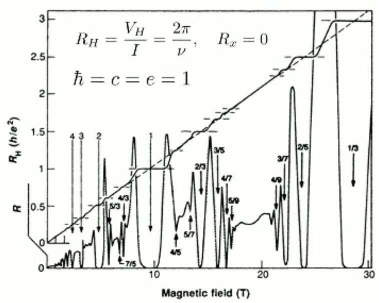
He's looking for a postdoc position

COLLABORATOR

TOPOLOGICALLY ORDERED MATTER PHASES

Fractional quantum Hall states





$$\nu = \frac{1}{3}, \frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{3}{7}, \dots$$

Very particular fractions

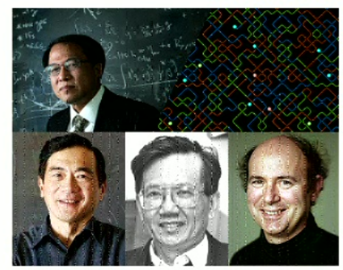
Distinct FQHS share the same symmetry

↓

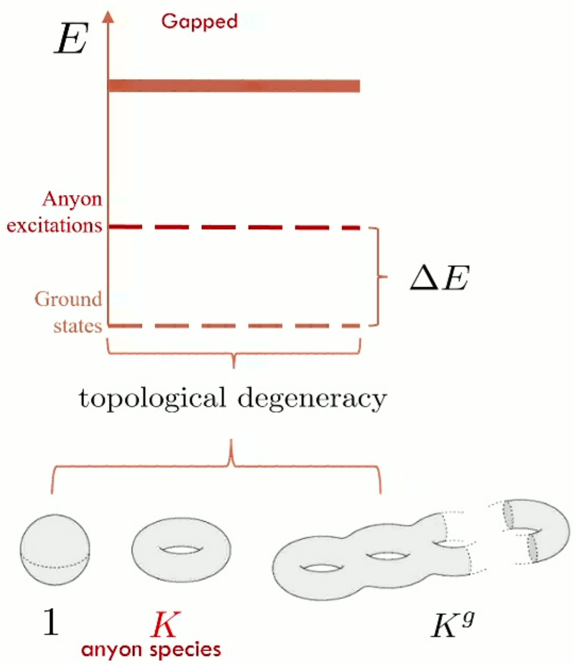
Landau-Ginzburg symmetry breaking **fails**

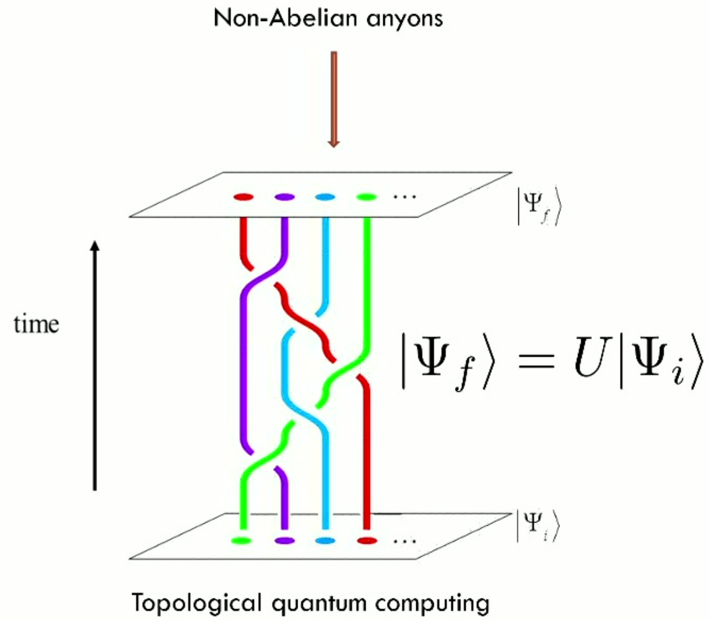
↓

Topological orders



TOPOLOGICAL ORDERS IN 2D

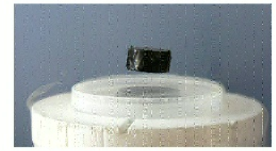




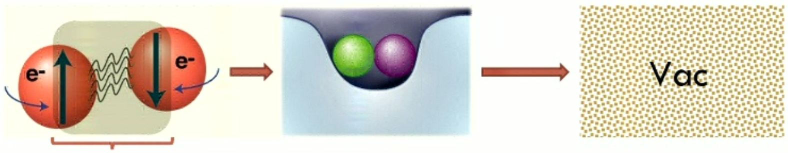
Certain anyons may condense like usual bosons do in the Higgs mechanism

Let’s briefly review the Cooper pair condensation and Higgs boson condensation

HIGGS MECHANISM: SUPERCONDUCTORS



Condense: Cooper pairs of electrons

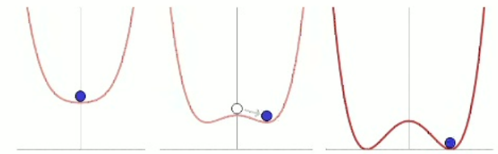


= Coherent State of Cooper Pairs

Charged boson

7

HIGGS MECHANISM: ELECTROWEAK BREAKING

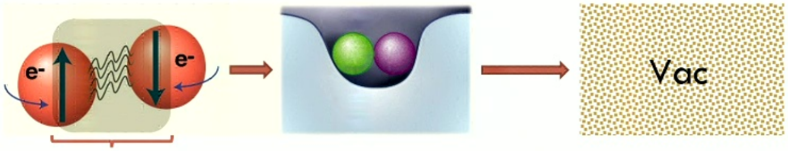


Higgs Boson: Complex doublet with the weak $\mathrm{U}(1)_{\mathrm{Y}} \times \mathrm{SU}(2)_{\mathrm{L}}$ symmetry---Four components!

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_0 + i\phi_3 \end{pmatrix}$$

HIGGS MECHANISM: SUPERCONDUCTORS

Condense: Cooper pairs of electrons

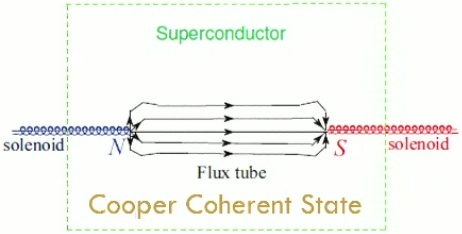


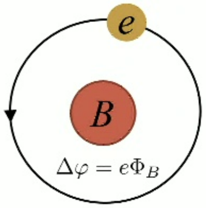
= Coherent State of Cooper Pairs

Charged boson

Gap: photons (gauge bosons) gapped (massive) by eating the Goldstone mode

Confinement: magnetic fluxes confined in Type-II superconductors:

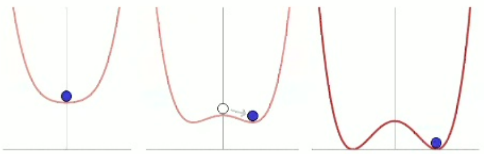




Aharonov-Bohm effect

7

HIGGS MECHANISM: ELECTROWEAK BREAKING



Higgs Boson: Complex doublet with the weak $\mathrm{U}(1)_{\mathrm{Y}} \times \mathrm{SU}(2)_{\mathrm{L}}$ symmetry---Four components!

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_0 + i\phi_3 \end{pmatrix}$$

Condensation: One component of Higgs Boson condenses (unitary gauge)

$$\langle \phi_0 \rangle = \frac{\sqrt{2}}{2} v, \quad \langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_3 \rangle = 0$$

Gap: $W^{\pm}, Z^0$ bosons "eat" Goldstone modes and are gapped (massive)

$$\mathcal{L}_{\mathrm{YMH}} = \left| \left(\partial_\mu - \frac{ig}{2} W_\mu^a \sigma_a - \frac{ig'}{2} B_\mu \right) \phi \right|^2$$

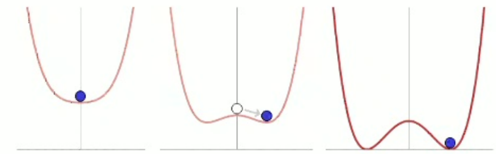
$$Z_\mu = \frac{g}{\sqrt{g^2 + g'^2}} W_\mu^3 - \frac{g'}{\sqrt{g^2 + g'^2}} B_\mu$$

Splitting: e^L and ν_e^L split

$$\mathcal{L}_{\mathrm{L}} = (\bar{\nu}_e^L \quad \bar{e}^L) \frac{ig}{2} W_\mu^a \sigma_a \begin{pmatrix} \nu_e^L \\ e_L \end{pmatrix}$$

8

HIGGS MECHANISM: ELECTROWEAK BREAKING



Higgs Boson: Complex doublet with the weak $\mathrm{U}(1)_\mathrm{Y} \times \mathrm{SU}(2)_\mathrm{L}$ symmetry---Four components!

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_0 + i\phi_3 \end{pmatrix}$$

Condensation: One component of Higgs Boson condenses (unitary gauge)

$$\langle\phi_0\rangle = \frac{\sqrt{2}}{2}v, \quad \langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_3\rangle = 0$$

Gap: $W^\pm, Z^0$ bosons "eat" Goldstone modes and are gapped (massive)

$$\mathcal{L}_\mathrm{YMH} = \left|\left(\partial_\mu - \frac{ig}{2}W_\mu^a\sigma_a - \frac{ig'}{2}B_\mu\right)\phi\right|^2$$

$$Z_\mu = \frac{g}{\sqrt{g^2+g'^2}}W_\mu^3 - \frac{g'}{\sqrt{g^2+g'^2}}B_\mu$$

Splitting: e^L and ν_e^L split

$$\mathcal{L}_\mathrm{L} = (\bar{\nu}_e^L \quad \bar{e}^L)\frac{ig}{2}W_\mu^a\sigma_a\begin{pmatrix} \nu_e^L \\ e_L \end{pmatrix}$$

Identification: e^L and e^R combine as a Dirac fermion

$$\mathcal{L}_\mathrm{Yukawa} = e_L^\dagger(\phi_0 + i\phi_3)e_R \rightarrow \mathcal{L}_e = m e_L^\dagger e_R$$

ANYON CONDENSATION VS BOSON CONDENSATION

Analogy

Anyon Condensation:
Anyons of certain types are condensed

Cooper-Pair/Higgs Boson Condensation

Child Topological Phase:
Ground States = Coherent states with condensed anyons

Vac = Coherent State of Cooper Pairs/Higgs bosons

Splitting:
An anyon may split into different sectors

e^L and ν_e^L splitting

Identification:
Different sectors from different anyons may be identified

e^L and e^R combine as a Dirac fermion

Confinement:
Certain sectors are confined in the child phase

Magnetic Fluxes Confinement

Gapped Gauge field:
Certain input dof (gauge fields) are gapped in the child phase

Photon, W, and Z become massive

TRADITIONAL CATEGORICAL DESCRIPTION OF ANYON CONDENSATION

Parent Topological Order	Unitary Modular Tensor Category (UMTC) $\mathcal{C}$
Parent Anyon Type J	Simple Objects $J \in \mathrm{Irr}(\mathcal{C})$
Condensate	Twist-Free Commutative Separable Frobenius Algebra (CSFA) Composite Object $\boldsymbol{A} = \bigoplus_{\mathrm{Condensed}\ J} J$ in $\mathcal{C}$
Child Topological Order	$\mathrm{Rep}_{\mathcal{C},0}(\boldsymbol{A})$
Child Vacuum	$\boldsymbol{A} = 1_{\boldsymbol{A}}$, trivial representation of $\boldsymbol{A}$
Child Anyon Types J_{Child}	Irreducible representations of $\boldsymbol{A}$ braiding trivially with $\boldsymbol{A}$

Cons: Only Describe relationships of topological properties of parent and child topological orders!

Ignore fundamental physical details, e.g., basic degrees of freedom of the underlying system!

Need basic field configurations! Need Hilbert space! Need Hamiltonian! Need a ***model!***

ANYON CONDENSATION: NEED A MODEL

Certain types of anyons in the parent phase are condensed:

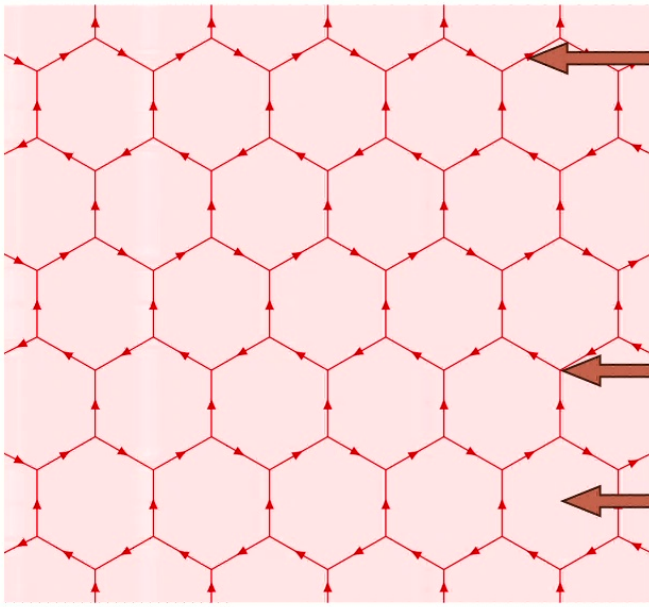
$H_{\text{Parent}} \Rightarrow H_{\text{Parent}} - \Lambda P,$ $\Lambda \in [0, +\infty)$: phase transition parameter.

Projector = Sum of Condensed Anyon Creation Operators

Child phase:

- Hilbert Space: $\mathcal{H}_{\text{Child}} = P\mathcal{H}_{\text{Parent}}$
- Hamiltonian: $H_{\text{Child}} = PH_{\text{Parent}}P$
- Ground States: $|\Phi\rangle_{\text{Child}} = P|\Phi\rangle_{\text{Parent}} =$ Coherent states with arbitrarily many condensed anyon types.
- Excited States: $|\psi\rangle_{\text{Child}} = P|\psi\rangle_{\text{Parent}}$

THE STRING-NET MODEL, BUT OUR VERSION



Edges: Carrying Basic Dofs
Dofs $\in$ Simple objects of $\mathcal{F}$

Vertices: A_v Operators
Detecting Charge Existence

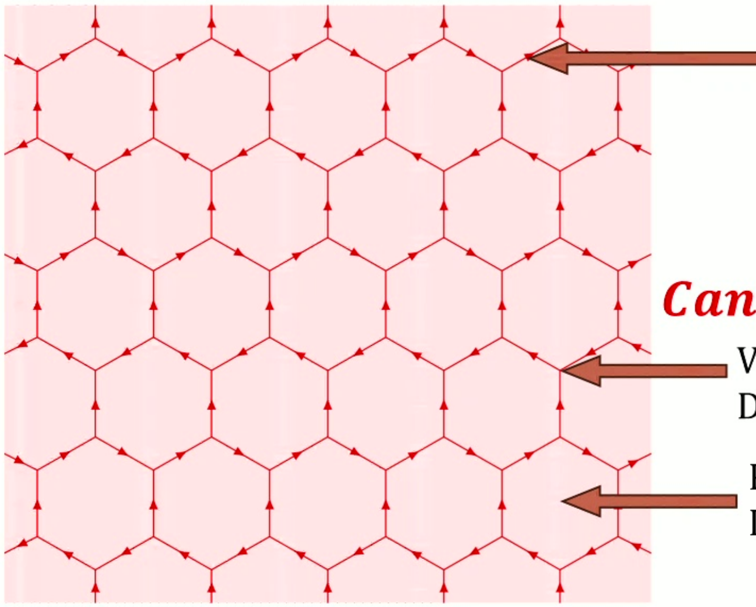
Plaquettes: B_P Operators
Detecting Flux Existence

Original String-Net Model; Input Fusion Category $\mathcal{F}$

$$H = -\sum_{\text{Vertices } V} A_V - \sum_{\text{Plaquettes } P} B_P$$

Sum of Commuting Projectors!
Exactly Solvable!

THE STRING-NET MODEL, BUT OUR VERSION



Edges: Carrying Basic Dofs
Dofs $\in$ Simple objects of $\mathcal{F}$

Cannot fully describe charge excitations!

Vertices: A_v Operators
Detecting Charge Existence

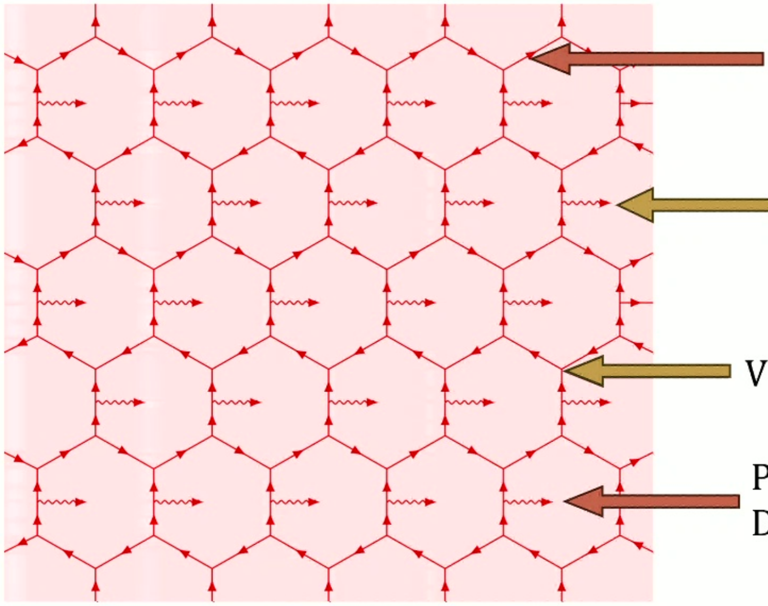
Plaquettes: B_P Operators
Detecting Flux Existence

Original String-Net Model; Input Fusion Category $\mathcal{F}$

$$H = -\sum_{\text{Vertices } V} A_V - \sum_{\text{Plaquettes } P} B_P$$

Sum of Commuting Projectors!
Exactly Solvable!

THE STRING-NET MODEL, BUT OUR VERSION



Edges: Carrying Basic Dofs
Dofs $\in$ Simple objects of $\mathcal{F}$

Tails: Carrying Basic Dofs as Charges
How charges and gauge fields couple

Vertices: In Hilbert space, $A_V = 1$ Always Satisfied

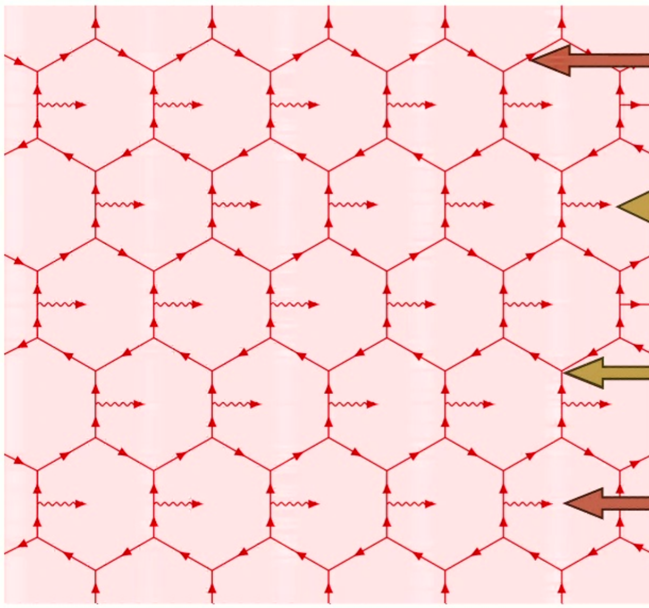
Plaquettes: B_P Operators
Detecting Flux Existence

String-Net Model, Our Version; Input Fusion Category $\mathcal{F}$

$$H = -\sum_{\text{Vertices } V} A_V - \sum_{\text{Plaquettes } P} B_P$$

Sum of Commuting Projectors!
Exactly Solvable!

THE STRING-NET MODEL, BUT OUR VERSION



Edges: Carrying Basic Dofs
Dofs $\in$ Simple objects of $\mathcal{F}$

Tails: Carrying Basic Dofs as Charges
How charges and gauge fields couple

Vertices: In Hilbert space, $A_V = 1$ Always Satisfied

Plaquettes: B_P Operators
Detecting Flux Existence

String-Net Model, Our Version; Input Fusion Category $\mathcal{F}$

$$H = \sum_{\text{Vertices } V} A_V - \sum_{\text{Plaquettes } P} B_P \, \delta_{\text{Dof on Tail in } P, 1}$$

Q_P Operator

Sum of Commuting Projectors!
Exactly Solvable!

THE STRING-NET MODEL: EXCITATIONS

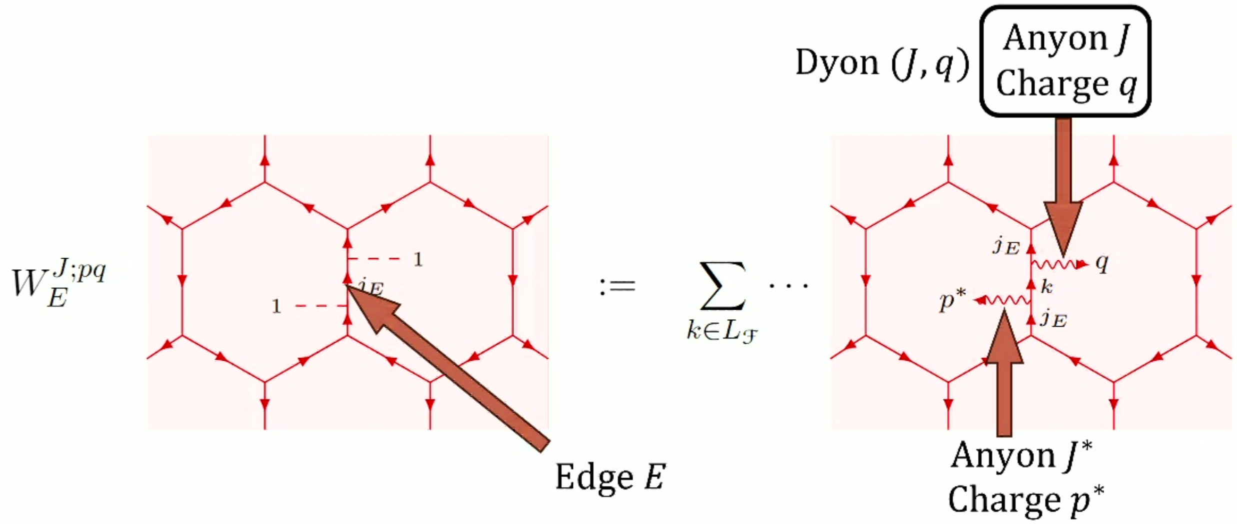
Ground States: $Q_P = 1$ for all P

Excited States: $Q_P = 0$ for certain P

Anyon $J \in \mathrm{Irr}(\mathcal{Z}(\mathcal{F}))$ in P, where $Q_P = 0$.

Creating anyons by $W_E^{J;pq}$

Represent anyons as dyons



THE STRING-NET MODEL: EXCITATIONS

Ground States: $Q_P = 1$ for all P

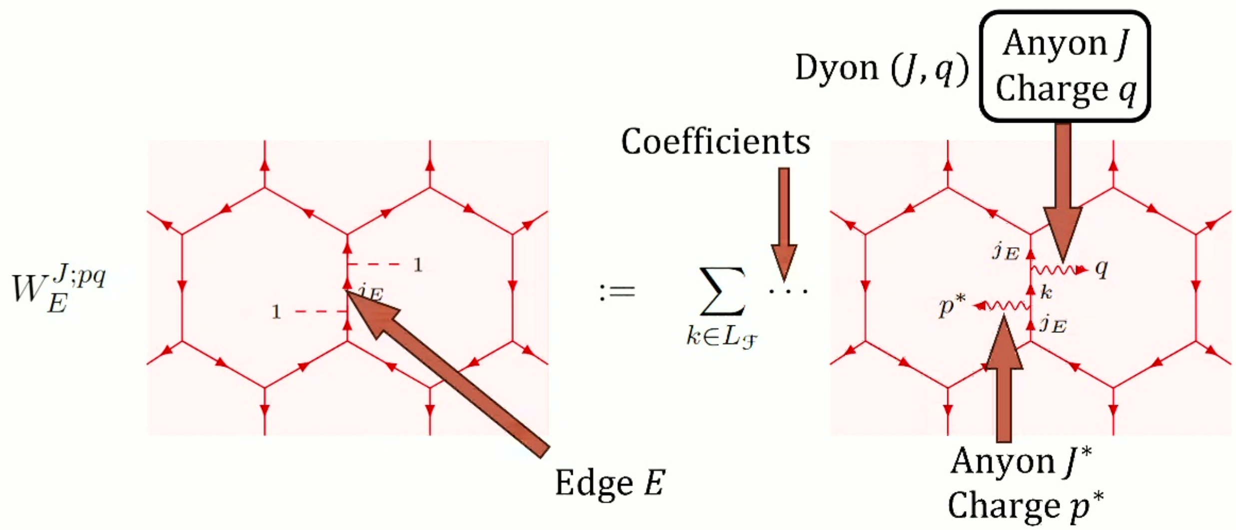
Excited States: $Q_P = 0$ for certain P

Anyon $J \in \mathrm{Irr}\big(\mathcal{Z}(\mathcal{F})\big)$ in P, where $Q_P = 0$.

Creating anyons by $W_E^{J;pq}$

Represent anyons as dyons

Manifest *Internal Spaces*



THE STRING-NET MODEL: EXCITATIONS

- Example: Fibonacci Model
- Input UFC Fibo: $\mathrm{Irr} = \{1, \tau\}$
- Anyon Types: $\mathrm{Irr}(\mathcal{Z}(\mathrm{Fibo}))$:

  $1\bar{1}, \quad 1\bar{\tau}, \quad \tau\bar{1}, \quad \tau\bar{\tau}$
- Dyon Type:

  $(1\bar{1}, 1), (\tau\bar{1}, \tau), (1\bar{\tau}, \tau), (\tau\bar{\tau}, 1), (\tau\bar{\tau}, \tau)$

Ground States: $Q_P = 1$ for all P

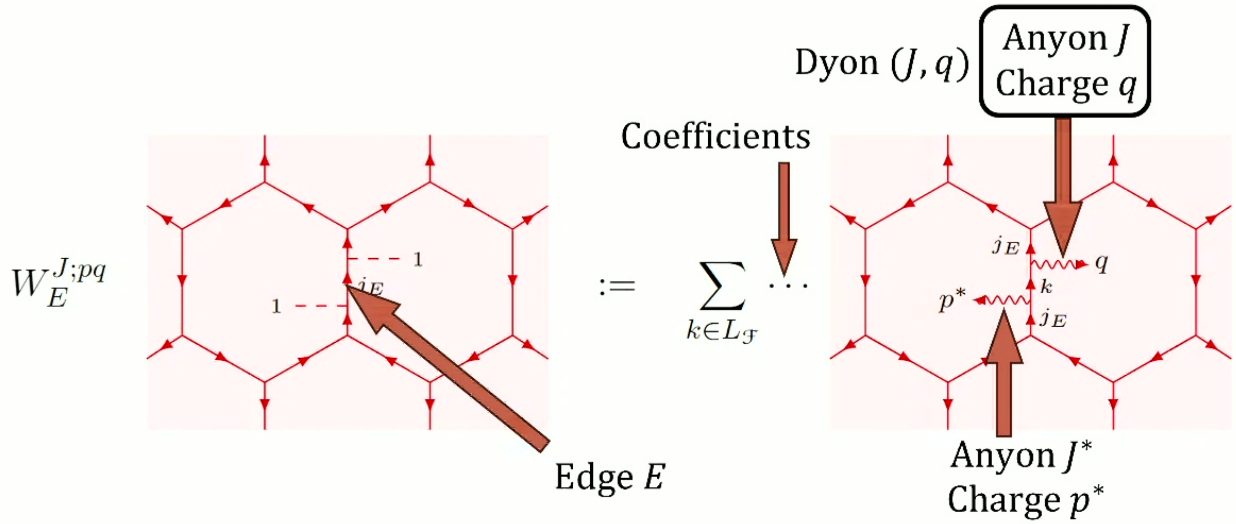
Excited States: $Q_P = 0$ for certain P

Anyon $J \in \mathrm{Irr}(\mathcal{Z}(\mathcal{F}))$ in P, where $Q_P = 0$.

Creating anyons by $W_E^{J;pq}$

Represent anyons as dyons

Manifest *Internal Spaces*



ANYON CONDENSATION: IN THE STRING-NET MODEL

Certain dyon types in the parent model are condensed:

$$H_{\mathcal{F}} \quad \Rightarrow \quad H_{\mathcal{F}} - \Lambda \sum_{\text{Edges } E} \boxed{\sum_{\substack{\text{Condensed} \\ \text{Anyon Type J}}} \sum_{p,q} \frac{\pi^J_{pq}}{d_p d_q} W_E^{J;pq}} \rightarrow \text{Local Projector } P_E$$

P_E: Project out certain basic degrees of freedom!

$$\mathcal{H}_E^{\mathcal{F}} = \{a \in \mathrm{Irr}(\mathcal{F})\} \rightarrow H_E^{\mathcal{S}} = P_E \mathcal{H}_E^{\mathcal{F}} \subset \mathcal{H}_E^{\mathcal{F}}$$

Child Model Input Data: Subcategory $\mathcal{S} \subset \mathcal{F}$

$$H_E^{\mathcal{S}} = \{r \in \mathrm{Irr}(\mathcal{S})\}$$

Question: How to calculate π^J_{pq}? What is the child input subcategory S?

ANYON CONDENSATION: IN THE STRING-NET MODEL

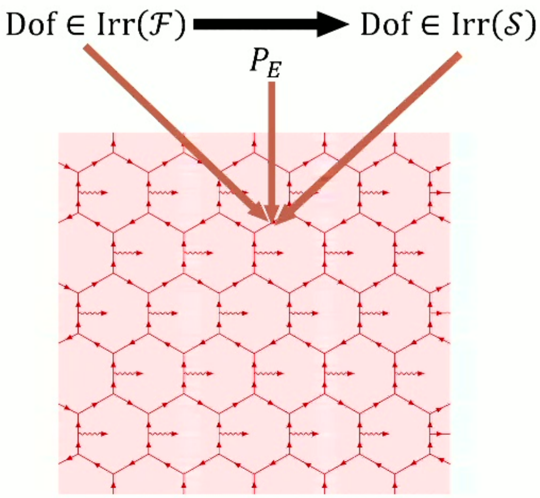
Certain dyon types in the parent model are condensed:

$$H_{\mathcal{F}} \quad \Rightarrow \quad H_{\mathcal{F}} - \Lambda \sum_{\text{Edges } E} \sum_{\substack{\text{Condensed} \\ \text{Anyon Type J}}} \sum_{p,q} \frac{\pi_{pq}^{J}}{d_p d_q} W_E^{J;pq}$$

→ Local Projector P_E

P_E: Project out certain basic degrees of freedom!

$$\mathcal{H}_E^{\mathcal{F}} = \{a \in \mathrm{Irr}(\mathcal{F})\} \to H_E^{\mathcal{S}} = P_E \mathcal{H}_E^{\mathcal{F}} \subset \mathcal{H}_E^{\mathcal{F}}$$



Child Model Input Data: Subcategory $\mathcal{S} \subset \mathcal{F}$

$$H_E^{\mathcal{S}} = \{r \in \mathrm{Irr}(\mathcal{S})\}$$

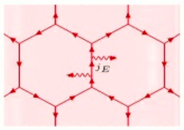
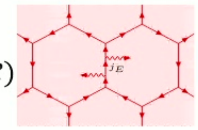
In general, $\mathrm{Irr}(\mathcal{S}) \not\subset \mathrm{Irr}(\mathcal{F})$

Question: How to calculate π_{pq}^{J}? What is the child input subcategory S?

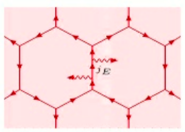
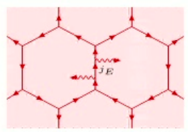
EASE WITH FLUXON CONDENSATION

$\Updownarrow$ IFF

Choosing Full Subcategory: $\mathrm{Irr}(\mathcal{S}) \subset \mathrm{Irr}(\mathcal{F})$

P_E  $= \delta_{j_E \in \mathrm{Irr}(\mathcal{C})}$ 

Fluxons: $(J, 1)$

$W_E^{J;11}$  $= w_J(j_E)$ 

$w_J : \mathrm{Irr}(\mathcal{F}) \to \mathbb{C}$

Solve Linear Equations (# unknowns = # eqs):

$$\sum_{\substack{\text{Condensed} \\ \text{Anyon Types } J}} \pi_J^{11} w_J(j_E) = \delta_{j_E \in \mathrm{Irr}(\mathcal{C})}$$

Condensing Anyons with $\pi_J^{11} \neq 0$!

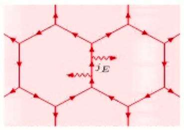
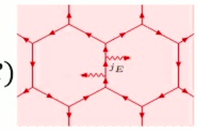
Example: Fibonacci UFC

$\mathrm{Irr}(\mathrm{Fibo}) = \{1, \tau\}$

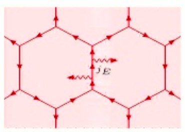
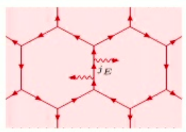
EASE WITH FLUXON CONDENSATION

$\Updownarrow$ IFF

Choosing Full Subcategory: $\mathrm{Irr}(\mathcal{S}) \subset \mathrm{Irr}(\mathcal{F})$

P_E  $= \delta_{j_E \in \mathrm{Irr}(\mathcal{C})}$ 

Fluxons: $(J, 1)$

$W_E^{J;11}$  $= w_J(j_E)$ 

$\downarrow$

$w_J : \mathrm{Irr}(\mathcal{F}) \to \mathbb{C}$

Solve Linear Equations (# unknowns = # eqs):

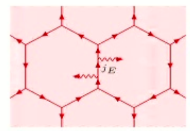
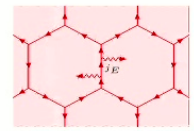
$$\sum_{\substack{\text{Condensed} \\ \text{Anyon Types } J}} \pi_J^{11} w_J(j_E) = \delta_{j_E \in \mathrm{Irr}(\mathcal{C})}$$

Condensing Anyons with $\pi_J^{11} \neq 0$!

Example: Fibonacci UFC

$\mathrm{Irr}(\mathrm{Fibo}) = \{1, \tau\}$

Full Subcategory: $\mathrm{Irr}(\mathcal{S}) = \{1\}$

P_E  $= \delta_{j_E,1}$  *Trivial!*

Fluxons: $(1\bar{1}, 1), (\tau\bar{\tau}, 1)$

$w_E^{1\bar{1}}(1) = w_E^{1\bar{1}}(\tau) = 1,$

$w_E^{\tau\bar{\tau}}(1) = 1, \quad w_E^{\tau\bar{\tau}}(\tau) = -\phi^{-2}.$

$P_E = \frac{1}{\phi^2+1}\left(W_E^{1\bar{1}} + \phi^2 W_E^{\tau\bar{\tau}}\right) = \delta_{j_E,1}$

Condensing Anyon $\tau\bar{\tau}$!

DIFFICULTIES WITH DYON CONDENSATION

There does exist non-fluxon condensation in the model!

They pose significant challenges:

Difficult to solve projector P_E!

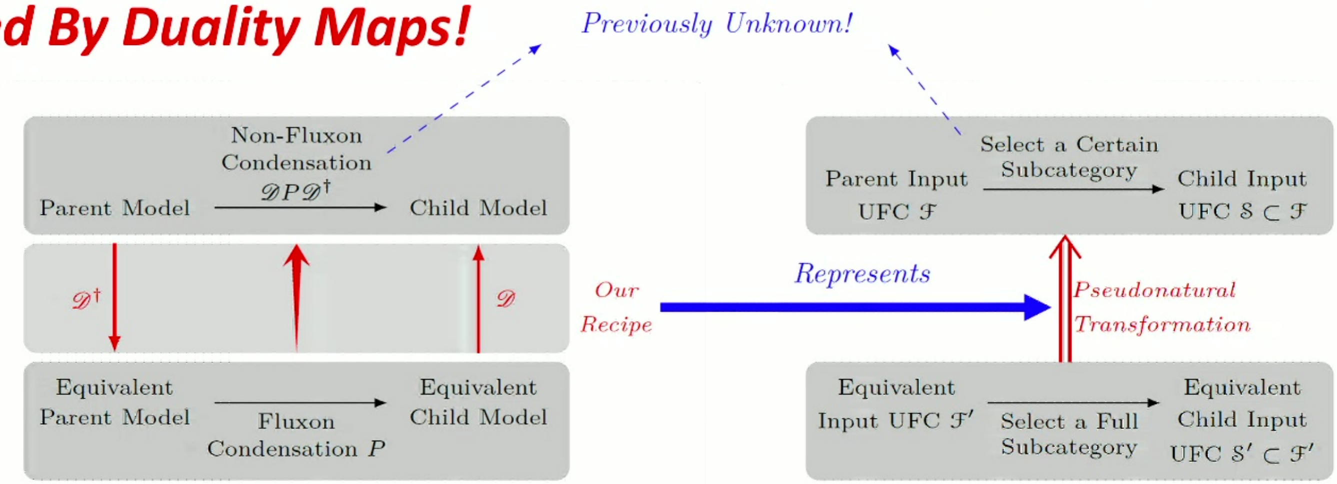
Difficult to identify the child model's degrees of freedom!

In general, $\mathrm{Irr}(\mathcal{S}) \not\subset \mathrm{Irr}(\mathcal{F})$!

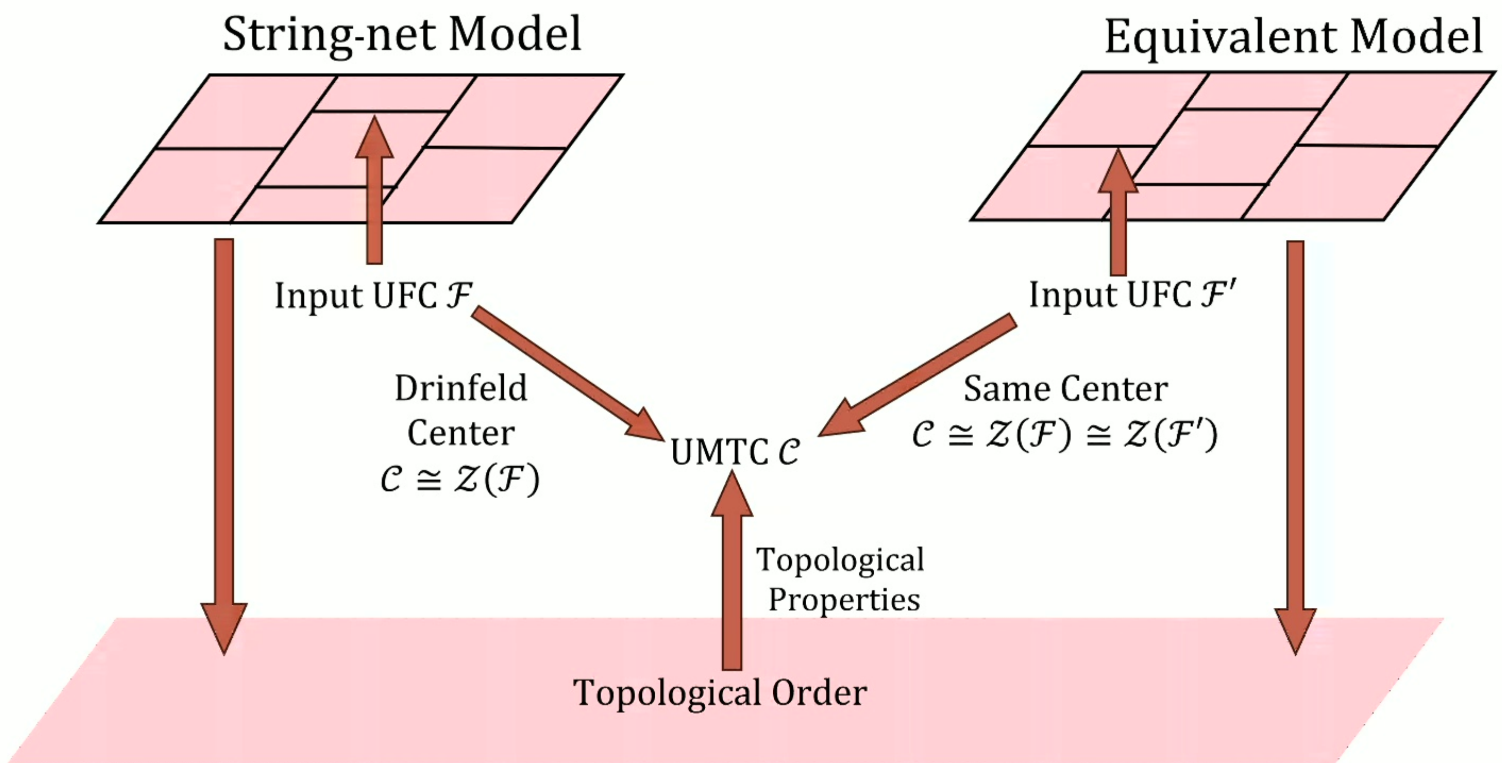
Degenerate Quadratic Multi-Variable Matrix Equation

$P_E^2 = P_E$

Solved By Duality Maps!



DUALITY MAPS: OVERVIEW



DUALITY MAPS: FROBENIUS ALGEBRA

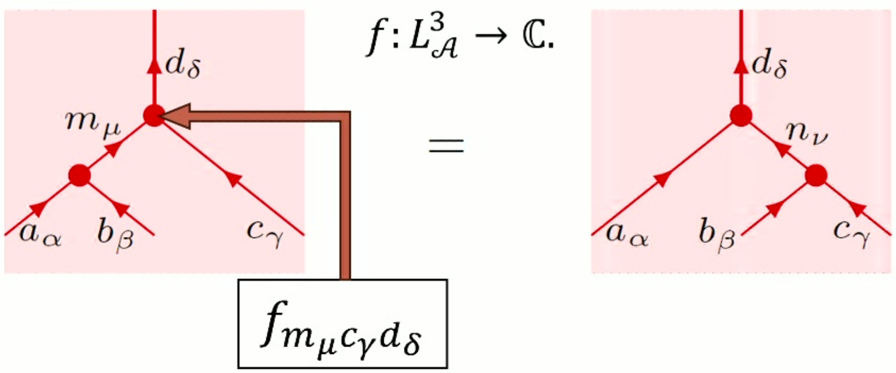
$\mathcal{Z}(\mathcal{F}) \cong \mathcal{Z}(\mathcal{F}') \quad \Leftrightarrow \quad \mathcal{F}' = \mathrm{Bimod}_{\mathcal{F}}(\mathcal{A}) \subseteq \mathcal{F}$

Etingof, et al. Tensor categories. Vol. 205. American Mathematical Soc., 2015.

Frobenius algebra: $\mathcal{A} = (L_{\mathcal{A}}, f)$:

$$L_{\mathcal{A}} = \{a_\alpha | a \in \mathrm{Irr}(\mathcal{F}), 1 \leq \alpha \leq n_a^{\mathcal{A}}\},$$

$f: L_{\mathcal{A}}^3 \to \mathbb{C}.$



DUALITY MAPS: FROBENIUS ALGEBRA

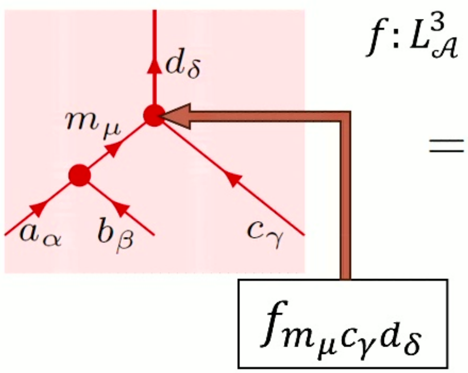
$\mathcal{Z}(\mathcal{F}) \cong \mathcal{Z}(\mathcal{F}') \quad \Leftrightarrow \quad \mathcal{F}' = \mathrm{Bimod}_{\mathcal{F}}(\mathcal{A}) \subseteq \mathcal{F}$

Etingof, et al. Tensor categories. Vol. 205. American Mathematical Soc., 2015.

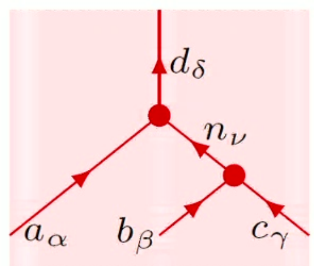
Frobenius algebra: $\mathcal{A} = (L_{\mathcal{A}}, f)$:

$$L_{\mathcal{A}} = \{a_\alpha | a \in \mathrm{Irr}(\mathcal{F}), 1 \leq \alpha \leq n_a^{\mathcal{A}}\},$$

$$f: L_{\mathcal{A}}^3 \to \mathbb{C}.$$



$=$



Example: Fibonacci UFC:

$$L_{\mathcal{A}} = \{1, \tau\},$$

$$f_{111} = f_{11\tau} = f_{1\tau 1} = f_{\tau\tau 1} = 1,$$

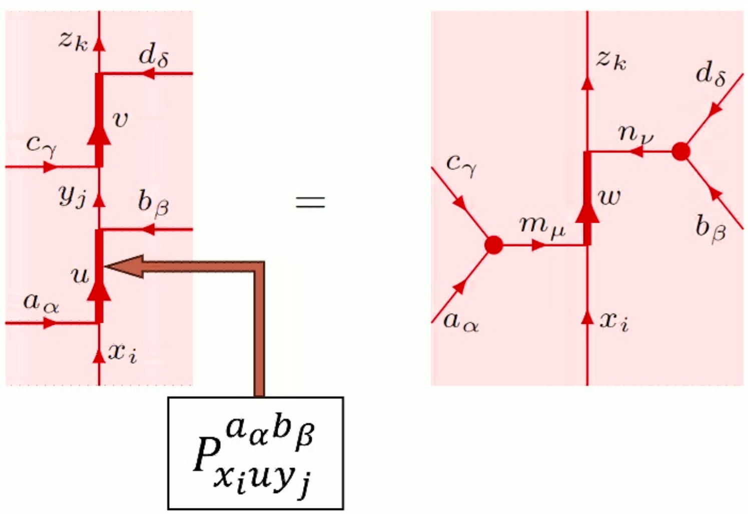
$$f_{\tau\tau\tau} = \phi^{-\frac{3}{4}}.$$

DUALITY MAPS: BIMODULES

Bimodules: $M = (L_M, P)$:

$$L_M = \{x_i | x \in \mathrm{Irr}(\mathcal{F}), 1 \leq i \leq n_x^M\},$$

$$P: L_{\mathcal{A}}^2 \otimes L_M \otimes L_{\mathcal{F}} \otimes L_M \to \mathbb{C}.$$



Example: Fibonacci UFC:

Two simple Bimodules over $\mathcal{A}$:

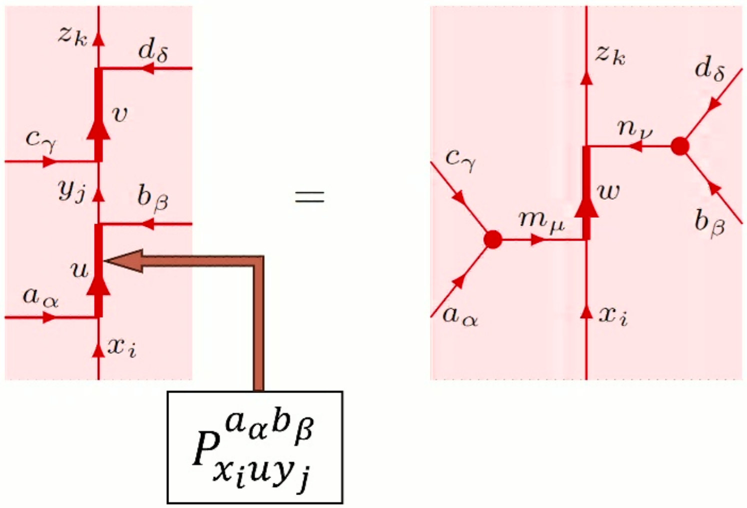
- Trivial Bimodule M_1:
  $L_{M_1} = \{1, \tau\}$
- Nontrivial Bimodule M_τ:
  $L_{M_\tau} = \{1, \tau_1, \tau_2\}$

DUALITY MAPS: BIMODULES

Bimodules: $M = (L_M, P)$:

$$L_M = \{x_i | x \in \mathrm{Irr}(\mathcal{F}), 1 \le i \le n_x^M\},$$

$$P: L_{\mathcal{A}}^2 \otimes L_M \otimes L_{\mathcal{F}} \otimes L_M \to \mathbb{C}.$$



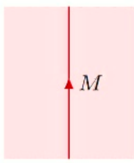
Example: Fibonacci UFC:

Two simple Bimodules over $\mathcal{A}$:

- Trivial Bimodule M_1:
  $L_{M_1} = \{1, \tau\}$
- Nontrivial Bimodule M_τ:
  $L_{M_\tau} = \{1, \tau_1, \tau_2\}$

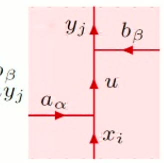
$\{M_1, M_\tau\}$ forms the bimodule category of $\mathcal{A} = 1 \oplus \tau$

DUALITY MAPS: CONSTRUCTION AND ENLARGEMENT

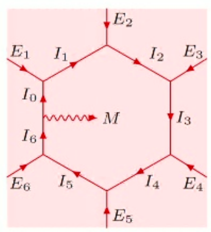


Dual Model
$M \in \mathrm{Bimod}_{\mathcal{F}}(\mathcal{A})$

$$\overset{\mathcal{D}}{\Longrightarrow} \sum_{a_\alpha b_\beta \in L_{\mathcal{A}}} \sum_{x_i, y_j \in M} \sum_{u \in L_{\mathcal{F}}} [P_M]^{a_\alpha b_\beta}_{x_i u y_j}$$

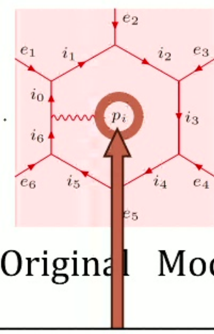


Original Model



Dual Model

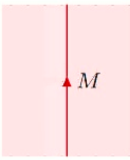
$$\Longrightarrow \sum_{\substack{i_k, e_k \in L \\ p_i \in L_M}} \cdots$$



Original Model

Internal Charges

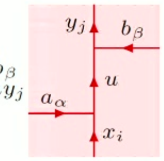
DUALITY MAPS: CONSTRUCTION AND ENLARGEMENT



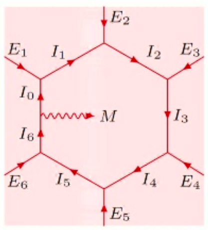
Dual Model

$M \in \mathrm{Bimod}_{\mathcal{F}}(\mathcal{A})$

$$\overset{\mathcal{D}}{\Longrightarrow} \sum_{a_\alpha b_\beta \in L_{\mathcal{A}}} \sum_{x_i, y_j \in M} \sum_{u \in L_{\mathcal{F}}} [P_M]^{a_\alpha b_\beta}_{x_i u y_j}$$

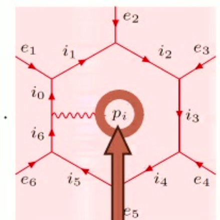


Original Model



Dual Model

$$\Longrightarrow \sum_{\substack{i_k, e_k \in L \\ p_i \in L_M}} \cdots$$



Original Model

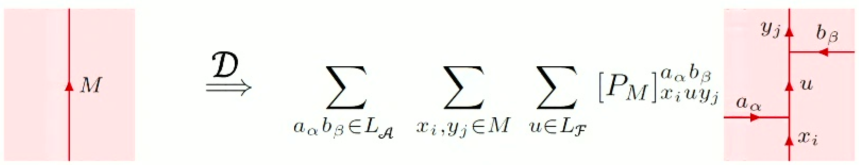
Internal Charges

Reflect $\mathcal{A}$ actions

$\mathcal{A}$ acts on p_i and p_j differently

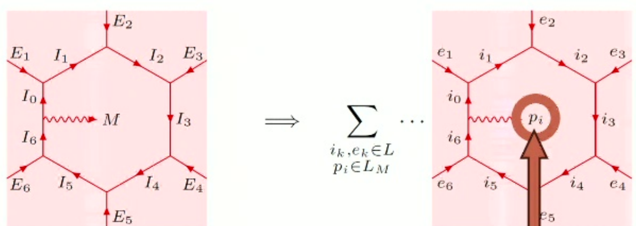
Enlarging Hilbert space!

DUALITY MAPS: CONSTRUCTION AND ENLARGEMENT



Dual Model
$M \in \text{Bimod}_{\mathcal{F}}(\mathcal{A})$

Original Model



Dual Model

Original Model

Internal Charges

Reflect $\mathcal{A}$ actions

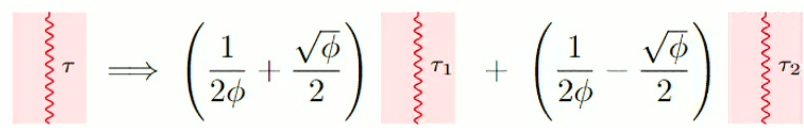
$\mathcal{A}$ acts on p_i and p_j differently

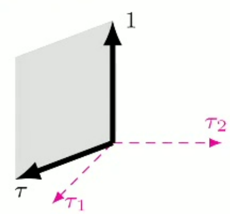
Enlarging Hilbert space!

Example: Fibonacci UFC:

$L_{M_1} = \{1, \tau\}, \qquad L_{M_\tau} = \{1, \tau_1, \tau_2\}$

Tail Dof Enlarging: $\{M_1, M_\tau\} \to \{1, \tau_1, \tau_2\}$





$\mathcal{D}$

Orthonormality:

$\langle M_1 | M_\tau \rangle = 0$ in dual model

DYON CONDENSATION: FIBONACCI EXAMPLE

Dual Model

Duality Map

Original Model

Fluxon Creation Operator

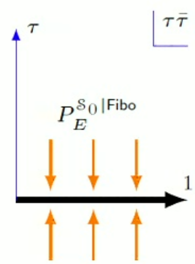
$$W_E^{M_\tau \overline{M_\tau}; M_1 M_1}$$

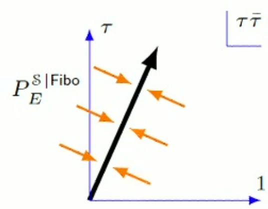
$$\frac{1}{\phi^4} W_E^{\tau\bar{\tau};11} + \frac{\sqrt[4]{5}}{\phi^4} W_E^{\tau\bar{\tau};1\tau} + \frac{\sqrt[4]{5}}{\phi^4} W_E^{\tau\bar{\tau};\tau 1} + \frac{\sqrt{5}}{\phi^4} W_E^{\tau\bar{\tau};\tau\tau}$$

$$P_E = \frac{1}{\phi^2 + 1}\left(W_E^{M_1 \overline{M_1}; M_1 M_1} + \phi^2 W_E^{M_\tau \overline{M_\tau}; M_1 M_1}\right)$$

$$P_E = \frac{1}{\phi^2(\phi^2+1)}(\phi^2 W_E^{1\bar{1};11} + W_E^{\tau\bar{\tau};11} + \sqrt[4]{5} W_E^{\tau\bar{\tau};\tau 1} + \sqrt[4]{5} W_E^{\tau\bar{\tau};1\tau} + \sqrt{5} W_E^{\tau\bar{\tau};\tau\tau})$$

Condensed Fluxon: $(M_\tau \overline{M_\tau}; M_1)$





Condensed Sector: $\frac{1}{\phi^2}(\tau\bar{\tau}, 1) + \frac{\sqrt[4]{5}}{\phi^2}(\tau\bar{\tau}, \tau)$

DYON CONDENSATION: FIBONACCI EXAMPLE

Duality Map →

Dual Model

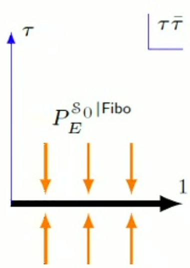
Fluxon Creation Operator

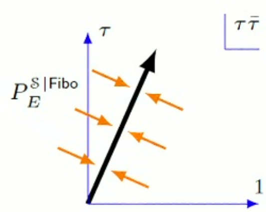
$$W_E^{M_\tau \overline{M_\tau}; M_1 M_1}$$

$$P_E = \frac{1}{\phi^2 + 1}\left(W_E^{M_1 \overline{M_1}; M_1 M_1} + \phi^2 W_E^{M_\tau \overline{M_\tau}; M_1 M_1}\right)$$

Condensed Fluxon: $(M_\tau \overline{M_\tau}; M_1)$

Child Input UFC: $\mathrm{Irr}(S) = \{1\}$
Trivial Subcategory of Fibo





Original Model

$$\frac{1}{\phi^4} W_E^{\tau\bar{\tau};11} + \frac{\sqrt[4]{5}}{\phi^4} W_E^{\tau\bar{\tau};1\tau} + \frac{\sqrt[4]{5}}{\phi^4} W_E^{\tau\bar{\tau};\tau 1} + \frac{\sqrt{5}}{\phi^4} W_E^{\tau\bar{\tau};\tau\tau}$$

$$P_E = \frac{1}{\phi^2(\phi^2+1)}\left(\phi^2 W_E^{1\bar{1};11} + W_E^{\tau\bar{\tau};11} + \sqrt[4]{5} W_E^{\tau\bar{\tau};\tau 1} + \sqrt[4]{5} W_E^{\tau\bar{\tau};1\tau} + \sqrt{5} W_E^{\tau\bar{\tau};\tau\tau}\right)$$

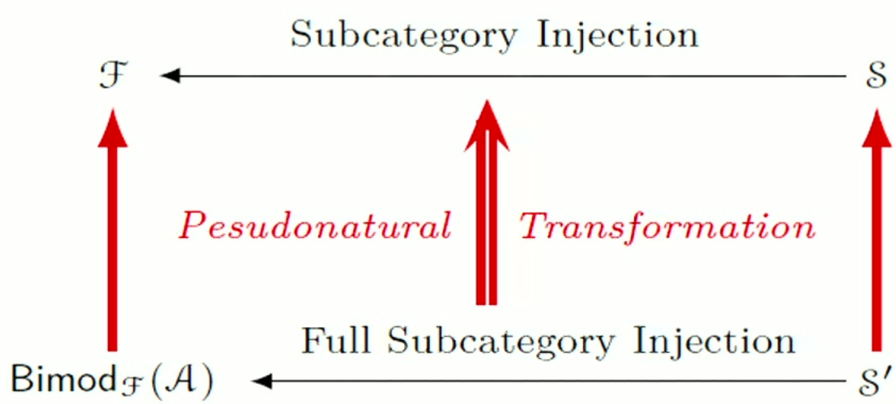
Condensed Sector: $\frac{1}{\phi^2}(\tau\bar{\tau}, 1) + \frac{\sqrt[4]{5}}{\phi^2}(\tau\bar{\tau}, \tau)$

Child Input UFC: $\mathrm{Irr}(S') = \{\mathcal{A} = 1 \oplus \tau\}$
Another Trivial Subcategory

PSEUDONATURAL TRANSFORMATION

A finer framework of anyon condensation than the UMTC-CSFA description

Gapping Certain basic degrees of freedom (gauge field) of the topological system



FROM GAPPING BASIC DEGREES OF FREEDOM TO CONDENSED ANYONS

$$P_E = \sum_{\substack{\text{Condensed} \\ \text{Anyon Type } J}} \sum_{p,q} \frac{\pi^J_{pq}}{d_p d_q} W_E^{J;pq} \xrightarrow{\text{Diagonalization}} P_E = \sum_{\substack{\text{Condensed} \\ \text{Sectors } J_i}} W_E^{J_i}$$

$$J_i = \sum_p u^p_{J_i}(J,p)$$

$$W_E^{J_i} = \sum_{p,q} u^p_{J_i} u^q_{J_i} W_E^{J;pq}$$

J_i: Condensed sectors

Absorbed by projector: $P_E W_E^{J_i} = W_E^{J_i} P_E = P_E$

Commutative Separable Frobenius Algebra A: Represented by $\{W_E^{J_i}\}$ $\quad \tilde{W}_E^{J_i} \tilde{W}_E^{K_j} = \sum_{I_k \in L_A} \frac{d_I}{d_J d_K} f^{I_k}_{J_i K_j} \tilde{W}_E^{I_k} \quad \forall J_i, K_j \in L_A$

Child basic dof: Invariant under condensed sectors $\quad W_E^{J_i} |r \in \mathrm{Irr}(\mathcal{S})\rangle = |r\rangle$

$\boldsymbol{A} = \{W_E^{J_i}\}$: ***Full Center*** *of* $\mathcal{S}$

Child Anyon Type: $J_{\text{Child}} \in \mathrm{Rep}_{\mathcal{F},0}(A)$ $\qquad W_E^{J_{\text{Child}}} = P_E W_E^{J_{\text{Parent}}} P_E$

FROM GAPPING BASIC DEGREES OF FREEDOM TO CONDENSED ANYONS

$$P_E = \sum_{\substack{\text{Condensed} \\ \text{Anyon Type } J}} \sum_{p,q} \frac{\pi^J_{pq}}{d_p d_q} W_E^{J;pq} \xrightarrow{\text{Diagonalization}} P_E = \sum_{\substack{\text{Condensed} \\ \text{Sectors } J_i}} W_E^{J_i}$$

$$J_i = \sum_p u^p_{J_i}(J,p)$$

$$W_E^{J_i} = \sum_{p,q} u^p_{J_i} u^q_{J_i} W_E^{J;pq}$$

J_i: Condensed sectors

Absorbed by projector: $P_E W_E^{J_i} = W_E^{J_i} P_E = P_E$

Commutative Separable Frobenius Algebra A: Represented by $\{W_E^{J_i}\}$ $\tilde{W}_E^{J_i} \tilde{W}_E^{K_j} = \sum_{I_k \in L_A} \frac{d_I}{d_J d_K} f^{I_k}_{J_i K_j} \tilde{W}_E^{I_k} \quad \forall J_i, K_j \in L_A$

Child basic dof: Invariant under condensed sectors $W_E^{J_i} |r \in \mathrm{Irr}(\mathcal{S})\rangle = |r\rangle$

$\boldsymbol{A} = \{W_E^{J_i}\}$: ***Full Center*** *of* $\mathcal{S}$

FROM GAPPING BASIC DEGREES OF FREEDOM TO CONDENSED ANYONS

$$P_E = \sum_{\text{Condensed Anyon Type } J} \sum_{p,q} \frac{\pi_{pq}^{J}}{d_p d_q} W_E^{J;pq} \xrightarrow{\text{Diagonalization}} P_E = \sum_{\text{Condensed Sectors } J_i} W_E^{J_i}$$

$$J_i = \sum_p u_{J_i}^p (J, p)$$

$$W_E^{J_i} = \sum_{p,q} u_{J_i}^p u_{J_i}^q W_E^{J;pq}$$

J_i: Condensed sectors   Absorbed by projector: $P_E W_E^{J_i} = W_E^{J_i} P_E = P_E$

Commutative Separable Frobenius Algebra A: Represented by $\{W_E^{J_i}\}$ $\tilde{W}_E^{J_i}\tilde{W}_E^{K_j} = \sum_{I_k \in L_A} \frac{d_I}{d_J d_K} f_{J_i K_j}^{I_k} \tilde{W}_E^{I_k}$ $\forall J_i, K_j \in L_A$

Child basic dof: Invariant under condensed sectors $W_E^{J_i}|r \in \mathrm{Irr}(\mathcal{S})\rangle = |r\rangle$

$\boldsymbol{A} = \{W_E^{J_i}\}$: ***Full Center*** *of* $\mathcal{S}$

Child Anyon Type: $J_{\text{Child}} \in \mathrm{Rep}_{\mathcal{F},0}(A)$ $W_E^{J_{\text{Child}}} = P_E W_E^{J_{\text{Parent}}} P_E$

THANK YOU FOR YOUR ATTENTION!