

Title: Quantum complementarity: A novel resource for exclusion

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Collection/Series: Quantum Foundations

Subject: Quantum Foundations

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Abstract:

Complementarity is a phenomenon explaining several core features of quantum theory, such as the well-known uncertainty principle. Roughly speaking, two objects are said to be complementary if being certain about one of them necessarily forbids useful knowledge about the other. Two quantum measurements that do not commute form an example of complementary measurements, and this phenomenon can also be defined for ensembles of states. Although a key quantum feature, it is unclear whether complementarity can be understood more operationally, as a necessary resource in some quantum information task. Here we show this is the case, and relates to a task which we term unambiguous exclusion. As well as giving complementarity a clear operational definition, this also uncovers the foundational underpinning of unambiguous exclusion tasks for the first time.

Quantum complementarity

A novel resource for exclusion tasks



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Quantum complementarity

A novel resource for exclusion tasks

Chung-Yun Hsieh, Roope Uola, Paul Skrzypczyk, arXiv:2309.11968



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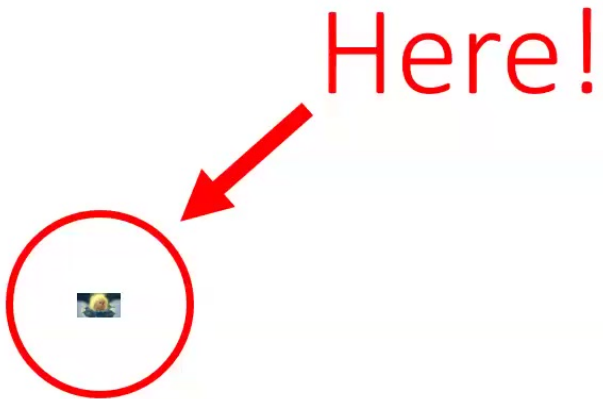
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What is complementarity











Details

Position

1



Details



Position



(momentum)
Details
of motion



Position



Spin X



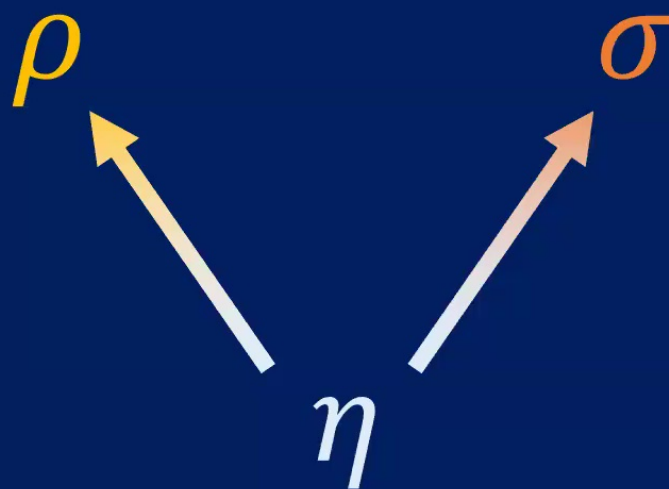
Spin Z

Complementarity



Carrying no common Q info

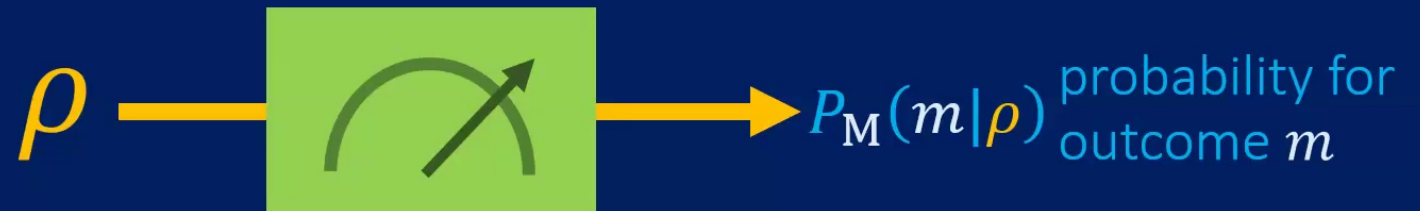
Can they carry common Q info?



(partially) describes measurement statistics

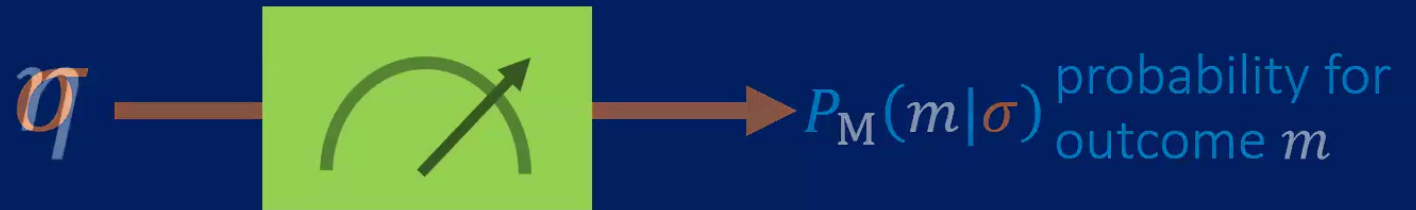
2

Measurement
(M)

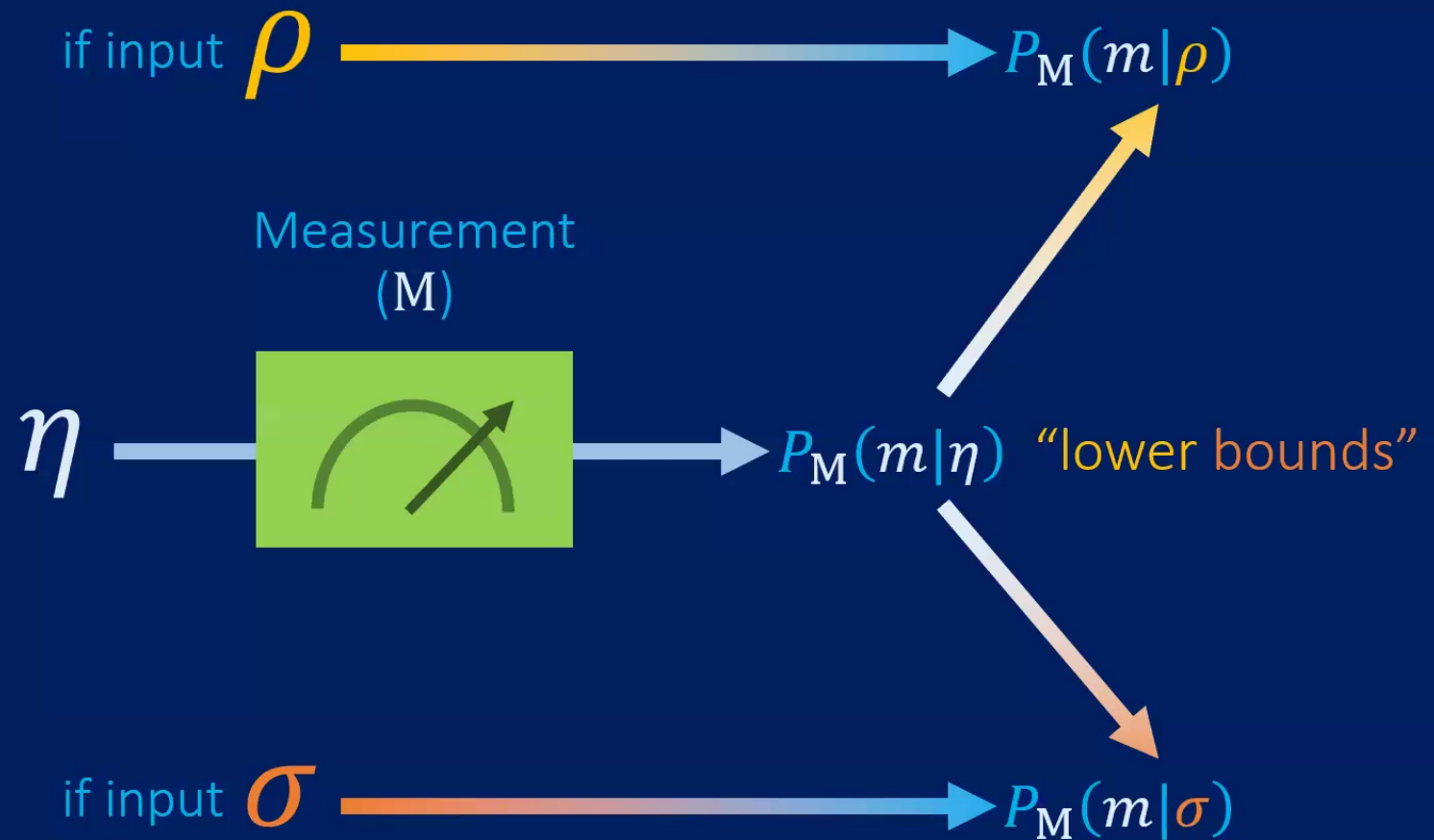


if input ρ  $P_M(m|\rho)$

Measurement
(M)



if input σ  $P_M(m|\sigma)$

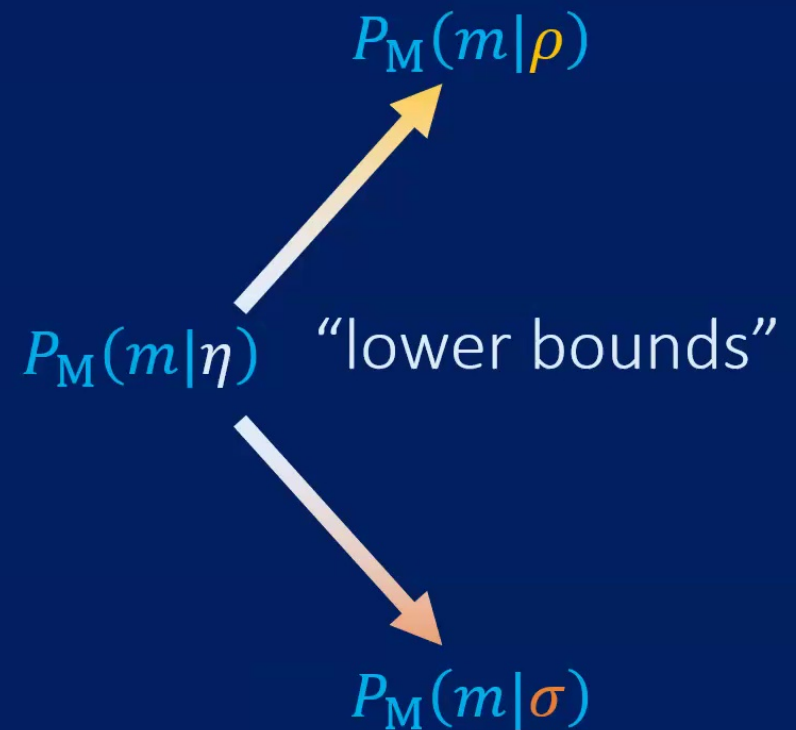


We say ρ & σ carry common Q info η
(quantified by $1 \geq \alpha > 0$) if

$$P_M(m|\rho) \geq \alpha P_M(m|\eta)$$

$$P_M(m|\sigma) \geq \alpha P_M(m|\eta)$$

for every measurement M and
outcome m



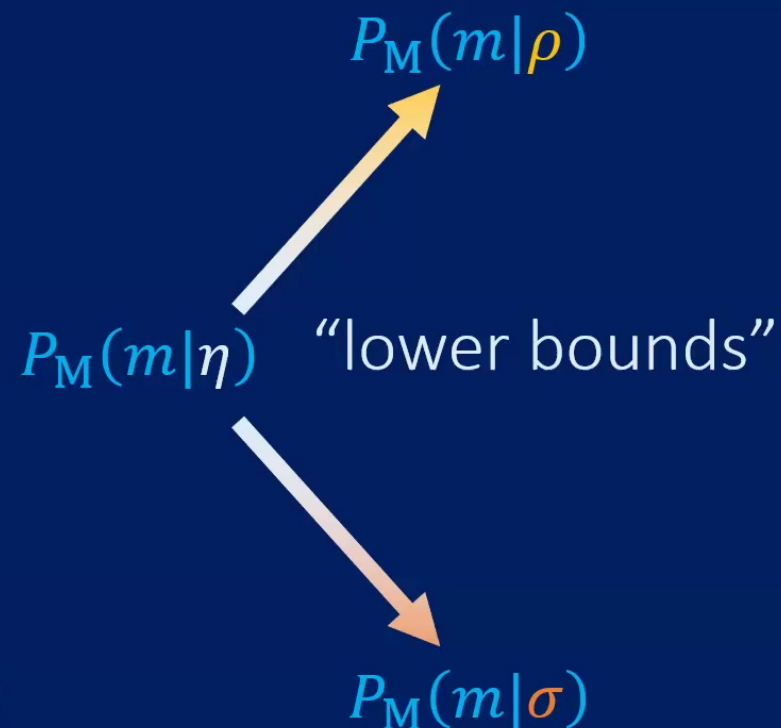
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We say ρ & σ are complementary if they
cannot carry common Q info (that is, $\alpha = 0$)

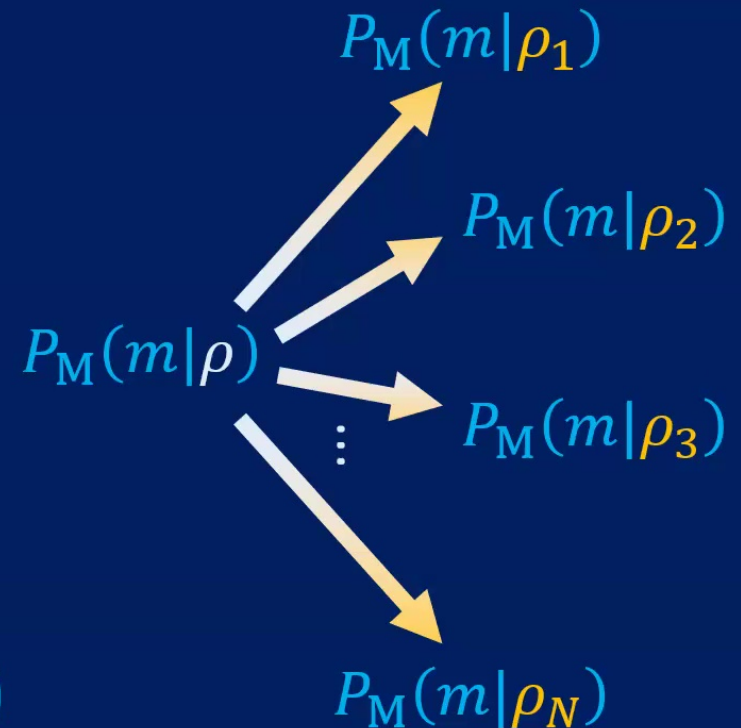


We say $\{\rho_x\}_{x=1}^N$ carry common Q info η (quantified by $1 \geq \alpha > 0$) if

$$P_M(m|\rho_x) \geq \alpha P_M(m|\eta) \quad \forall x$$

for every measurement M and outcome m

We say $\{\rho_x\}_{x=1}^N$ are complementary if they cannot carry common Q info (that is, $\alpha = 0$)



Complementary Q states are useful in tasks excluding classical info

Sending x , aiming to always output an index $\neq x$



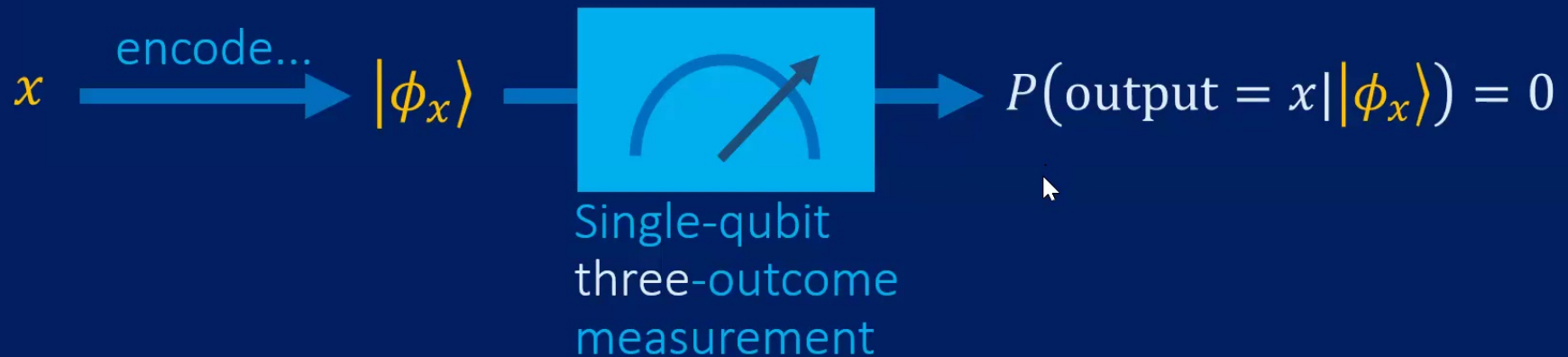
Complementary Q states are useful in tasks excluding classical info

$$|\phi_0\rangle$$

$$|\phi_1\rangle$$

$$|\phi_2\rangle$$

Example: 3 qubit-states carry **no** common Q info



Example: Effective qubit in a qutrit

$$|\phi_x\rangle = (|0\rangle - e^{2\pi xi i/3}|1\rangle)/\sqrt{2} \quad x = 0,1,2$$

$$|\omega_a\rangle = (|0\rangle + e^{2\pi ai/3}|1\rangle + e^{4\pi ai/3}|2\rangle)/\sqrt{3} \quad a = 0,1,2$$

By considering the qutrit projective measurement $|\omega_a\rangle\langle\omega_a|$, we have...

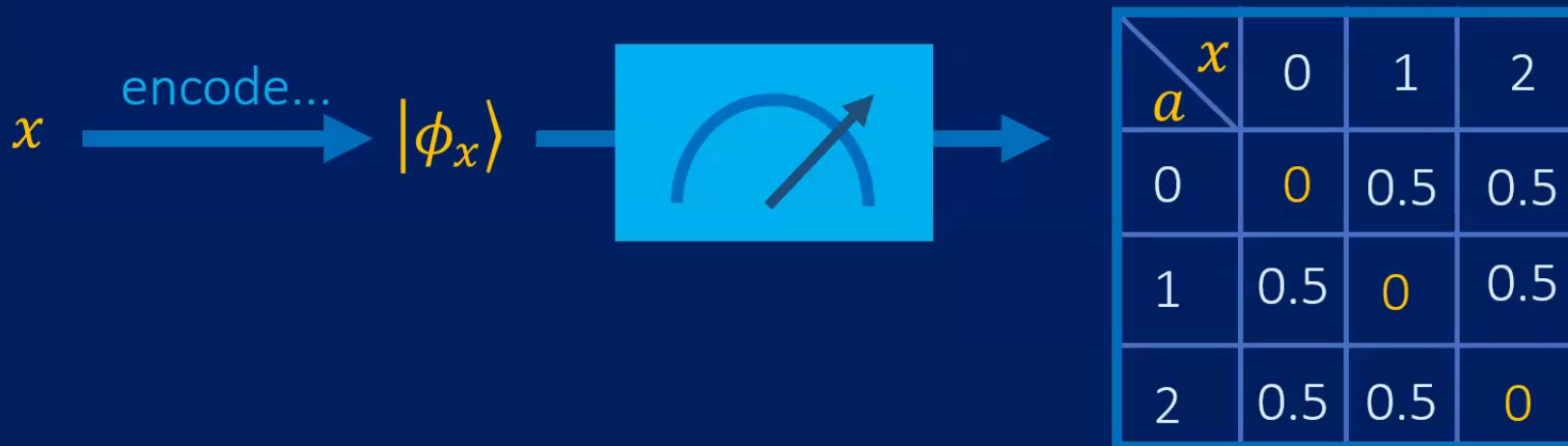


Example: Effective qubit in a qutrit

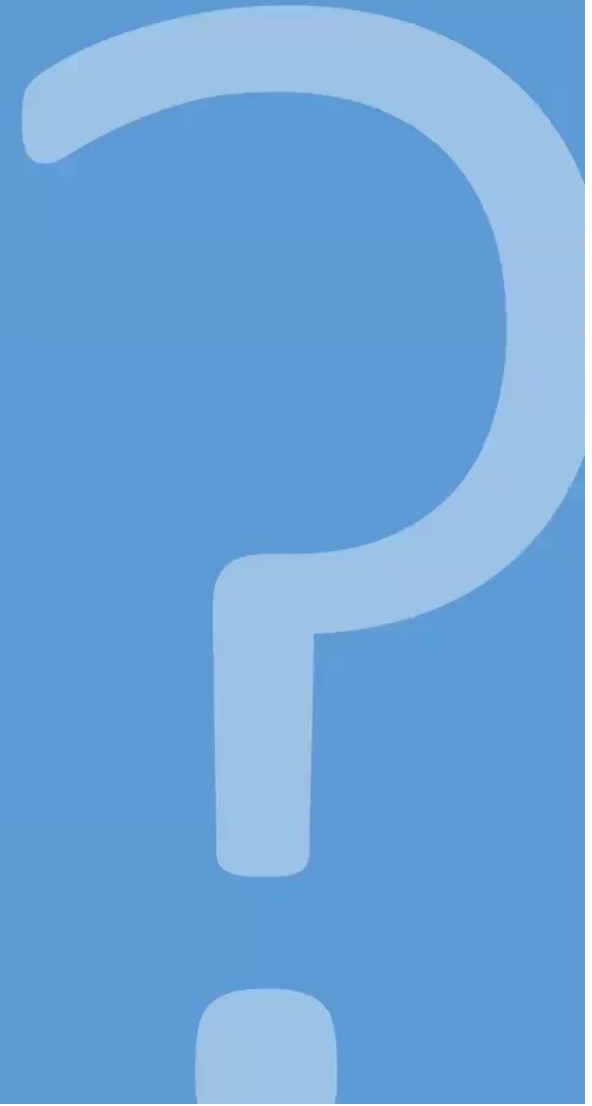
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Is complementarity useful
operationally?

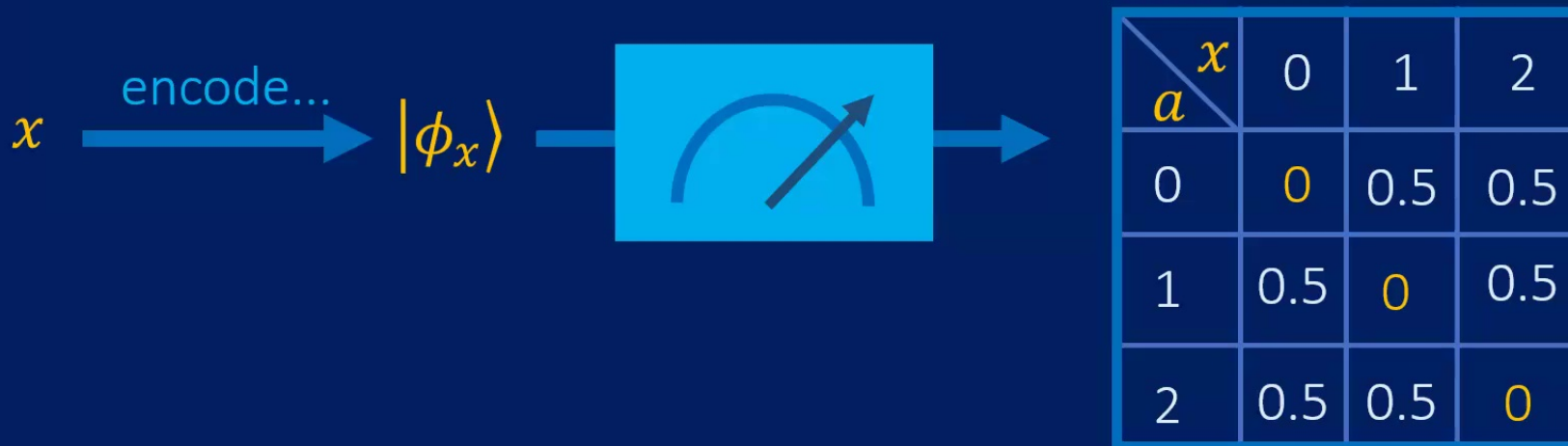


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Complementary Q states are useful in tasks excluding classical info



Example: 3 qubit-states carry **no** common Q info

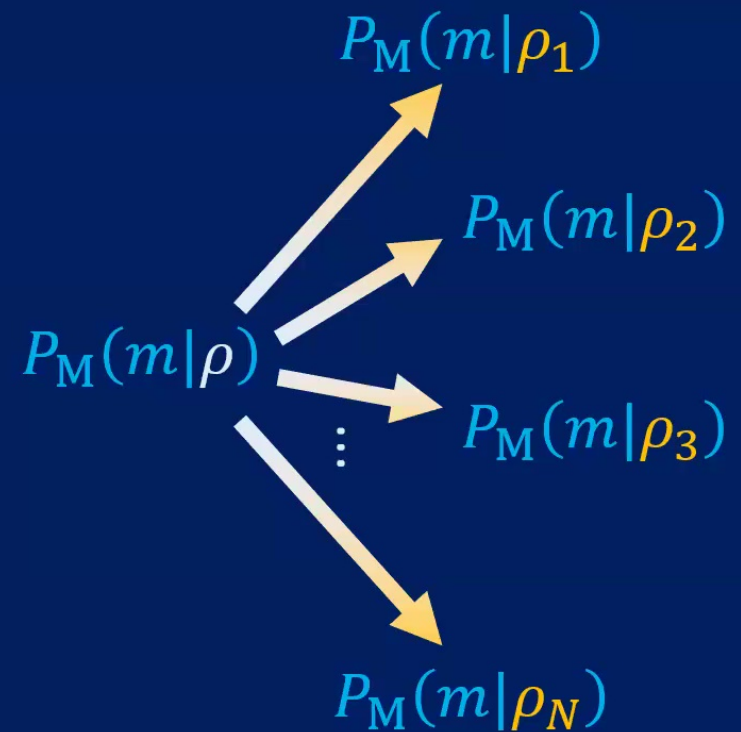


Q complementarity is a resource for **exclusion tasks**

Complementarity is useful
in exclusion tasks

We say $\{\rho_x\}_{x=1}^N$ carry common Q info if there is $P \neq 0$ such that

$$\rho_x \geq P \quad \forall x$$

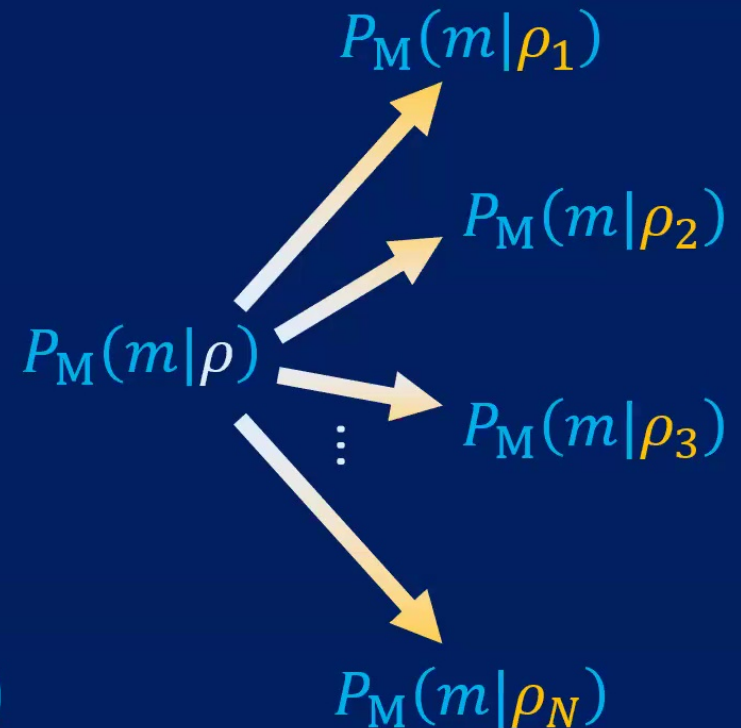


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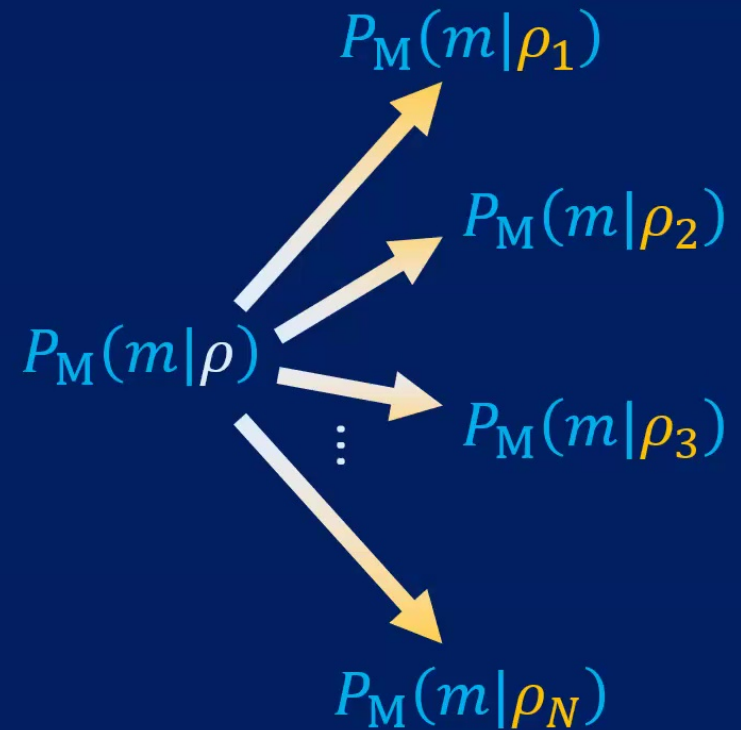


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Quantifier:

$$q_{\text{exc}}(\{\rho_x\}) := \max_{\substack{\rho_x \geq P \quad \forall x \\ P \geq 0}} \text{tr}(P)$$



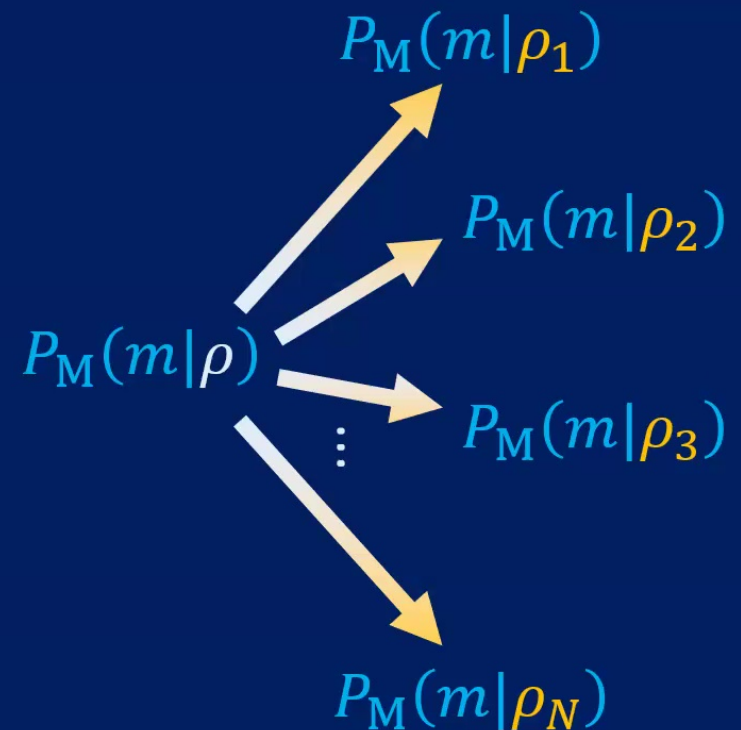
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$q_{\text{exc}}(\{\rho_x\}) = 0$ iff $\{\rho_x\}$ is complementary



Complementary Q states are useful in tasks excluding classical info

$$|\phi_0\rangle$$

$$|\phi_1\rangle$$

$$|\phi_2\rangle$$

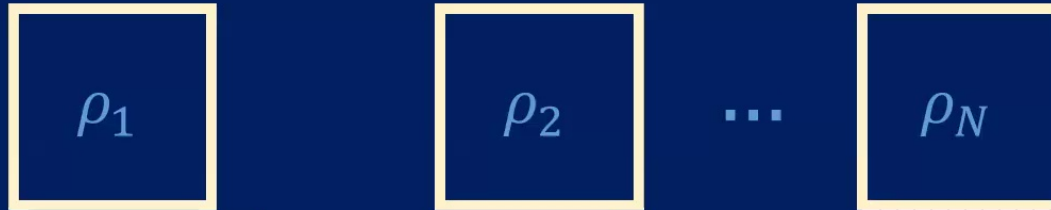
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Complementary Q states are useful in tasks excluding classical info



Complementary Q states are useful in tasks excluding classical info



$$P_{\text{error}}(\{\rho_x\}, \epsilon) := \min \left\{ \sum_{x=1}^N \frac{1}{N} \text{tr}(Q_x \rho_x) \mid (1 - \epsilon)\mathbb{I} \leq \sum_{x=1}^N Q_x \leq \mathbb{I}, Q_x \geq 0 \right\}$$



Single-qudit
(N+1)-outcome
measurement

minimise $P(\text{output} = x | \rho_x)$
 $= \text{tr}(Q_x \rho_x)$
 &
 allow $P(\text{output} = \emptyset) \leq \epsilon$

Result

For every $\{\rho_x\}$, there exists $\epsilon_* < 1$ such that

$$q_{\text{exc}}(\{\rho_x\}) = \frac{P_{\text{error}}(\{\rho_x\}, \epsilon)}{(1 - \epsilon)/N} \quad \forall \epsilon_* \leq \epsilon < 1$$

Complementarity = exclusion advantage!

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Complementarity = carrying no common Q info

$$q_{\text{exc}}(\{\rho_x\}) := \max_{\substack{\rho_x \geq P \forall x \\ P \geq 0}} \text{tr}(P)$$

Complementarity = unambiguous exclusion advantage

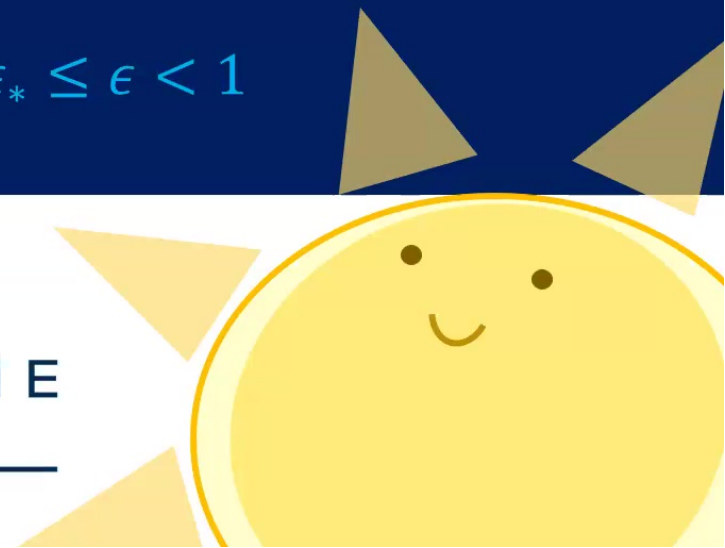
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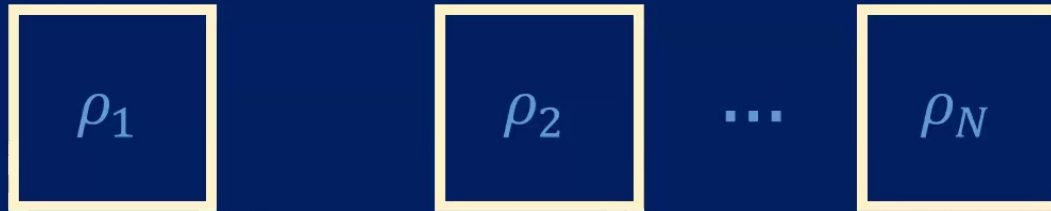


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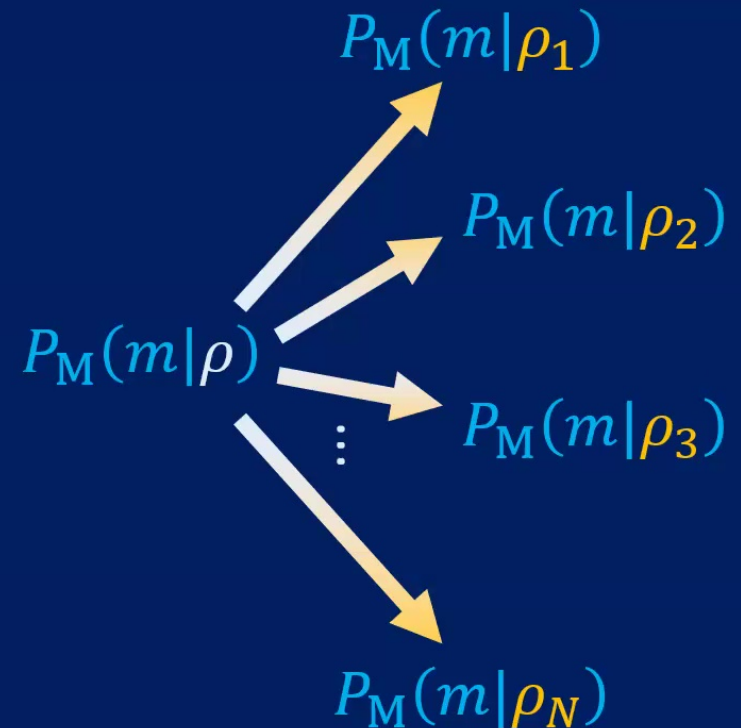


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x $\xrightarrow{\text{encode...}}$ ρ_x



Single-qudit
(N+1)-outcome
measurement

minimise $P(\text{output} = x | \rho_x)$
 $= \text{tr}(Q_x \rho_x)$
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 allow $P(\text{output} = \emptyset) \leq \epsilon$