Title: Hidden symmetries of gravity on the Carrollian boundary

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Collection/Series: Quantum Gravity

Subject: Quantum Gravity

Date: February 18, 2025 - 10:00 AM

URL: https://pirsa.org/25020039

Abstract:

Gravity possesses hidden symmetries that emerge upon dimensional reduction. One of the first examples being the Elhers SL(2,R) group revealed when reducing four-dimensional Einstein gravity to three-dimensions. However useful such symmetries are, especially to design solution generating techniques, they act in a highly non-local way on the gravitational data. On the other hand it is known that the solution space of asymptotically Flat spacetimes can be expressed covariantly in terms of an infinite number of tensors defined on the null conformal boundary. Therefore it is expected that hidden symmetries will act on the boundary data. In this talk, focusing on the simpler case of Petrov algebraically special spacetimes, I want to show that at the level of the boundary, the action becomes local, therefore much simpler, and makes explicit gravitational electric/magnetic dualities.

Hidden symmetries of gravity
on the Carrollian boundary

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Based on 2212.14062
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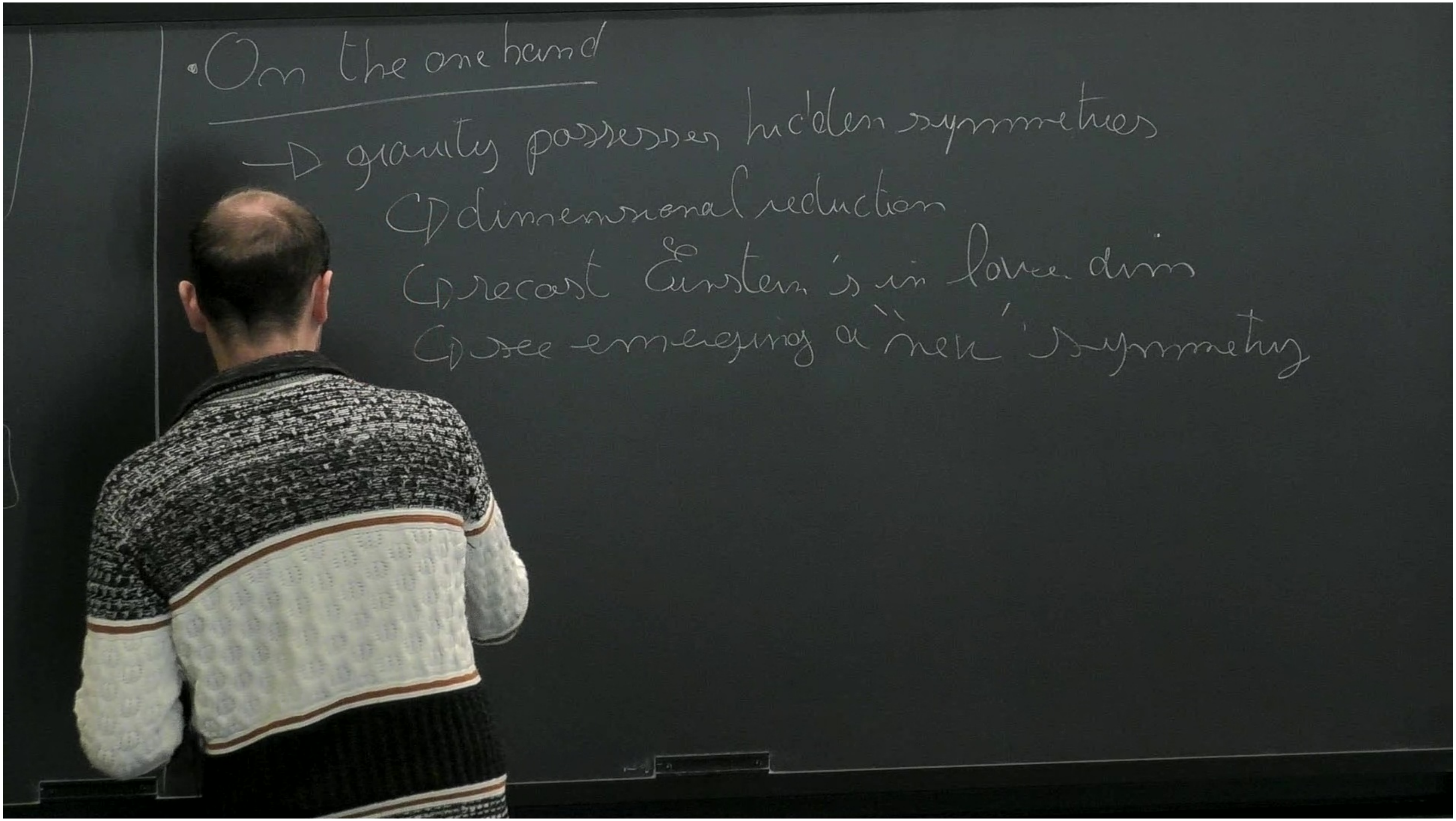

• On the one hand
→ gravity possesses hidden symmetries
↳ dimensional reduction
↳ recast Einstein's in lower dim
↳ see emerging a "new" symmetry

→ recast Einstein's in lower dim

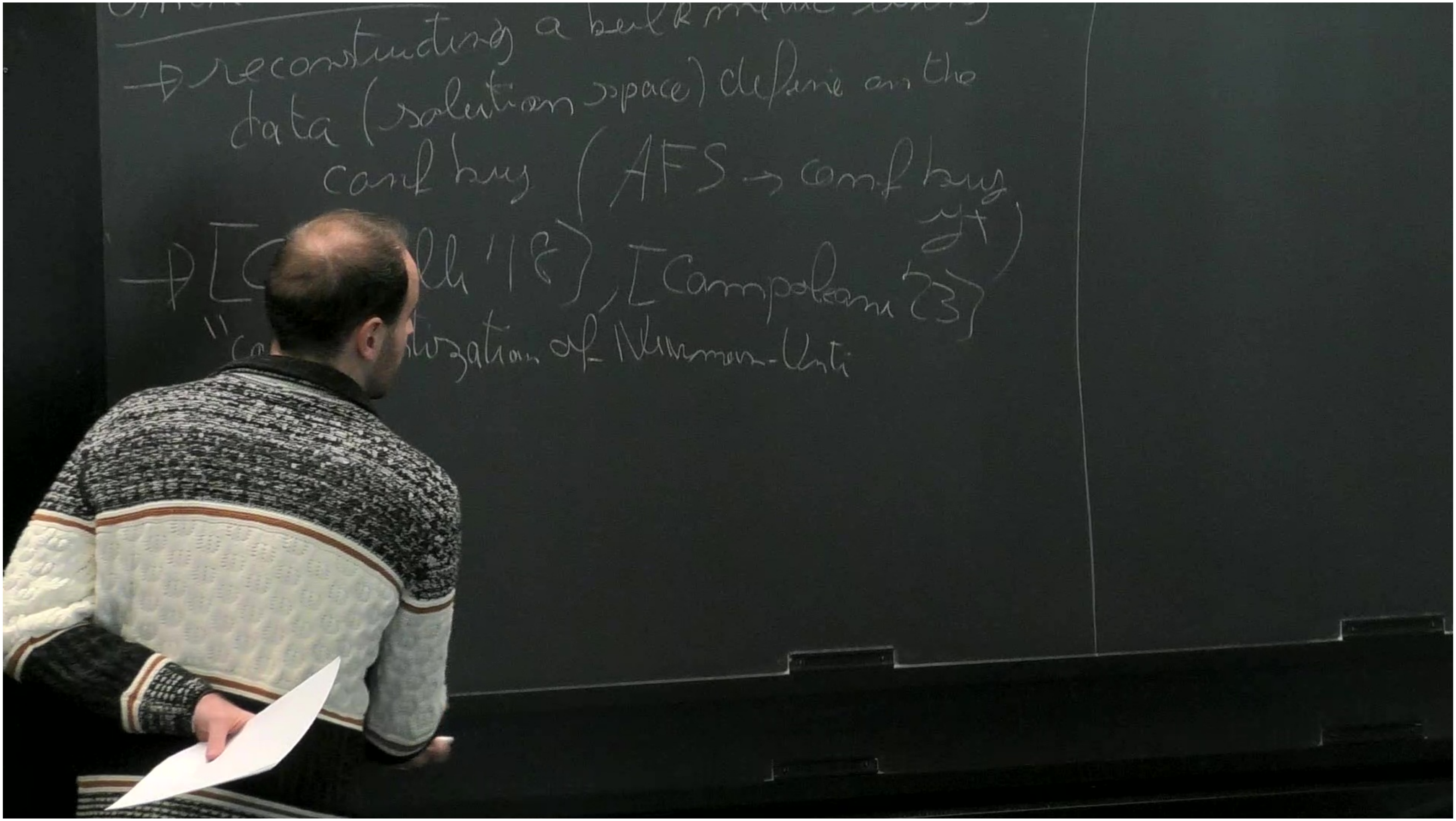
→ see emerging a "new" symmetry

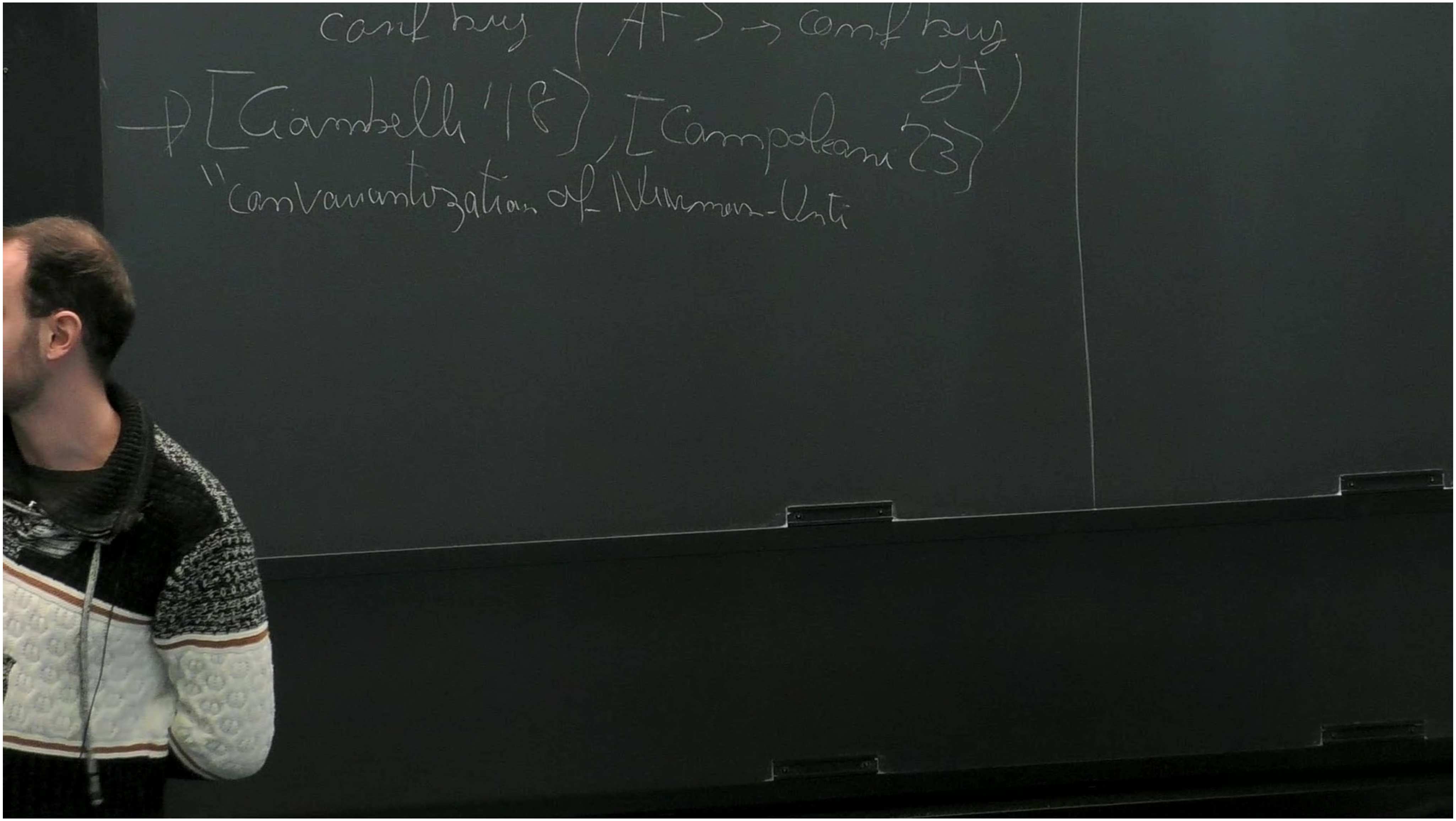
e.g. 4d Ricci-flat gravity in 4d → 3d

reduce 11d supergravity Ehlers $SL(2,\mathbb{R})$ sym.

$\Longrightarrow$ exceptional groups E_9

→ important feature of gravity


→ reconstructing a bulk
data (solution space) define on the
conf bdy (AFS → conf bdy
→ [Campoleoni '23]
zation of Newman-Unti


conf bry (AFS → conf bry
[Gambelli '18], [Campoleoni '23]
"covariantization of Newman-Unti

On the other hand:

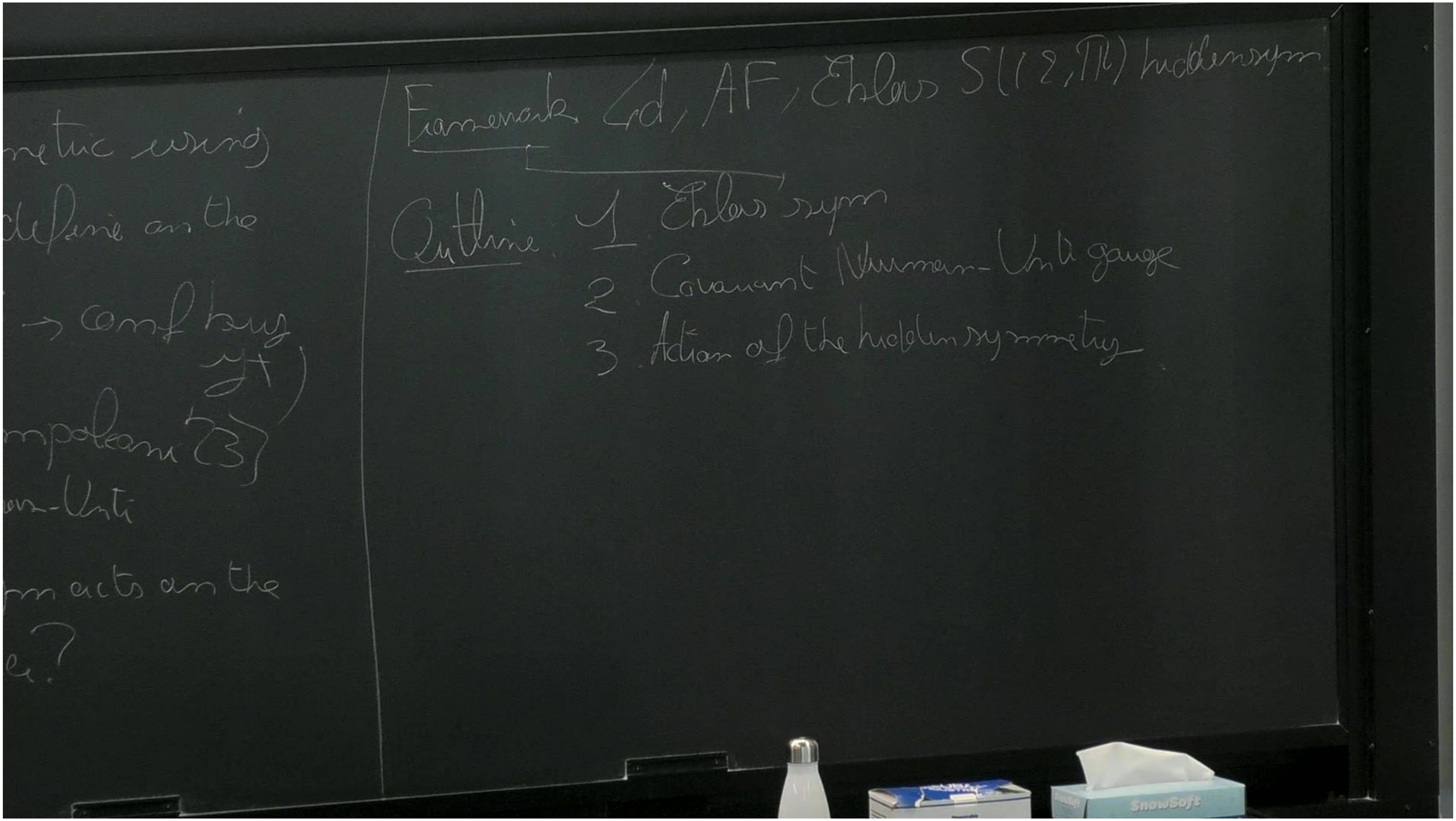
→ reconstructing a bulk metric using data (solution space) define on the conf bdy (AFS → conf bdy $\mathscr{J}^+$)

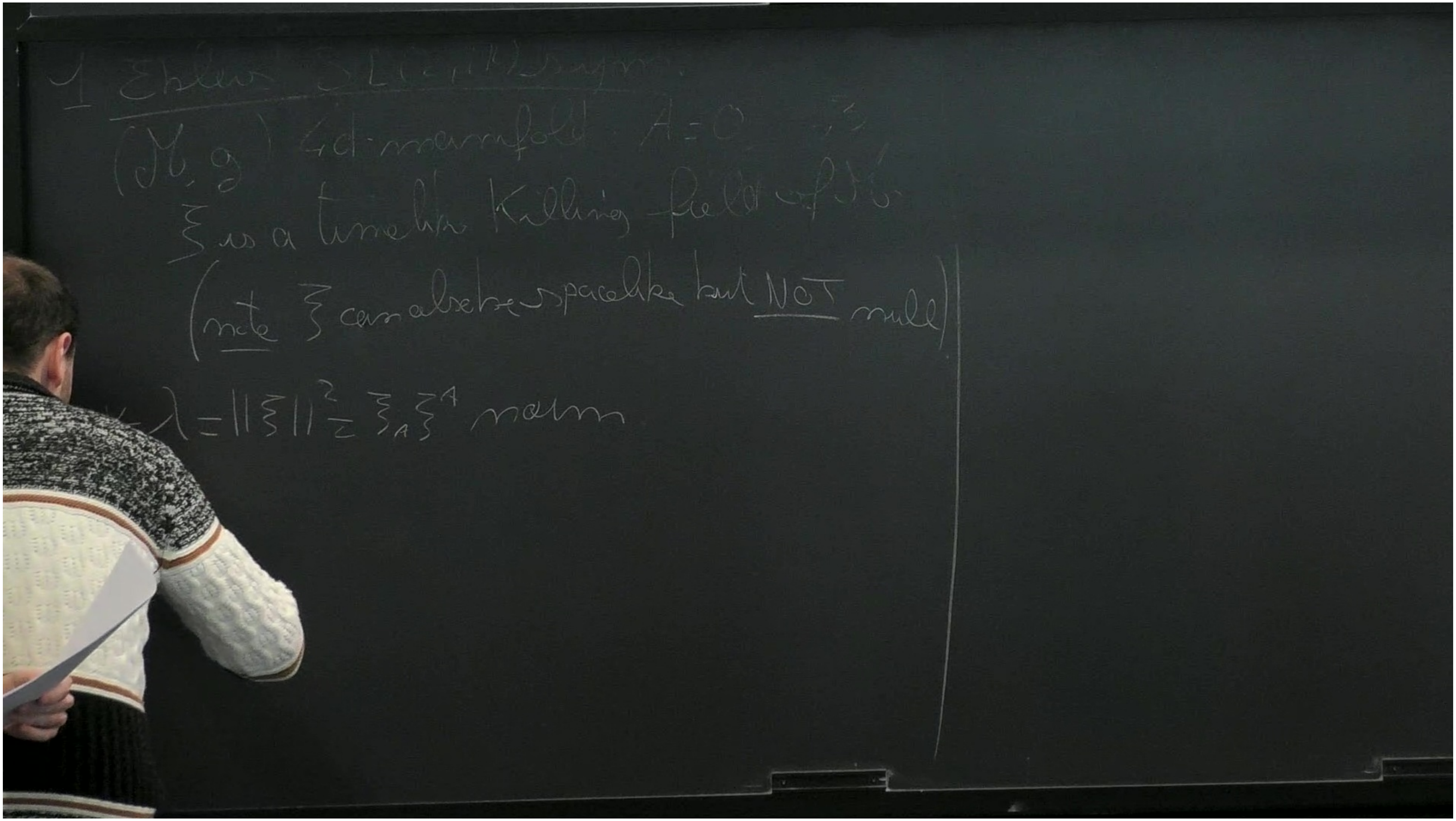
→ [Gambelli '18], [Campoleoni '23]

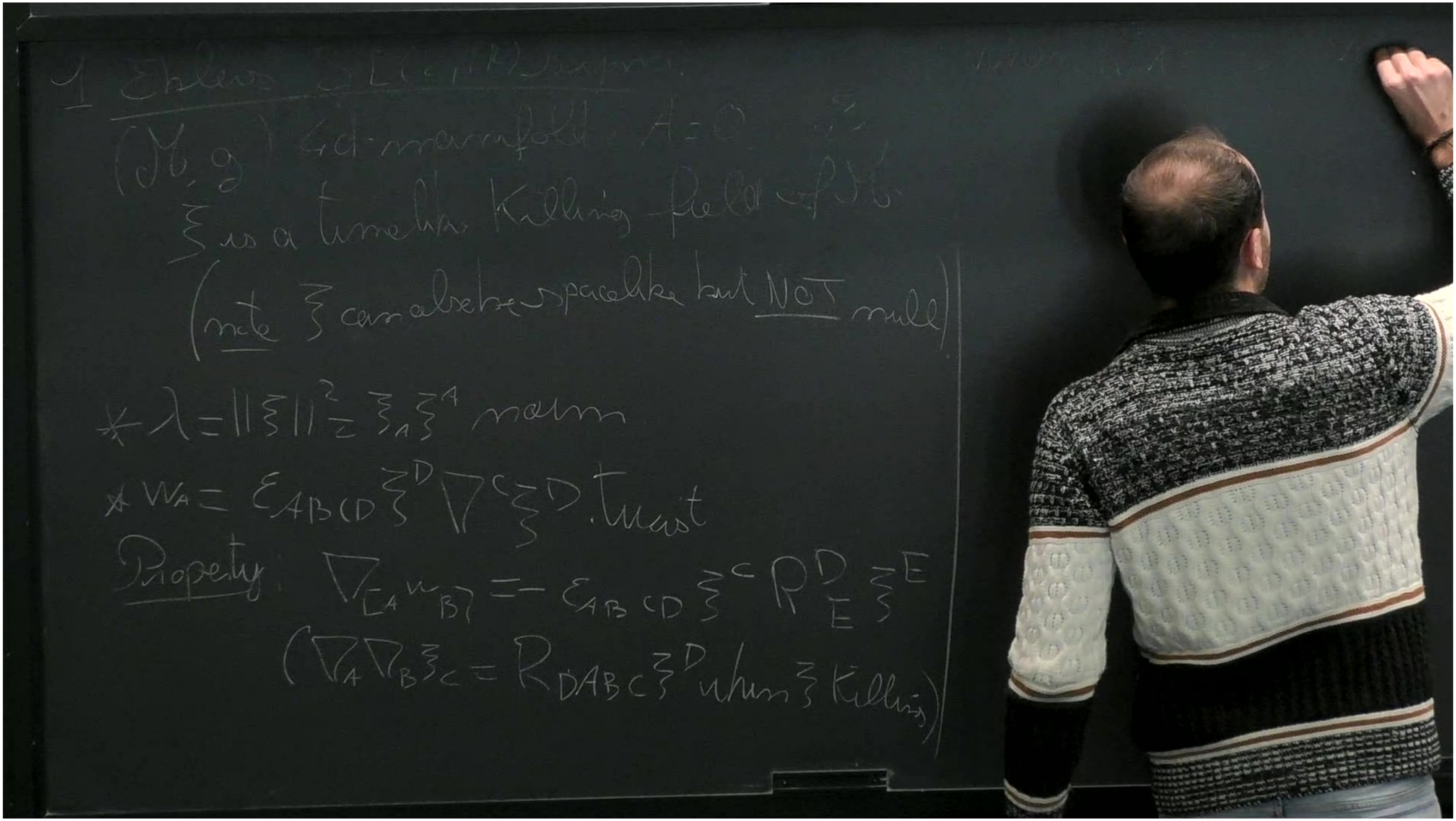
"conformalization of Newman-Unti"

in this talk: how does hidden symm acts on the AFS solution space?

Framework:


Framework: 4d, AF, Ehlers $Sl(2,\mathbb{R})$ hidden sym
Outline: 1 Ehlers' sym
2 Covariant Newman-Unti gauge
3 Action of the hidden symmetry
SnowSoft


1 Ehlers SL(2,R) sym.
(M, g) 4d-manifold Λ=0
ξ is a timelike Killing field of M.
(note ξ can also be spacelike but NOT null)
λ = ||ξ||² = ξ_A ξ^A norm


1 Ehlers
(M, g) 4d-manifold, $\Lambda = 0$, ξ
ξ is a timelike Killing field of M
(note ξ can also be spacelike but NOT null)
* $\lambda = \|\xi\|^2 = \xi_A \xi^A$ norm
* $w_A = \varepsilon_{ABCD} \xi^D \nabla^C \xi^D$ twist
Property: $\nabla_{[A} w_{B]} = -\varepsilon_{ABCD} \xi^C R^D{}_E \xi^E$
$(\nabla_A \nabla_B \xi_C = R_{DABC} \xi^D$ when ξ Killing$)$

→ reduce to 3d. $S = \mathcal{M}/\mathrm{Orb}(\xi)$

3d dim manifold

A metric $h_{AB} = g_{AB} - \frac{\xi_A \xi_B}{\lambda}$.

projector $h^A{}_B = \delta^A{}_B - \frac{\xi^A \xi_B}{\lambda}$

Prop: $T^{\mu\nu\ldots}{}_{\rho\sigma\ldots}$ tensor of M

$\mathcal{L}_\xi T = 0$
$\xi^\rho T^{\mu\nu\ldots}{}_{\rho\sigma\ldots} = 0$
$\Longrightarrow$ T is a tensor on S.

Let ∇ is L-C fcn $(\mathcal{M}, g)$

D is L-C fcn $(\mathcal{S}, h)$

$$\boxed{\mathcal{D}_C T_A{}^B = h_C{}^L h_A{}^M h^B{}_N \nabla_L T_M{}^N}$$

in particular

$$\mathcal{R}_{ABCD} = h_{(A}{}^P h^Q{}_{B]} h_{[C}{}^R h_{D)}{}^S \left(R_{PQRS} + \frac{2}{\lambda} [\nabla_P \xi \nabla_Q \xi + \nabla_P \xi_R \nabla_Q \xi_S] \right)$$

Let ∇ is L-C for $(\mathcal{M}, g)$

D is L-C for $(\mathcal{S}, h)$

$$\boxed{\mathcal{D}_C T_A{}^B = h_C{}^L h_A{}^M h^B{}_N \nabla_L T_n{}^N}$$

in particular

$$\mathcal{R}_{ABCD} = h_{(A}{}^P h^Q{}_{B]} h_{[C}{}^R h_{D)}{}^S \left(R_{PQRS} + \frac{2}{\lambda} \left[\nabla_P \xi \nabla_Q \xi + \nabla_P \xi_R \nabla_Q \xi_S \right] \right)$$

$$\ldots D^{B}{}_{N}\, \nabla_{L}\, T_{n}{}^{N}$$

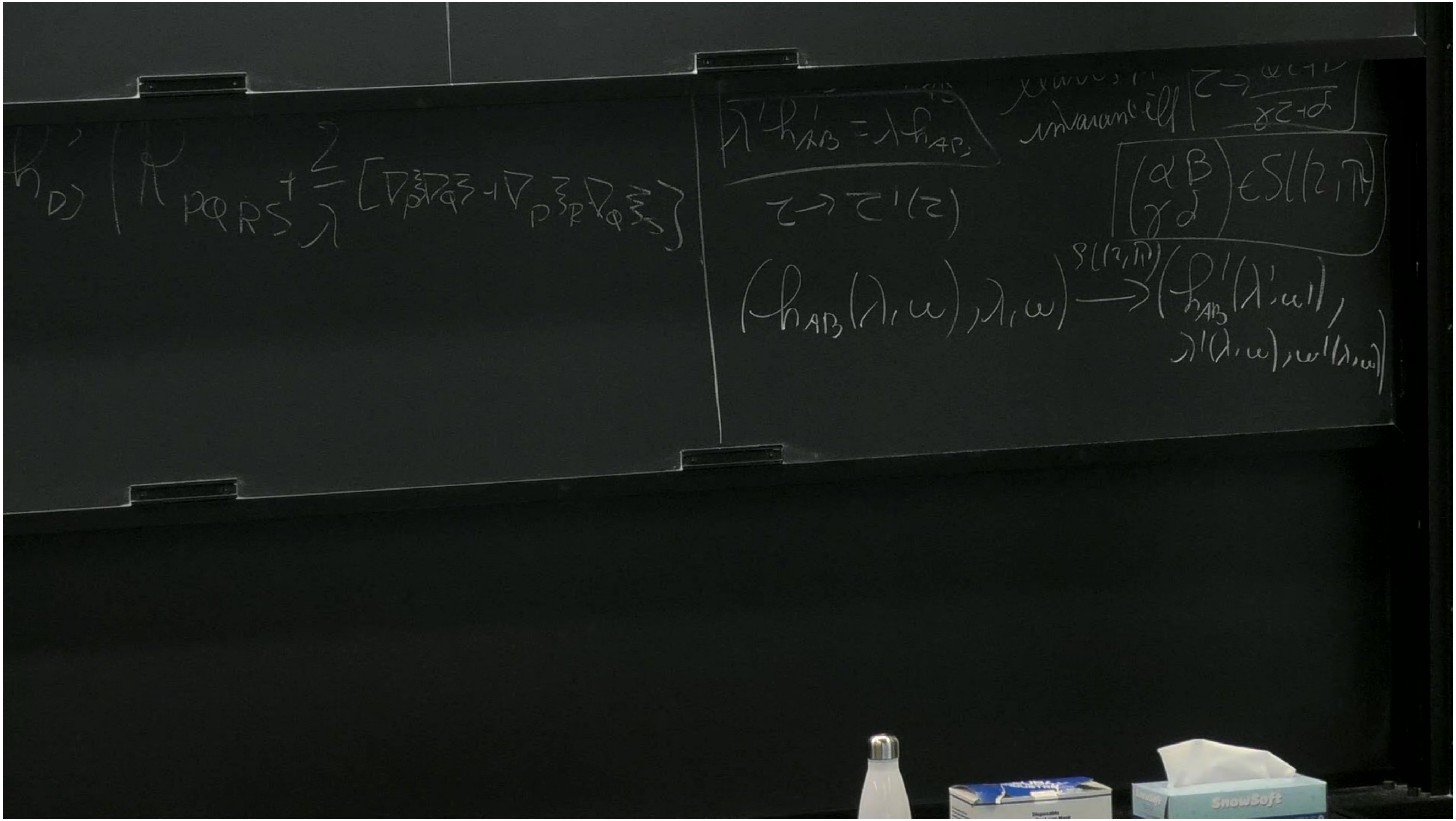
$$\ldots h_{D)} \left(R_{PQRS} + \frac{2}{\lambda} \left[\nabla_P \xi \nabla_Q \xi + \nabla_P \xi_R \nabla_Q \xi_S \right] \right.$$

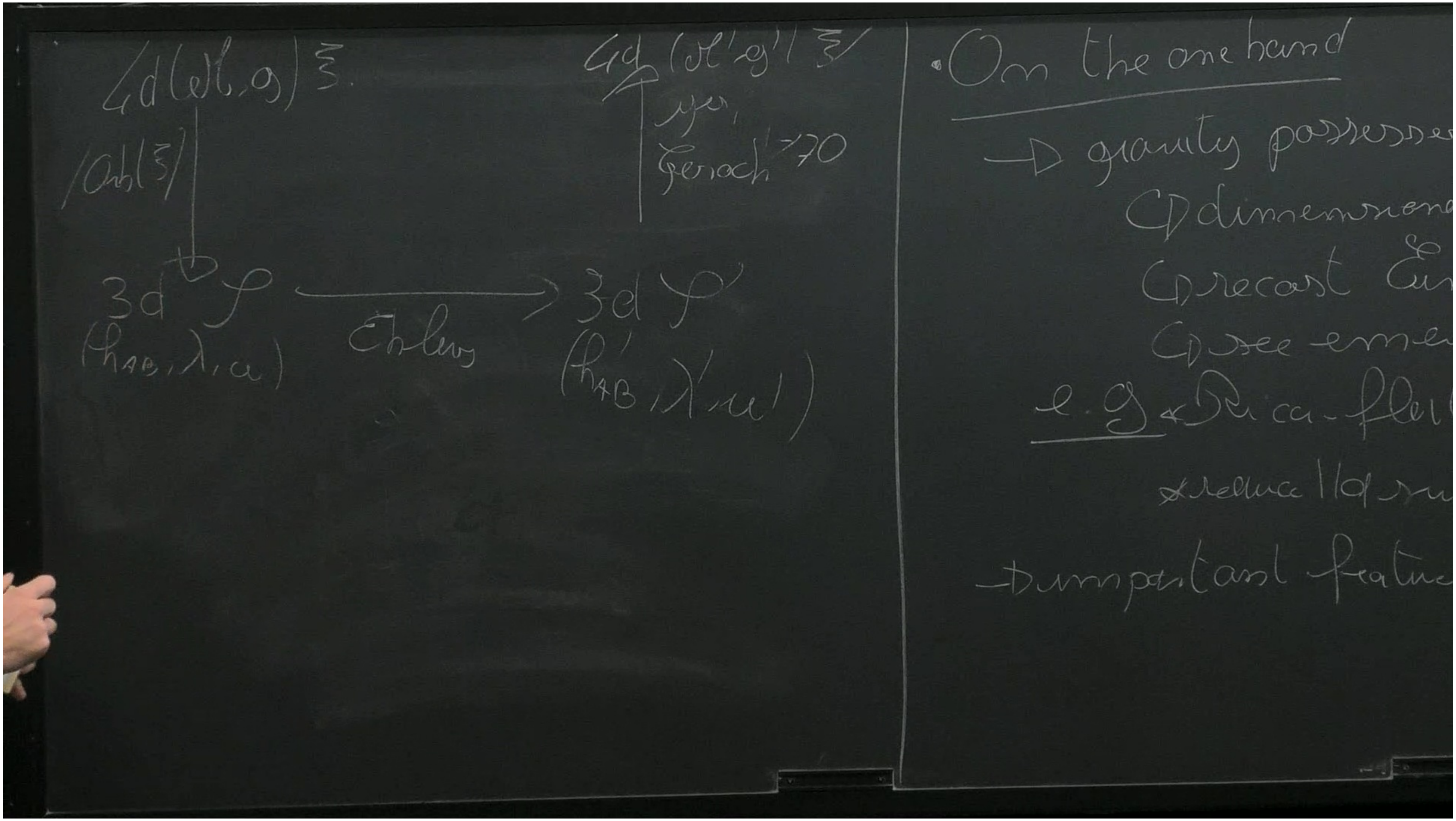
Einstein's eq on T:

$$\tilde{h}_{AB} = \lambda\, h_{AB}$$

$$\tau = \omega + i\lambda$$

$$\Longrightarrow \boxed{\begin{aligned} \tilde{R}_{AB} &= -\frac{2}{(\tau-\bar{\tau})^2}\, \tilde{D}_{(A}\tau\, \tilde{D}_{B)}\bar{\tau} \\ \tilde{D}^2 \tau &= \frac{2}{\tau-\bar{\tau}}\, \tilde{D}_{M}\tau\, \tilde{D}_{N}\tau\, \tilde{h}^{MN} \end{aligned}}$$


$R_{PQRS} + \frac{2}{\lambda}[\nabla_P\xi_R\nabla_Q\xi_S - \nabla_P\xi_R\nabla_Q\xi_S]$
$\lambda' h'_{AB} = \lambda h_{AB}$
$\tau \to \tau'(\tau)$
invariant iff $\tau \to \frac{\alpha\tau+\beta}{\gamma\tau+\delta}$
$\begin{pmatrix}\alpha & \beta \\ \gamma & \delta\end{pmatrix} \in SL(2,\mathbb{Z})$
$(h_{AB}(\lambda,\omega),\lambda,\omega) \xrightarrow{SL(2,\mathbb{Z})} (h'_{AB}(\lambda',\omega'), \lambda'(\lambda,\omega), \omega'(\lambda,\omega))$



Ehlers
Geroch
On the one hand
gravity
recast
e.g.
important feature

4d $(\mathcal{M}, g)$ ξ.

/ Orb(ξ) /

3d $\mathcal{S}$ $(h_{AB}, \lambda, \omega)$ $\xrightarrow{\text{Ehlers}}$ 3d $\mathcal{S}'$ $(h'_{AB}, \lambda', \omega')$

4d $(\mathcal{M}', g', \xi')$ yes, Geroch '70

Back to 4d:
from (*) $\Rightarrow F'_{AB} = \frac{1}{(-\lambda')^{3/2}} \sqrt{h}\, \varepsilon_{ABC} \mathcal{D}^C \omega'$

is closed

$\underline{F} = d\underline{\eta} \Longrightarrow g'_{AB} = h'_{AB} + \frac{\xi'_A \xi'_B}{\lambda}$

$\xi'^A \eta_A = 1$

- On the one hand

→ gravity possesses

↳ dimensional

&

↳ recast Ein

↳ see emer

e.g. & Ricci-flat

& reduce 1/d

→ important feature

ξ'

70

ξ'_B

example ; (M, g) Schwarzschild $\xi = \partial_t$

"3 pages"

Taub-NUT solution $\xi' = \partial_t$.

ξ'

70

example: $(\mathcal{M}, g)$ Schwarzschild $\xi = \partial_t$

"3 pages" Taub-NUT solution $\xi' = \partial_t$.

② AFS in covariant N-U gauge

ⓐ Gauge fixing procedure

ξ'_B

② AFS in covariant N-U gauge

ⓐ Gauge fixing procedure

Select a gauge

↓ express the line element in terms of arbitrary f^n of ALL coordinates (u, r, x^i)

→ impose bry conditions fall-off in $1/r$ (log terms)

↓ Einstein's equations

set of required f^n now depending on (u, x^i)

→ dynamical eq ("flux balance" laws) $(\partial_u M = \dots)$

conserv. of M

$\mathcal{L}_\xi T = 0 \Longrightarrow T$ is a tensor on S.

$\xi^\rho T^{\mu\nu}{}_{\rho\sigma} = 0$

(ξ Killing)

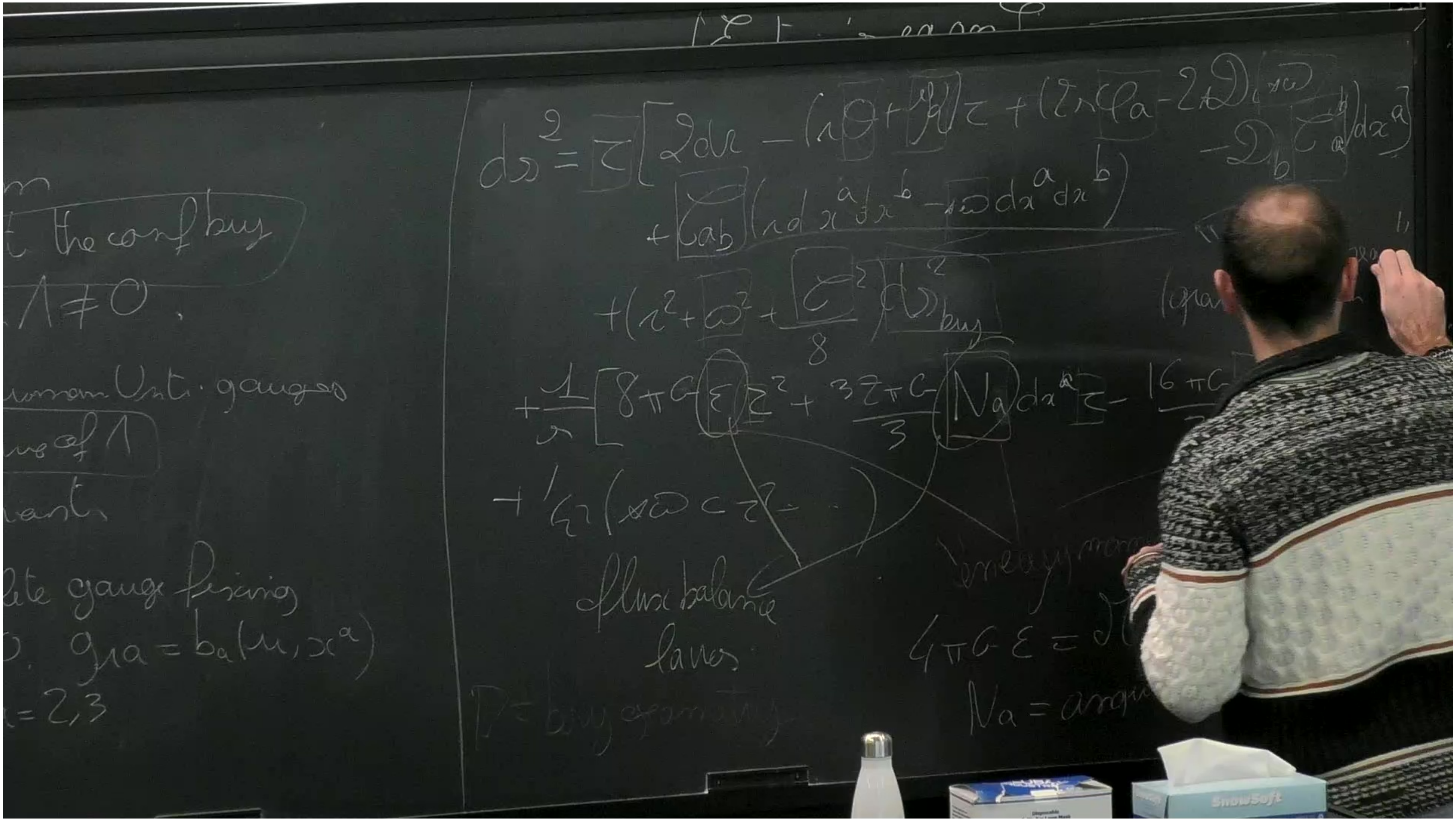
examples:

* AdS → Fefferman-Graham
  * covariant w.r.t. the conf bdy
  * valid only when $\Lambda \neq 0$.
* AF → Bondi and Newman-Unti gauges
  * valid irrespective of Λ
  * not bdy covariant.

Cov NU gauge: incomplete gauge fixing

$$g_{rr} = 0, \quad g_{ra} = b_a(u, x^a)$$

$$a = 2, 3$$


the conf bdy
Λ ≠ 0
gauge fixing
flux balance
laws
4πG ε =

t. the conf bry

$\Lambda \neq 0$.

...unmam. Unti. gauges

...ue of Λ

...ant.

...lete gauge fixing

$g_{ra} = b_a(u, x^a)$

$= 2,3$

$ds^2 = \ldots$

$+ \boxed{C_{ab}}\left(r\, dx^a dx^b - \ldots\, dx^a dx^b\right)$

"Bondi shear" (grav-radiation)

$+\left(r^2 + \boxed{\omega}^2 + \frac{\boxed{C}^2}{8}\right)\boxed{ds^2_{bry}}$

$+\frac{1}{r}\left[8\pi G\, \varepsilon\, \boxed{\tau}^2 + \frac{32\pi G}{3} \boxed{N_a}\, dx^a \boxed{\tau} - \frac{16\pi G}{3} \boxed{T_{ab}}\, dx^a dx^b\right]$

$+ 1/r^2\left(\ldots\omega\, c\, \tau^2 - \ldots\right)$

flux balance laws

"energy momentum" tensor

$4\pi G\, \varepsilon = \partial b$ covariant mass

N_a = angular momentum aspect

D = bry geometry

$$ds^2 = \boxed{\Sigma}\Big[2\,du - (r\Theta + \varphi_a)\Sigma + (2r\varphi_a - 2r D_a(\ldots) - D_b C_a{}^b)\,dx^a\Big]$$

$$+\, C_{ab}\,(r\,dx^a dx^b - \ldots\, dx^a dx^b)$$

"Bondi shear" (grav-radiation)

$$+\,\Big(r^2 + \omega^2 + \frac{C^2}{8}\Big)\,ds^2_{bdy}$$

$$+\,\frac{1}{r}\Big[8\pi G\,\mathcal{E}\,\Sigma^2 + \frac{32\pi G}{3} N_a\,dx^a\,\Sigma - \frac{16\pi G}{3} T_{ab}\,dx^a dx^b\Big]$$

$$+\,\frac{1}{r^2}\big(\ldots\big)$$

flux balance laws

"energy momentum tensor"

$4\pi G\,\mathcal{E} = M$ covariant mass

N_a = angular momentum aspect

D = bdy geometry

Bry geo on $\mathscr{I}^+$: conformal Carrollian structure

$$(\bar{v},\ ds^2 = 0 \times \underline{\tau}^2 + q_{ab}\, dx^a dx^b)$$

$$\underline{\tau}(\bar{v}) = 1$$

in coor. $\bar{v} = \frac{1}{\Omega}\partial_t$

$$\underline{\tau} = \Omega\, dt - b_a dx^a$$

$$ds^2 = \boxed{\underline{\tau}}\,[\,2dr - (\ldots$$

$$+ \boxed{q_{ab}}$$

$$+ (r^2 + \boxed{\omega}^2 + \ldots$$

$$+ \frac{1}{r}\left[8\pi G\, \varepsilon\, \underline{\tau}^2 + \ldots\right.$$

$$+ \frac{1}{r^2}(\star\omega\, c\, \tau \ldots$$

flux balance laws

D = bry geometry

$(\vec{v}, ds^2 = 0 \times \underline{\tau}^2 + q_{ab}\, dx^a dx^b)$

$\underline{\tau}(\vec{v}) = 1$

in coor. $\vec{v} = \frac{1}{\Omega} \partial_t$

$\underline{\tau} = \Omega dt - b_a dx^a \rightarrow g_{ra} = b_a \neq 0$

$ds^2 = \ldots$

$+ \boxed{C_{ab}} (\ldots$

$+ (r^2 + \boxed{\omega}^2 + \ldots$

$+ \frac{1}{r} [8\pi G \boxed{\varepsilon}\, \underline{\tau}^2 + \ldots$

$+ \frac{1}{\varepsilon^2} (\ldots \omega \ldots$

flux balance laws

∃ a subspace of solutions of Einstein's eq which admits a resummable line elements

$\Longrightarrow$ Petrov - algebraically special sol

$* \; \sigma_{ab} = 0$

$* \; N_a = 0 \Longrightarrow$ $\boxed{\text{a null geodesic shear-less}}$ $D = $ bulk geometry

$* \; E_{ij} = 0$

Rq: $\Psi^0 = 0$; $\Psi^1 = 0$

in Weyl-scalars notation

$$ds^2_{\text{Alg Special}} = \Sigma \left[2 d\tau + 2 (r \mathcal{Q}_a - * \mathcal{D}_a * \omega) dx^a - (r\Theta + {}^*\!\mathcal{K}) \Sigma \right] + (r^2 + * \omega^2) ds^2_{\text{sy}} + \frac{\Sigma^2}{r^2 + * \omega^2} (8\pi G \varepsilon r + * \omega C)$$

example: "3 pages" { $(\mathcal{M}, g)$ / Sch

Taub

② AFS in covariant N-

ⓐ Gauge fixing

Select a gauge

$\downarrow$ express the line element in terms of arbitrary f^n of All coordinates (u, r, x^i)

set

f^n

$= \Xi \left[2 d r + 2 h \mathcal{Q}_a - * \mathcal{Q}_a * \varpi \right) dx^a - (\ldots) \Xi \int$

$+ (r^2 + *\varpi^2) ds^2_{\partial y} + \frac{\Xi^2}{r^2 + *\varpi^2} \left(8\pi G \Xi r + *\varpi C \right))$

$* \Rightarrow$ transverse Hodge dual
x^a

Cotton tensor
C^{00}

$* V^a = \sqrt{q}\, \underbrace{\varepsilon^a{}_b}_{\text{L-C tensor}} V^b$

$C \supset$ NUT charge

$(\nabla_A \nabla_B \xi_C = R_{DABC} \xi^D$ when ξ Killing$)$

② AFS in covariant N-...

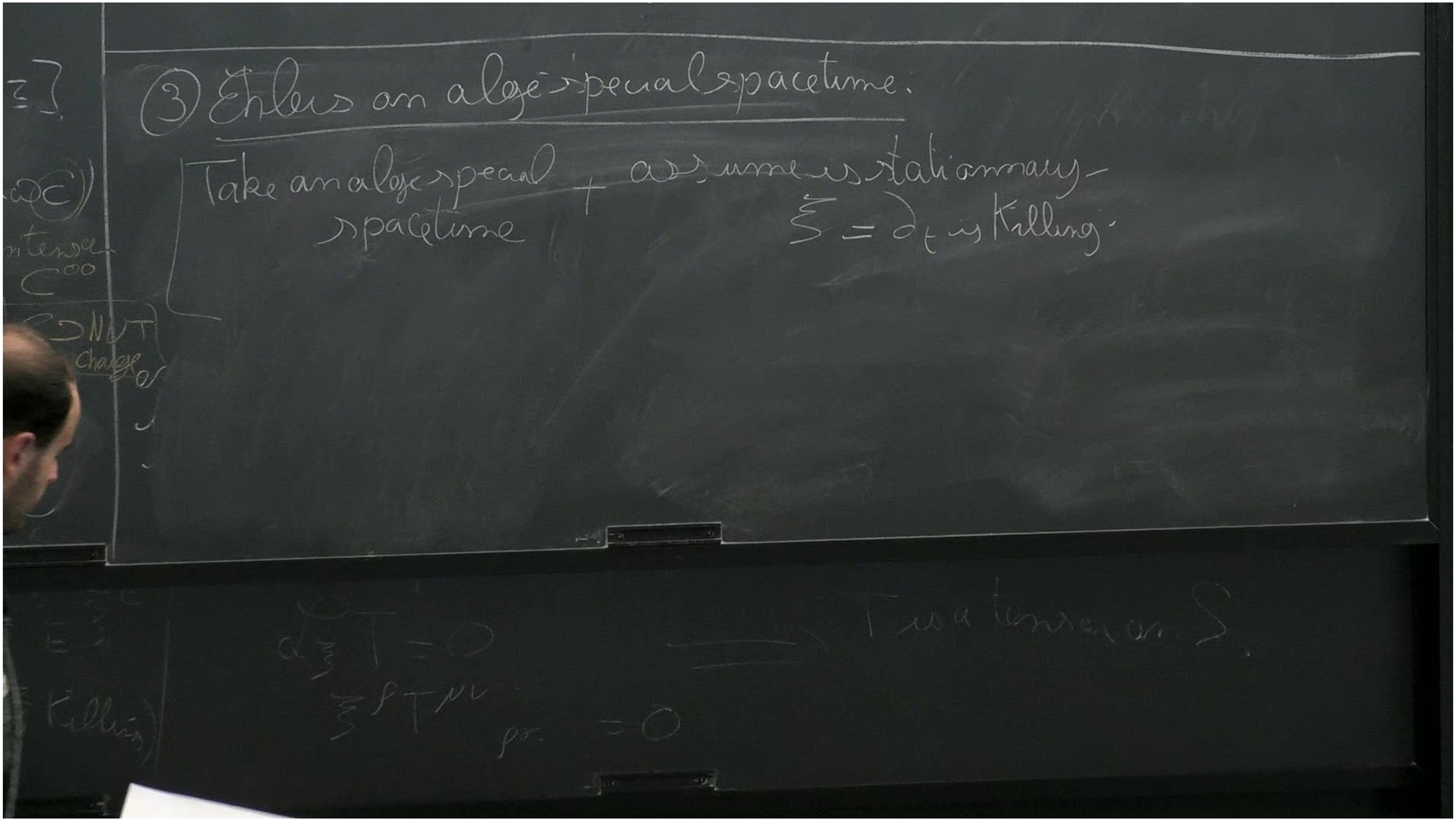
ⓐ Gauge fixing

Select a gauge
$\downarrow$
express the line element
in terms of arbitrary
f^n of ALL coordinates
(u, ρ, x^i)

set
f^n

$\xi \rho T^{\mu\nu}$

$= C$


③ Ehlers on alg. special spacetime.
Take an alg. special spacetime + assume is stationary –
ξ = ∂_t is Killing.
T is a tensor on S.

③ Ehlers on alge. special spacetime:

Take an alge. special spacetime + assume is stationary –
$\xi = \partial_t$ is Killing.

$$\lambda = \frac{8\pi G \varepsilon_r + \star\omega r}{r^2 + \star\omega^2} - \underbrace{K}_{\text{curvature}} \qquad \omega = \frac{8\pi G \varepsilon \star\omega - \omega r}{r^2 + \star\omega^2} + K^*$$

$\boxed{h_{AB}}$

ξ]

$\omega(C)$)

intensive

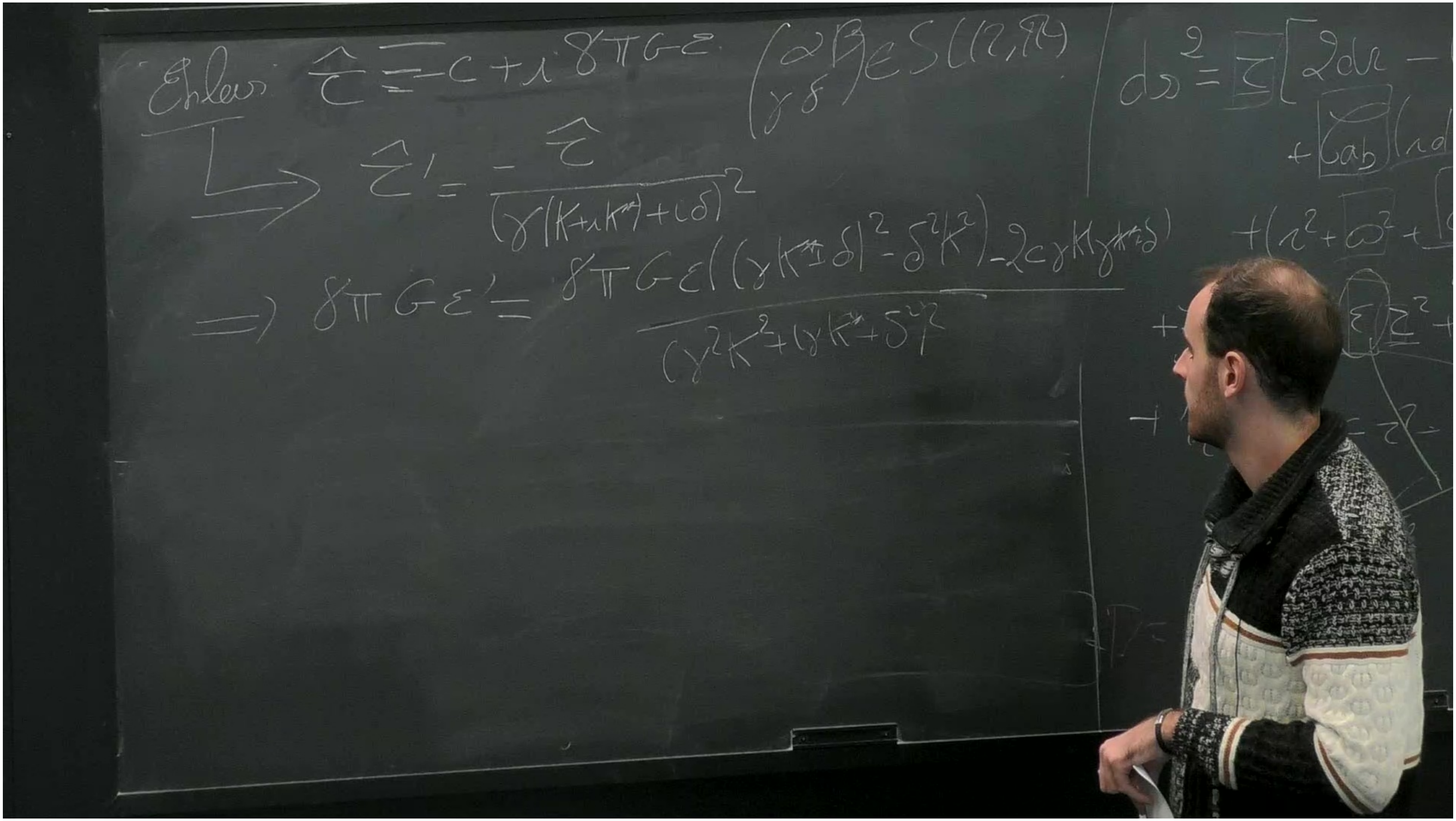
C^∞

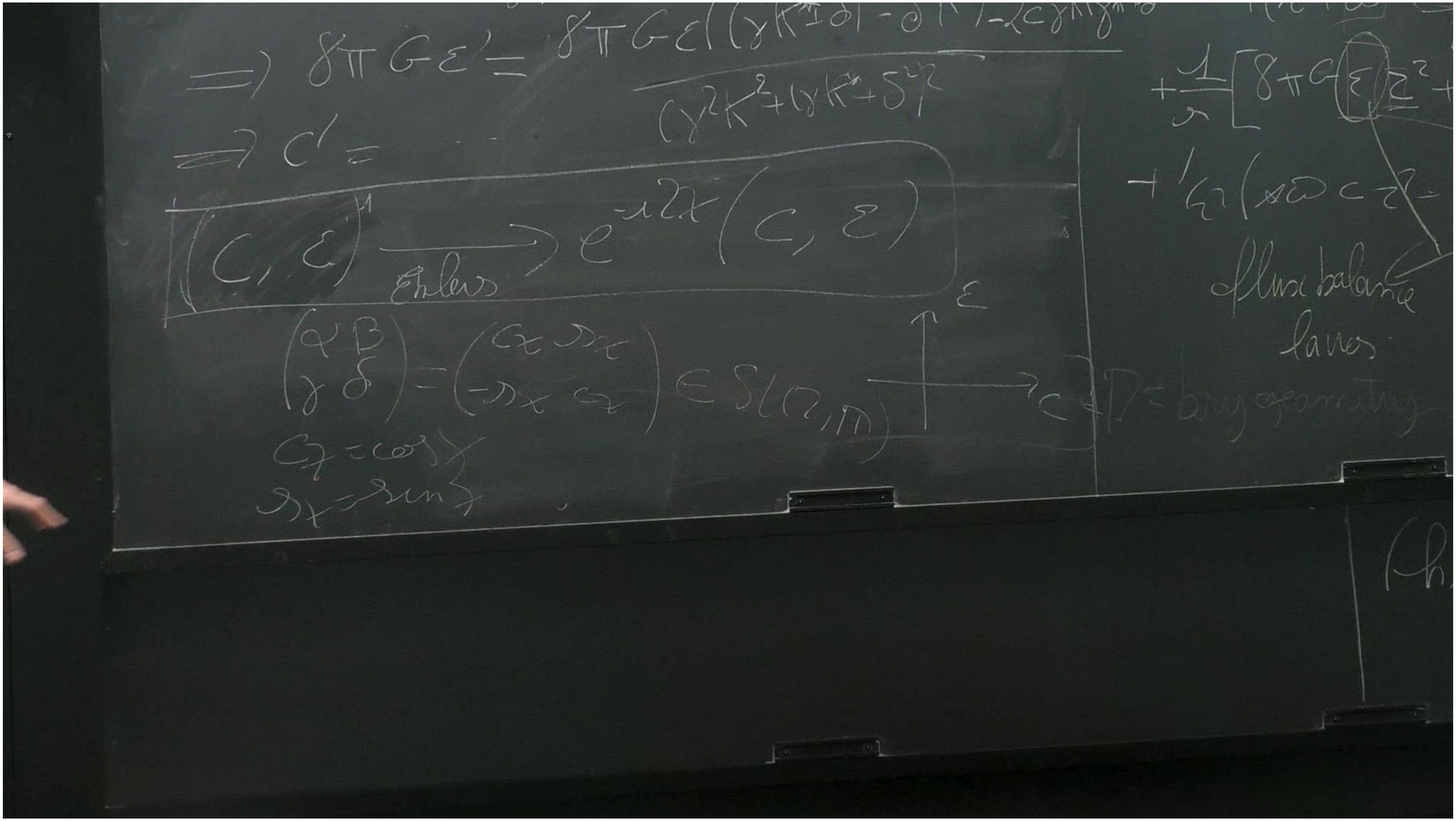
$C \supset$ NUT charge

$d_\xi T = 0 \Longrightarrow T$ is a tensor on S.

$\xi^\rho T^{\mu\nu}{}_{;\rho} = 0$

(ξ Killing)


Ehlers


Ehlers
flux balance
laws