Title: Applications and prospects of Lorentzian path integrals in quantum gravity

Speakers: Johanna Borissova

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Abstract:

Lorentzian path integrals exhibit profoundly different properties from Euclidean ones due to the oscillatory integrand which weighs different configurations through interference. Key troubles encountered in Euclidean quantum gravity are the conformal factor problem of Euclidean quantum GR and divergences due to spike configurations in Euclidean quantum Regge calculus. The first part of this talk will focus on how these troubles are resolved in Lorentzian quantum Regge calculus. I will emphasize the unambiguous choice of contour for the integral over the conformal mode in a saddle-point expansion and furthermore show that bulk-length expectation values are finite for spike and spine configurations away from the classical regime.

The second part of this talk will focus on properties of Lorentzian path integrals beyond GR. I will illustrate that higher-derivative and non-local actions can be expected to suppress spacetime configurations with curvature singularities. Finally, I will revisit the long-standing question of global symmetries in quantum gravity by providing examples for non-local actions designed to suppress global-symmetry-violating black-hole configurations in the Lorentzian path integral.

Applications and prospects of Lorentzian path integrals in quantum gravity

Johanna Borissova

PI Graduate Student Seminar
3rd February 2025

Motivation

Path integral for quantum gravity

$$Z = \int_{\mathscr{C}} \mathscr{D}[\text{gravity}]\mathscr{D}[\text{matter}]\, e^{iS}$$

- configuration space $\mathscr{C}$ (fundamental d.o.f.):
  - [gravity] = geometry? topology? continuum vs discrete?
  - signature: Lorentzian vs Euclidean?*
  - measure $\mathscr{D}$?*
- action S (dynamics):
  - classical GR vs higher-derivative or non-local?*
  - coupling to matter?
- evaluation:
  - semiclassical or saddle-point approximation?*
  - symmetry reduction a.k.a. minisuperspace?*
  - gauge fixing?*
  - regularization?*

This talk:
- [gravity] = fundamentally continuous Lorentzian metric geometries
- aspects of *'s in different frameworks

Conformal factor problem in Euclidean quantum GR

$S_E[g] = -\int d^4x\sqrt{g}R$ **not positive semi-definite** — **conformal transformation:** $S_E[\Omega^2 g] \supset -\int d^4x\sqrt{g}\left(\Omega^2 R + 6\partial_\mu\Omega\partial^\mu\Omega\right)$
[Gibbons, Hawking & Perry 1978]

Expansion to 2nd order: $S_E[g+\delta g] = S_E[g] + \delta S_E + \frac{1}{2}\delta^2 S_E + \ldots$ with $\delta g_{\mu\nu} = h_{\mu\nu} = h^T_{\mu\nu}(\text{tracefree}) + \frac{1}{4}h g_{\mu\nu}(\text{trace})$ — onshell:
$\delta^2 S_E\Big|_{\delta S_E=0} \supset -h^T\nabla^2 h^T, +h\nabla^2 h$

$Z_E^{(2)}$* **divergent Gaussian integral**

mathematical resolution: $h \to ih$ **physical ad-hoc procedure** — $\mathscr{C}_E = ?$

*note: saddle-point expansion

Conformal factor (non-)problem in Lorentzian quantum Regge calculus

$S_L[g] = \int d^4x\sqrt{-g}R \xrightarrow{\text{discretization} \triangleq \text{regularization}} S_{L_{Regge}}[s_e] = -i\sum_{h\in\triangle}\sqrt{\mathbb{V}_h}\epsilon_h$ [Regge 1961, Sorkin 2019, Asante, Dittrich & Padua-Argüelles 2021]

$\mathscr{C}_L = \{[g]\} \rightarrow \mathscr{C}_{L_{Regge}} = \{s_e \,|\, \sigma\text{-inequalities}\}$

σ: d-simplex
h (hinge): codimension-2 subsimplex
ϵ_h: deficit angle $\leftrightarrow$ curvature
$\mathbb{V}_h$: signed squared volume of h
$s_e = \mathbb{V}_e$: signed squared length of edge e

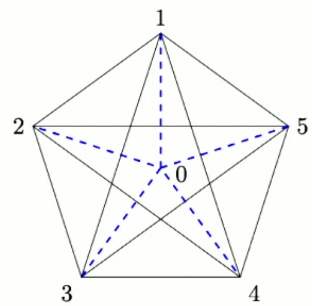
conformal mode d.o.f. encoded in $(d+1)-1$ Pachner moves: [JB & Dittrich 2023]

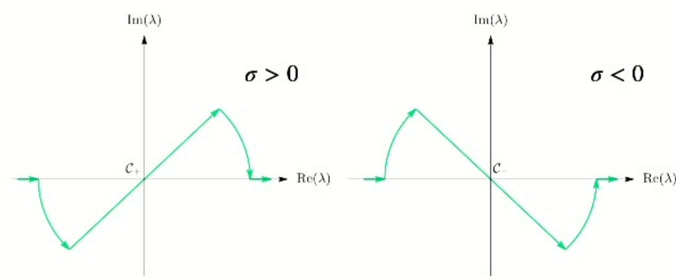
Expansion to 2nd order: $S_{L_{Regge}}[s_e + \delta s_e] = S_{L_{Regge}}[s_e] + \delta S_{L_{Regge}} + \frac{1}{2}\delta^2 S_{L_{Regge}} + \ldots$ with $\delta s_e = \lambda_e$ — onshell:

$Z^{5-1\;(2)}_{L_{Regge}}$* $\supset \int_{\mathbb{R}} d\lambda e^{\frac{i}{2}\sigma\lambda^2}$, σ Hessian eigenvalues

$Z^{5-1\;(2)}_{L_{Regge}}$* convergent **Gaussian integral**

5 − 1 (4d)





*note: saddle-point expansion

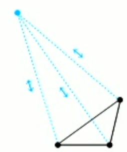
Spike problem in Euclidean quantum Regge calculus

$\mathscr{C}_{E_{Regge}} = \{s_e \,|\, \sigma\text{-inequalities in Euclidean signature: } \mathbb{V}_\sigma > 0 \;\;\&\;\; \mathbb{V}_\rho > 0 \;\forall \rho \subset \sigma\}$

σ: d-simplex
$\mathbb{V}_\rho$: signed squared volume of subsimplex $\rho \subset \sigma$,
computed as Caley-Menger determinant
($\mathbb{V}_\rho < 0$, $\mathbb{V}_\rho > 0$, $\mathbb{V}_\rho = 0$ if ρ timelike, spacelike, null)

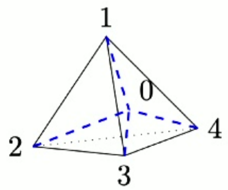
$2d$ **Euclidean example:** $\mathbb{V}_{\triangle=(012)} > 0$ & $s_{e=(01),(02),(12)} > 0$

e.g. possible: $s_{e=(12)} > 0$ fixed, $s_{e=(01),(02)} \to +\infty \;\Rightarrow$ **spike**

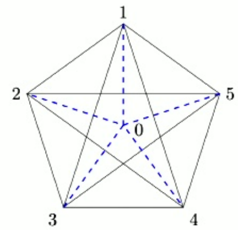


Example for spikes in d-dim: initial configuration of $(d+1)-1$ **Pachner moves** with $|$bulk edges$| \to \infty$

$4-1$ $(3d)$



$5-1$ $(4d)$



bulk length expectation values: $\langle l^n \rangle \;\to \infty$ **infinite** for sufficiently high n
[Ambjørn et al 1997]

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Spike problem in Euclidean quantum Regge calculus

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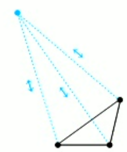
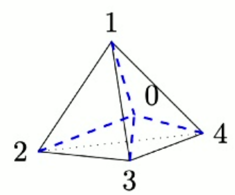
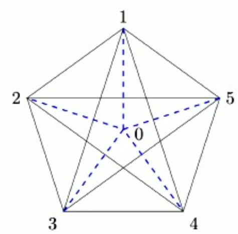


Example for spikes in d-dim: initial configuration of $(d+1)-1$ **Pachner moves** with |bulk edges| $\to \infty$

$4-1$ ($3d$)



$5-1$ ($4d$)



bulk length expectation values: $\langle l^n \rangle \to \infty$ **infinite** for sufficiently high n
[Ambjørn et al 1997]

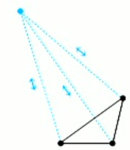
Spike and spine (non-)problem in Lorentzian quantum Regge calculus

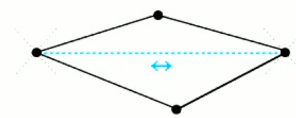
$\mathscr{C}_{L_{Regge}} = \{s_e \,|\, \sigma\text{-inequalities in Lorentzian signature: } \mathbb{V}_\sigma < 0 \;\&\; (\rho \subset \sigma, \mathbb{V}_\rho \leq 0 \Rightarrow \forall \rho' \supset \rho, \rho' \subset \sigma : \mathbb{V}_{\rho'} \leq 0)$ [Tate, Visser 2012]$\}$

$2d$ **Lorentzian example:** $\mathbb{V}_{\triangle=(012)} < 0$ e.g. for $\triangle = (st_1t_2), (ts_1s_2)$ always satisfied

e.g. possible: $s_{e=(12)} < 0$ fixed, $s_{e=(01),(02)} \to +\infty \;\Rightarrow$ **spike**

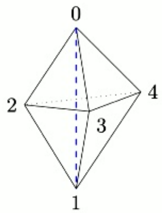
e.g. possible: $s_{e=(02),(12)} > 0$ fixed, $s_{e=(01)} \to -\infty \;\Rightarrow$ **spine**



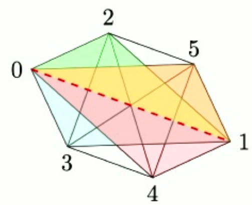


Example for spines in d-dim: initial configuration of $d-2$ **Pachner moves** with |bulk edge| $\to \infty$

$3-2\;(3d)$



$4-2\;(4d)$



bulk length expectation values: $\langle \lambda^n \rangle$, $m = n + M$, $\mathscr{D}\mu \simeq \lambda^M$ **measure** [JB & Dittrich 2023]

$\langle \lambda^n \rangle^{5-1*} \supset \int_{\lambda_0}^{\infty} d\lambda \lambda^m e^{i\beta\sqrt{\lambda}} = 2\lambda_0^{m+1}\mathbf{EI}_{-2m-1}(-i\pi\beta\sqrt{\lambda_0})$ **finite** for all m [bulk configuration: $(sssss)$ or $(ttttt)$, $s_{0i} \to \pm\lambda$, $\beta(\text{bdry}) \in \mathbb{R}$]

$\langle \lambda^n \rangle^{4-2*} \simeq \int_{\lambda_0}^{\infty} d\lambda \lambda^m e^{i\pi\lambda} = \lambda_0^{m+1}\mathbf{EI}_{-m}(-i\pi\lambda_0)$ **finite** for all m [bulk configuration: (s) or (t), $s_{01} \to \pm\lambda$] [JB, Dittrich, Qu & Schiffer 2024]

*note: asymptotic regime far away from classical saddle point

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Properties of Lorentzian path integrals beyond GR

So far:

I) Properties of Lorentzian vs Euclidean path integrals for GR (continuum + Regge calculus) — 2 types of approximations:

- saddle-point approximation vs far-from-saddle-point approximation *within* quantum GR
- quantum GR ≙ semiclassical approximation of an unknown quantum-gravity theory including higher-derivative & non-local terms

Now:

II) Properties of Lorentzian vs Euclidean path integrals beyond GR (continuum)

- **destructive interference** between neighbouring off-shell "paths" with rapidly oscillating phase factors iS
- action S (dynamics) dictates which configurations have $|S[g]| \gg 1$ and are therefore suppressed dynamically

Questions:

- How does the presence of higher-derivative or non-local terms affect the contribution of (off-shell) spacetime configurations with **curvature singularities**?
- Are there dynamical mechanisms for the suppression of **global-symmetry-violating configurations**?

Infinite-action principle

$$Z = \int_{\mathscr{C}} \mathscr{D}[g] e^{iS[g]}$$

dynamical suppression mechanism: $|S[g]| \gg 1$ for g and "neighbours" $\Rightarrow$ $\{[g]\}$ **suppressed in** Z

$\mathscr{C} \equiv \{[g]$ Lorentzian four-geometries$\} \supset \{$**geometries with curvature singularity**$\}$, $\{$**generic black-hole geometries**$\}$

Applications:

1. **singularity suppression** in quantum amplitudes as **selection principle** for action S and gravitational dynamics [JB & Eichhorn 2021; JB 2023]

2. required dynamics for the **suppression of virtual black-hole configurations** in the context of the **no-global-symmetries conjecture** [JB, Eichhorn, Ray 2024]

Suppression of spacetimes with curvature singularity

Viewpoint

- **singularity theorems** [Penrose 1965] $\Rightarrow$ breakdown of predictivity of GR as **classical theory**
- ultimate goal: **singularity-free quantum theory**

Question: Significance of classical spacetime singularities for quantum gravity?

Goal: find S satisfying $\left| S[\{[g]\ \textbf{singular}\}] \right| = \infty \ \hat{=}$ **infinite-action principle (quantum)** [JB & Eichhorn 2021; JB 2023]

Logic:

S **satisfies quantum singularity-suppression principle (QSSP)** $:\Leftrightarrow \ \forall\, g$ off-shell **singular:** $\left| S[g] \right| = \infty$

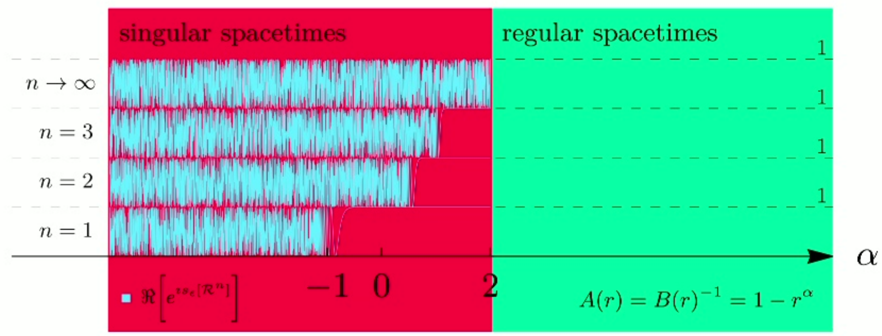
reversed: $\exists\, g$ off-shell **singular:** $\left| S[g] \right| =$ finite $\Rightarrow$ S **does not satisfy QSSP**

Consequence
allows restriction to **"minisuperspace"** for ruling out actions S incompatible with QSSP [JB 2023]

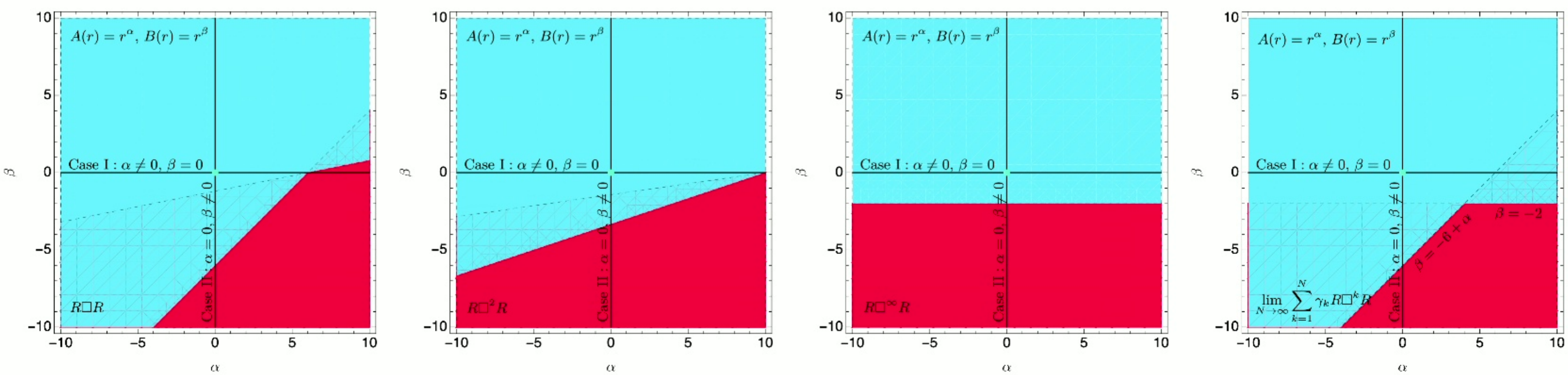
Suppression of spacetimes with curvature singularity

focus: spacetimes with **curvature singularity + static spherically symmetry**

Quantum suppression of classical spacetime singularities guiding principle for building and constraining quantum-gravitational dynamics



- full singularity suppression can not be achieved via $S = S[\mathscr{R}^n, \mathscr{R}\,\Box^k\,\mathscr{R}]$ for <u>any finite</u> $n, k \in \mathbb{N}$ $\Rightarrow$ ~~S_{GR}~~, ~~$S_{quadratic\ gravity}$~~, … as EFTs [JB 2023]
- **structural differences** in suppression powers of curvature invariants $\supset \mathscr{R}$ versus curvature-derivative invariants $\supset \mathscr{R}, \nabla$



Virtual black-hole configurations in the path integral

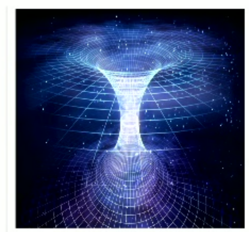
No-global-symmetries conjecture [Kallosh et al 1995, Banks & Seiberg 2011,…]

Hawking evaporation of a black hole incompatible with global charge conservation (remnants problematic too)

What if virtual black holes do not contribute to quantum-gravitational transition amplitudes?

Question: What dynamics is required to **suppress global-symmetry violating black-hole configurations** in the Lorentzian path integral? [JB, Eichhorn, Ray 2024]

Goal: find S satisfying $\left| S[\{g \text{ black hole}\}] \right| = \infty$



q ?

[Picture credits: EHT, Shutterstock]

Invariant characterization of black-hole horizons

General dynamical spherically symmetric black-hole metric

$ds^2 = -e^{2\beta(r,v)}\left(1 - \frac{2m(r,v)}{R(r)}\right)dv^2 + 2e^{\beta(v,r)}dvdr + R(r)^2 d\Omega^2$, **apparent horizon(s)** at r satisfying: $R(r) - 2m(r,v) = 0$

Horizon-detecting scalar invariant* [McNutt et al 2021]

$\chi \equiv 4C^2\left(\nabla_\mu C\right)^2 - \left(\nabla_\mu C^2\right)^2 = (R(r) - 2m(r,v)) \cdot \mathcal{F}(m(r,v), \beta(r,v), R(r)), \quad \mathcal{F} > 0$ ($C_{\mu\nu\rho\sigma}$ Weyl tensor)

$\Rightarrow \quad \chi\Big|_{r_h} = 0 \;\Leftrightarrow\; r_h$ apparent horizon i.e. **black hole** (or wormhole)

*note: apparent horizon = quasi-local surface (vs event horizon = teleological), $\chi \propto \theta_+\theta_-$ with $\theta_\pm$ expansions of the two radial null geodesic vector fields

Destructive interference of virtual black-hole configurations

Non-local action suppressing global-symmetry-violating black-hole configurations
[JB, Eichhorn, Ray 2024]

$$S = \int d^4x \sqrt{-g}\mathscr{L} \text{ with } \mathscr{L} \propto \frac{1}{\chi} \quad \Rightarrow \quad \left| S[\{g \text{ black hole (spherically symmetric)}\}] \right| \; = \; \infty$$

What are black-hole "neighbours"?

$$\text{BH}_1 \text{ with } (m_1, R_1, \beta_1) \text{ s.t. } \left(R_1 - 2m_1\right)\Big|_{r_{h_1}} = 0 \sim \text{BH}_2 \text{ with } (m_2, R_2, \beta_2) \text{ s.t. } \left(R_2 - 2m_2\right)\Big|_{r_{h_2}} = 0$$

Deformations preserving $\chi_{1,2} = \infty$:

- infinitesimal deformation of horizon surface: $\beta_1 = \beta_2$, $(m_1, R_1) \mapsto (m_2, R_2)$
- infinitesimal deformation away from horizon: $m_1 = m_2$, $R_1 = R_2$, $\beta_1 \mapsto \beta_2$

Destructive interference of virtual black-hole configurations

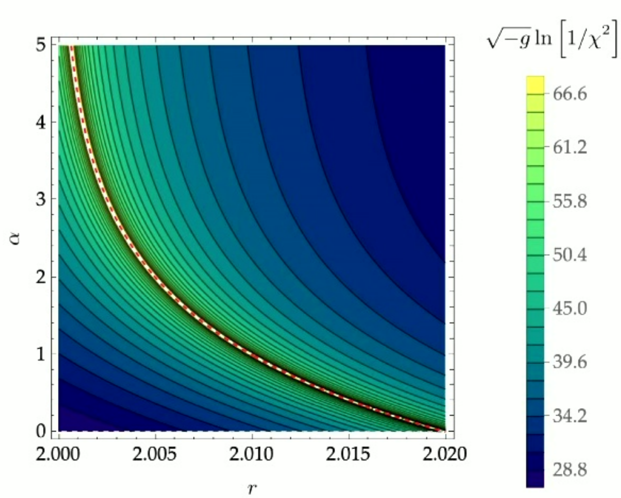
Example — one-parameter deformations of given black hole configuration $(m(r,v) = m(r), \beta(r,v) = 0, R(r) = r)$

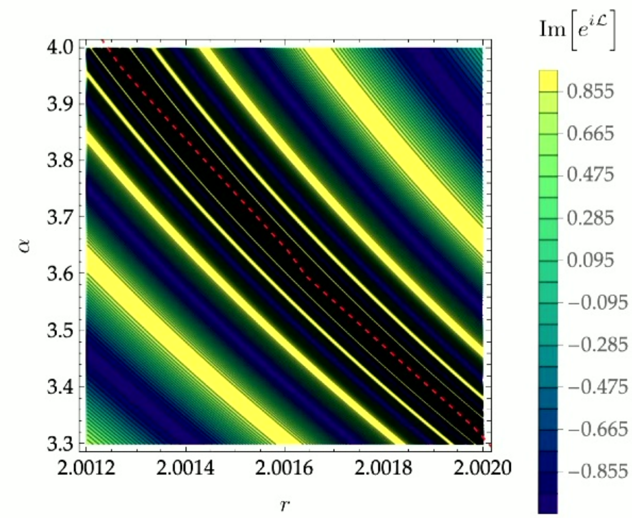
$m_1(r) = M \;\mapsto\; m_2(r) = M + m_\alpha r^{-\alpha}$ s.t. "configuration space" $\mathscr{C} = \{(m_\alpha)\}$
(one-parameter deformations of Schwarzschild labeled by α)

$m_\alpha = 0 \mapsto m_\alpha \ll 1$ fixed $\leftrightarrow$ "infinitesimally close" (to Schwarzschild) in $\mathscr{C}$

$$\sqrt{-g}\mathscr{L} = \sqrt{-g}\,\ln\left(\frac{1}{\chi^2}\right)$$

rapidly oscillating phase factor leads to **destructive interference** between black-hole neighbours [JB, Eichhorn, Ray 2024]





red dashed line: $(r_{h_\alpha} - 2M)r_{h_\alpha}^{\alpha} - 2m_\alpha = 0$
$\hat{=}$ horizon locations of configurations with different α

What is left in $\mathscr{C}$? $\Rightarrow$ no black holes — horizonless compact objects?

Conclusion

Message: Lorentzian path integrals **distinct** from Euclidean path integrals

$$Z_L = \int_{\mathscr{C}_L} \mathscr{D}[g]\, e^{iS_L[g]} \ \text{(quantum interference)} \ \text{vs} \ Z_E = \int_{\mathscr{C}_E} \mathscr{D}[g]\, e^{-S_E[g]} \ \text{(statistical weighting)}$$

oscillatory integrand…

- **evades conformal factor problem** of Euclidean quantum GR (in framework of Lorentzian quantum Regge calculus)
- ensures **finiteness of bulk length expectation values** for **spikes & spines** in Lorentzian quantum Regge calculus

- leads to **destructive interference** between neighbouring configurations with infinitely fast oscillating phase factors
- thereby provides **selection principles for gravitational dynamics** consistent with **quantum singularity suppression** and allowance of **global symmetries** in a path integral for quantum gravity