Title: The BV-Logic of Spacetime Interventions

Speakers: James Hefford

Collection/Series: Quantum Foundations

Subject: Quantum Foundations

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Abstract:

I will give a general method for producing a process theory of local spacetime events and higher-order transformations from any base process theory of first-order maps. This process theory models events as intervention-context pairs, uniting the local actions by agents with the structure of the spacetime around them. I will show how this theory is richer than a standard process theory by permitting additional ways of composing agents beyond the usual tensor product, thereby capturing various strengths of possible spatio-temporal correlations. I will also explain the connection between these compositions and the logic "system BV".

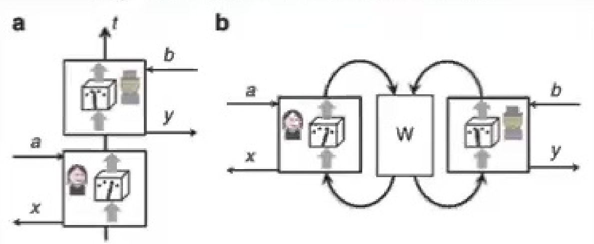
The BV-Logic of Spacetime Interventions

James Hefford (joint work with Matt Wilson)

February 4, 2025

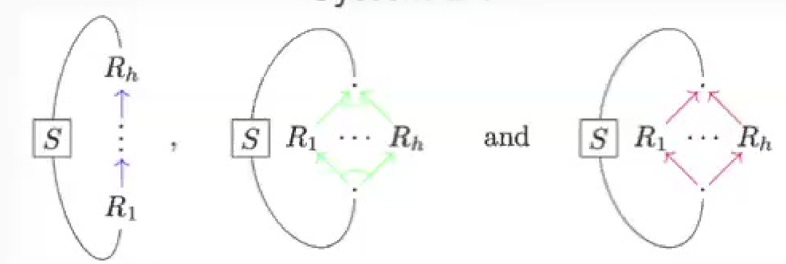
Overview

Spacetime Interventions



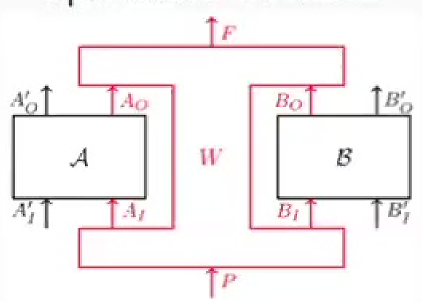
2012: Oreshkov, Costa, Brukner

System BV



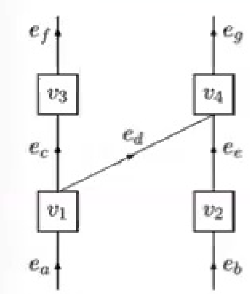
2007: Guglielmi

Spacetime Contexts



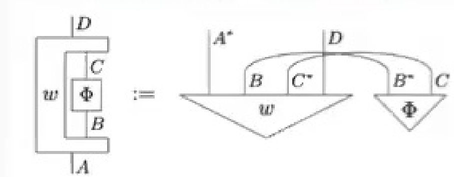
2020: Araújo, Feix, Navascués, Brukner

Discrete Quantum Causal Dynamics



2001: Blute, Ivanov, Panangaden

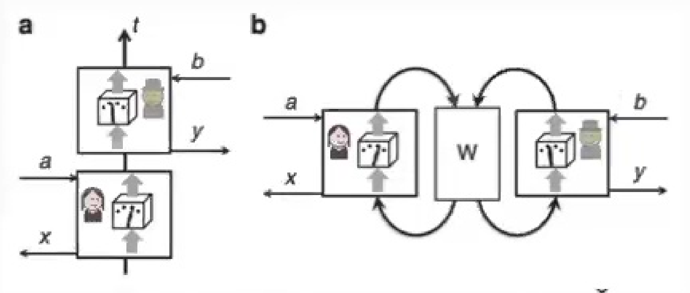
Caus-Construction



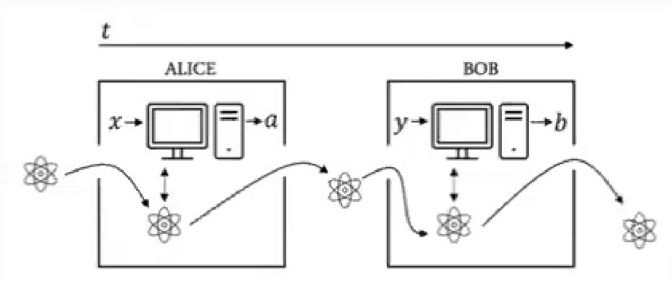
2018: Kissinger, Uijlen

2022: Kissinger, Simmons

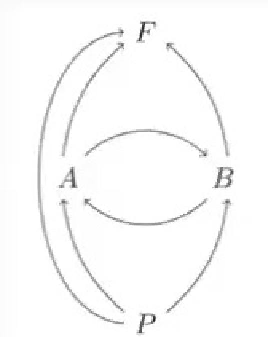
Interventions



2012: Oreshkov, Costa, Brukner

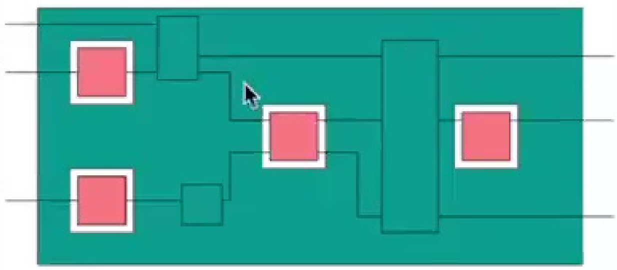


2021: Purves, Short

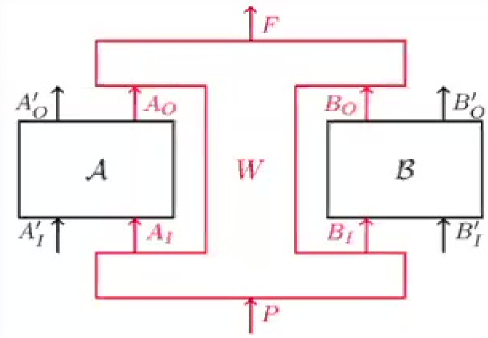


2021: Barrett, Lorenz, Oreshkov

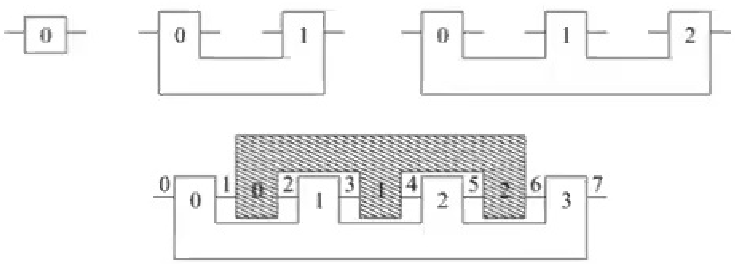
Contexts and Supermaps



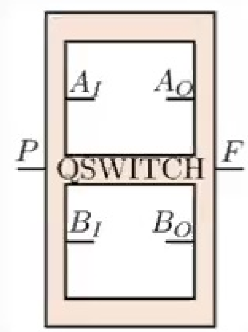
2008: Chiribella, D'Ariano, Perinotti



2020: Araújo, Feix, Navascués, Brukner



2009: Chiribella, D'Ariano, Perinotti



2021: Yokojima, Quintino, Soeda, Murao

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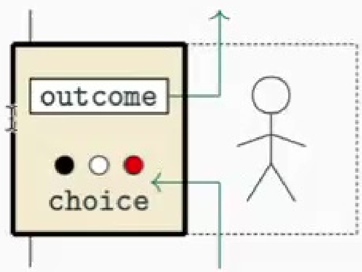
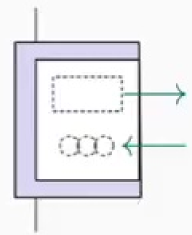
Interventions and Contexts





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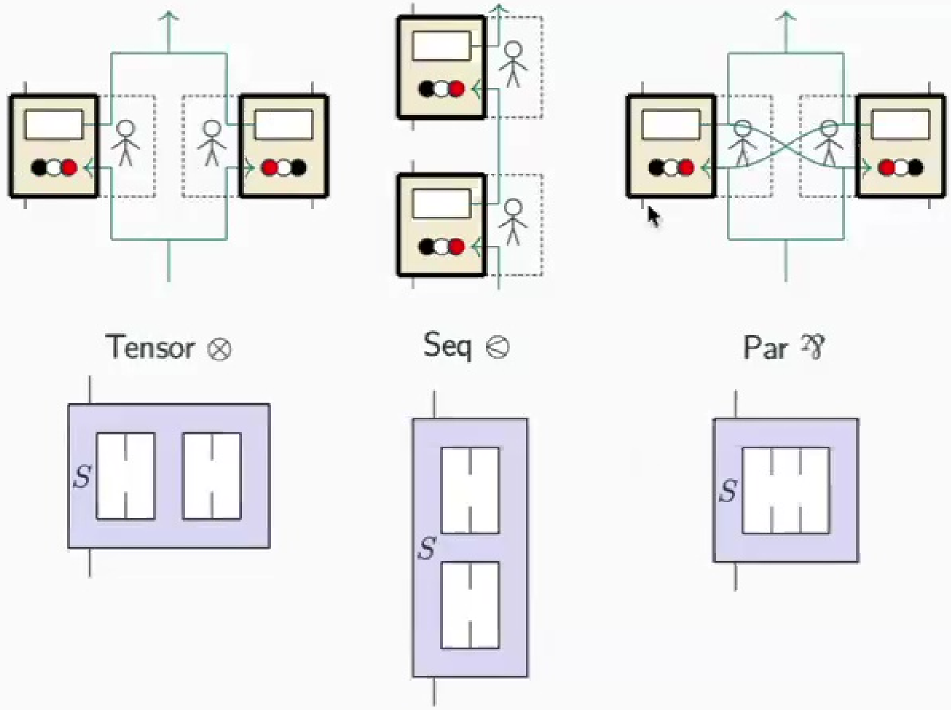
Interventions and Contexts



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System BV

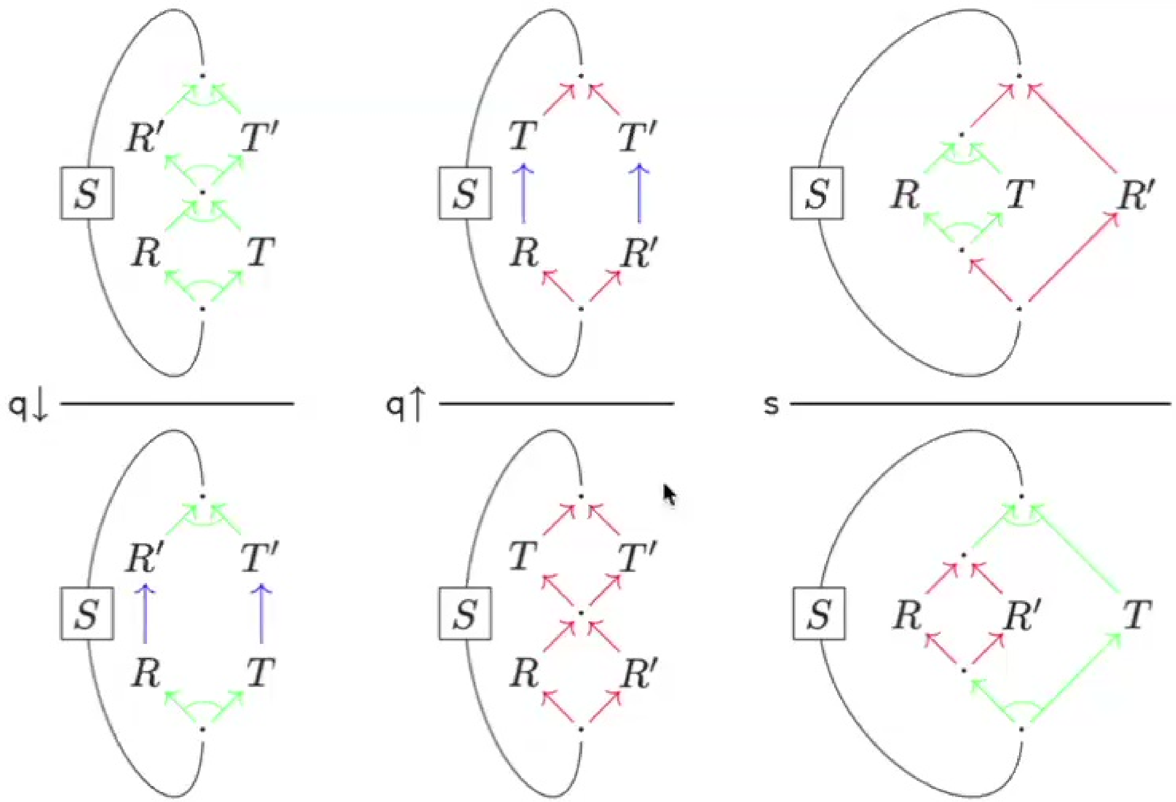
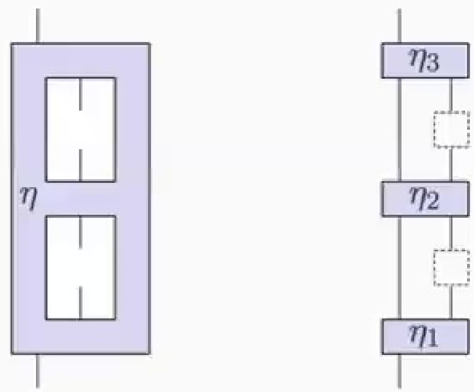


Fig. 7 *Seq, coseq and switch rules (system* SBVc)

2007: Guglielmi

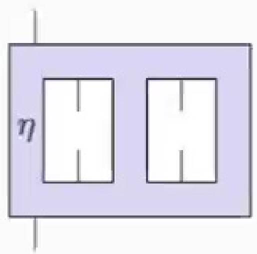
Black-Box Contexts



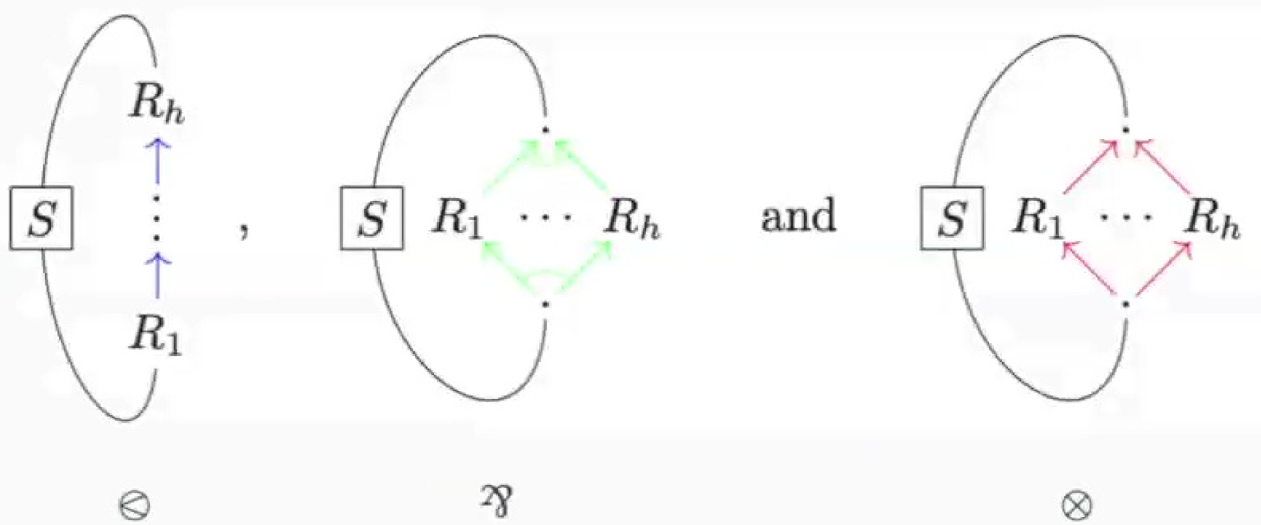
Black-Box Contexts

There exist maps with no decomposition as a comb:

- Quantum Switch (2013: Chiribella, D'Ariano, Perinotti, Valiron)
- Lugano process (2014: Baumeler, Feix, Wolf)
- OCB process (2012: Oreshkov, Costa, Brukner)
- Grenoble process (2021: Wechs, Dourdent, Abbott, Branciard)

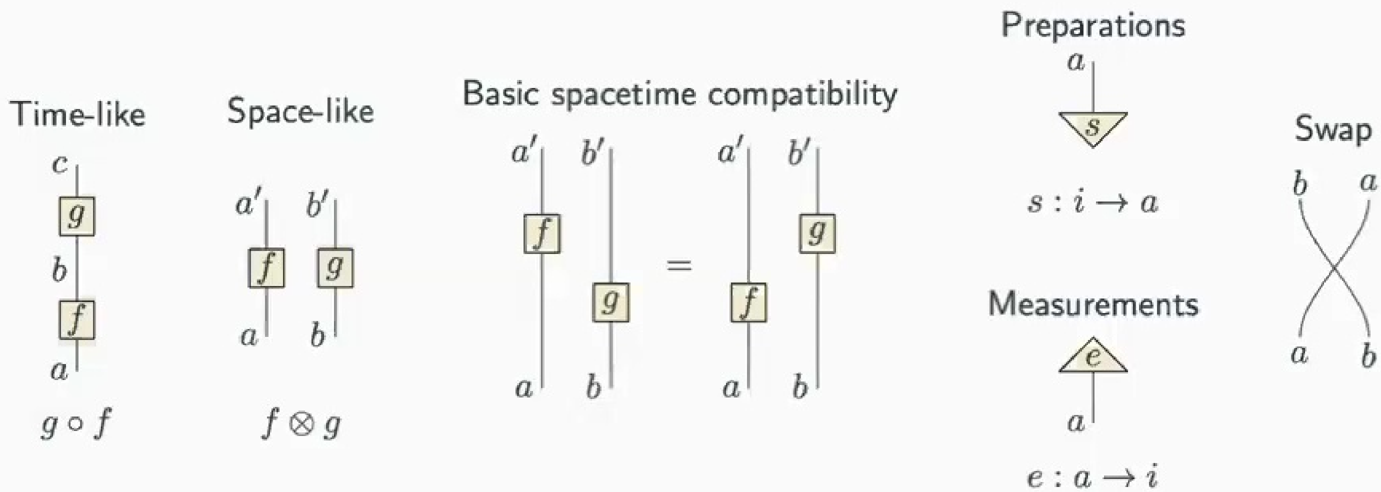


System BV



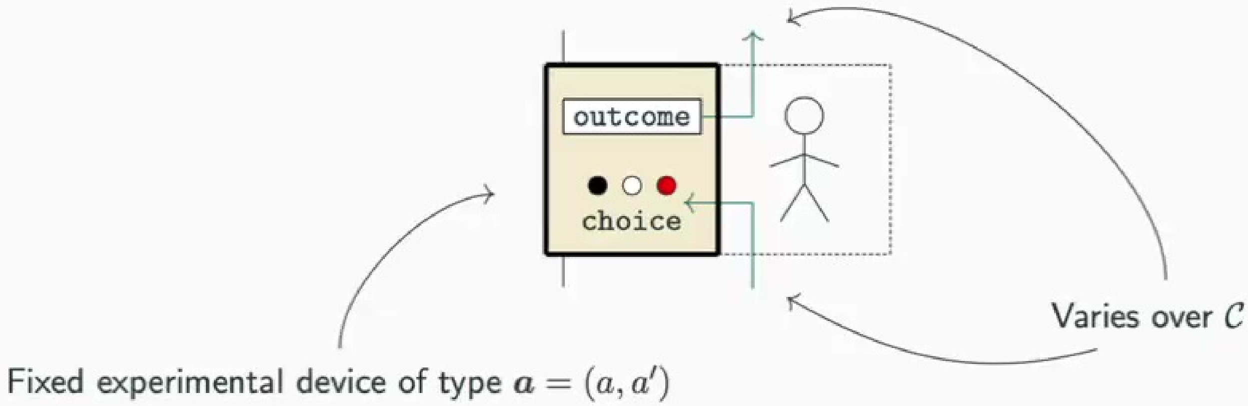
2007: Guglielmi

Process Theories



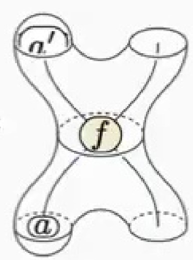
Process Theory = Symmetric Monoidal Category $\mathcal{C}$

Interventions



For instance, the collection of all tensorially extended processes at $\boldsymbol{a}$:

$$\mathcal{C}(a \otimes -, a' \otimes =) =$$



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Interventions

- $\mathcal{C}(a \otimes -, a' \otimes =)$ is an example of a profunctor
- General profunctor $P : \mathcal{C}^{\text{op}} \times \mathcal{C} \to \mathsf{Set}$
  - Sets $P(x, x')$ of interventions varying over x and x' in the process theory $\mathcal{C}$

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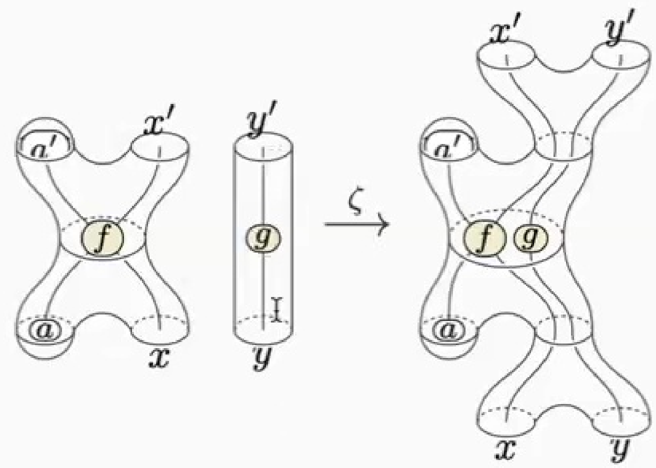
Interventions



- Profunctors with arrows $\zeta : P(x, x') \times \mathcal{C}(y, y') \to P(x \otimes y, x' \otimes y')$ are *strong*

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Interventions

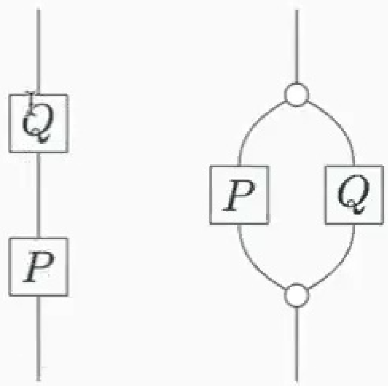
$\mathsf{StProf}(\mathcal{C})$ has

- objects: strong profunctors $P : \mathcal{C} \nrightarrow \mathcal{C}$, i.e. functors $P : \mathcal{C}^{\mathrm{op}} \times \mathcal{C} \to \mathsf{Set}$
- morphisms: strong natural transformations $\eta : P \to Q$

(SPOILER: The morphisms are the supermaps!)

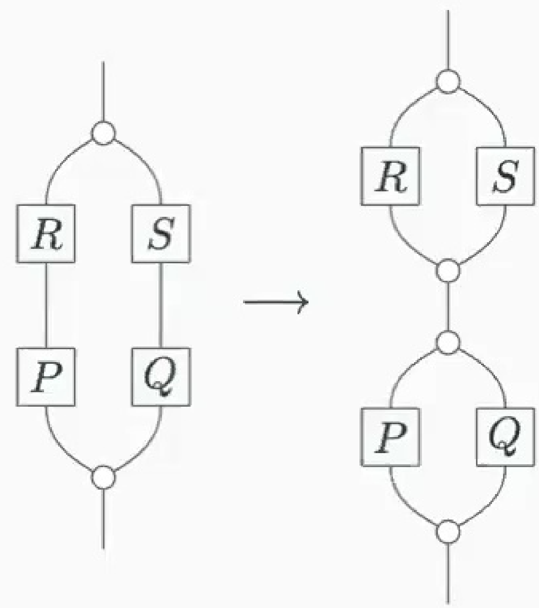
Interventions

Two tensors:



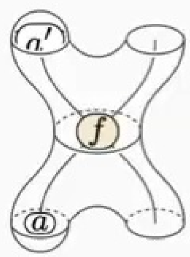
$P \varolessthan Q$ $\quad$ $P \otimes Q$

Distributive law:

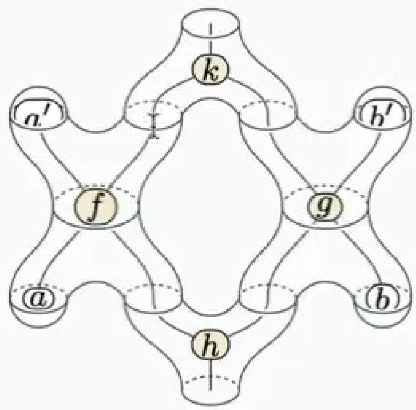


2008: Pastro, Street; 2016: Garner, López Franco; 2023: Earnshaw, H, Román

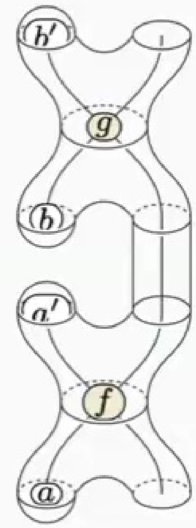
Interventions



$\mathcal{C}_{\boldsymbol{a}} := \mathcal{C}(a \otimes -, a' \otimes =)$

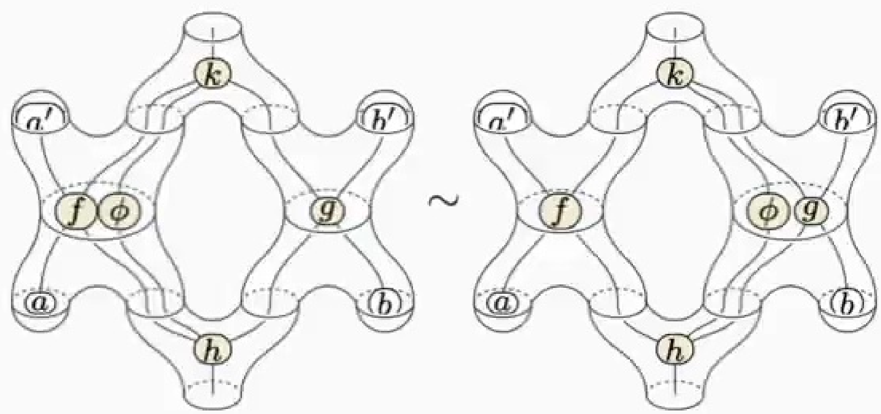


$\mathcal{C}_{\boldsymbol{a}} \otimes \mathcal{C}_{\boldsymbol{b}}$

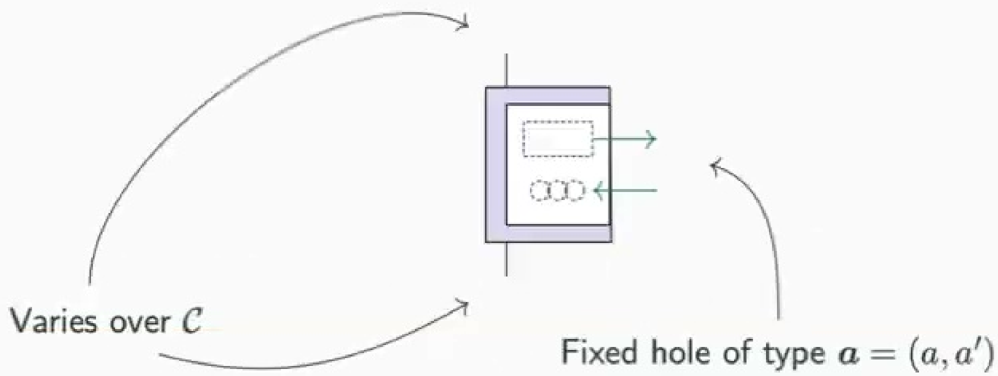


$\mathcal{C}_{\boldsymbol{a}} \oslash \mathcal{C}_{\boldsymbol{b}}$

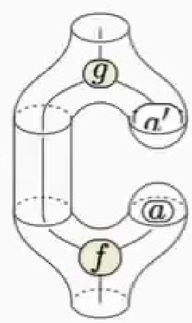
Interventions



Contexts



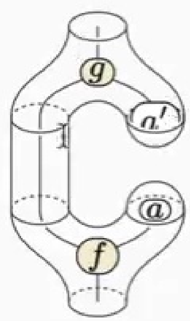
For instance, the collection of all combs at $\boldsymbol{a}$:

$y_{\boldsymbol{a}} =$ 

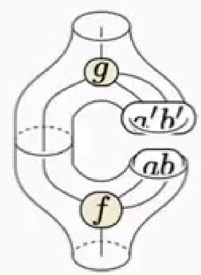
Contexts

- y_a is also a strong profunctor!
- Interventions and contexts are united in $\mathsf{StProf}(\mathcal{C})$

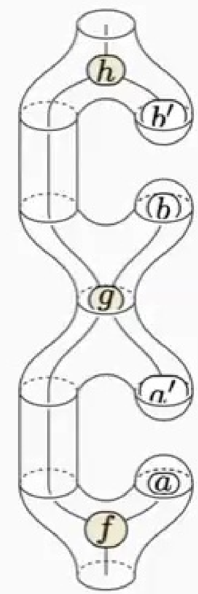
Contexts



$y_{\boldsymbol{a}}$

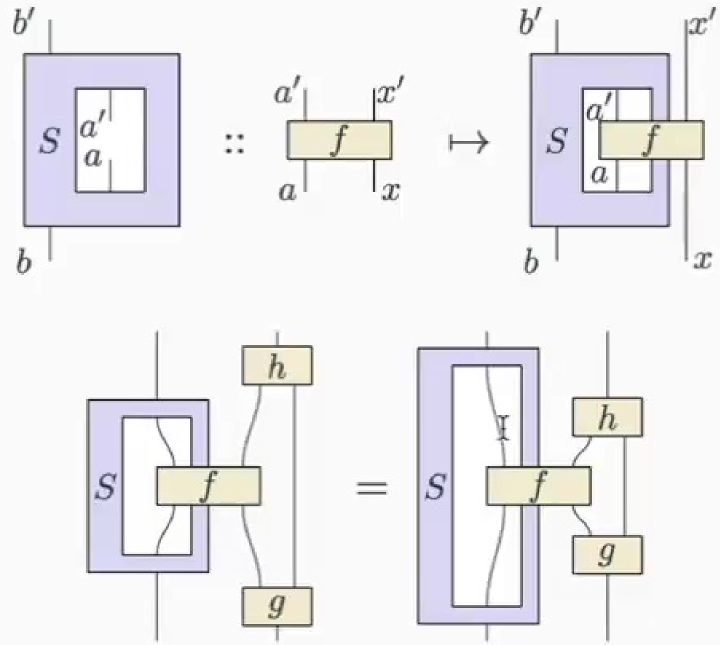


$y_{\boldsymbol{a}} \otimes y_{\boldsymbol{b}} \cong y_{\boldsymbol{a} \otimes \boldsymbol{b}}$



$y_{\boldsymbol{a}} \oslash y_{\boldsymbol{b}}$

Supermaps



2022: Wilson, Chiribella, Kissinger; 2022: Wilson, Chiribella

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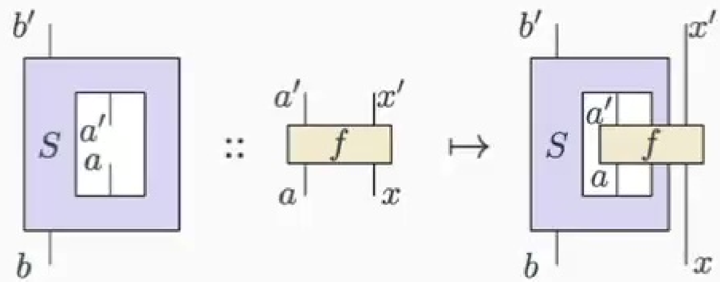
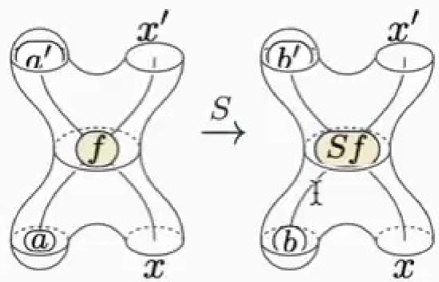
Supermaps





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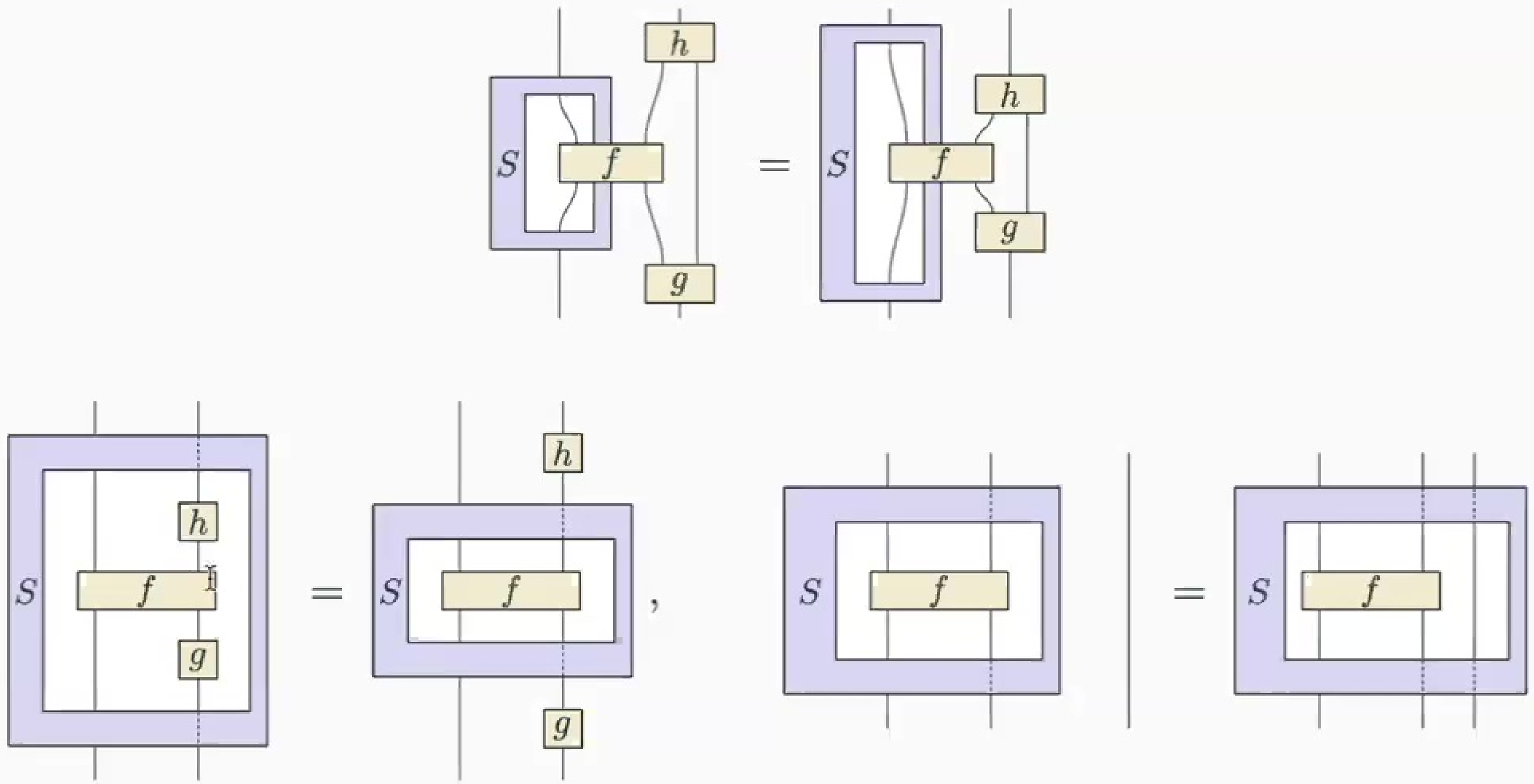
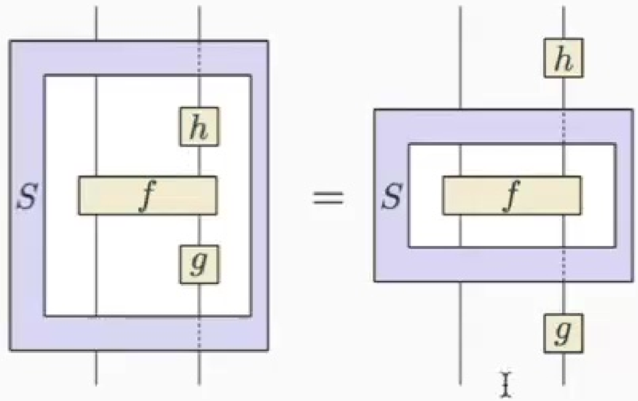
Supermaps



2022: Wilson, Chiribella, Kissinger; 2022: Wilson, Chiribella

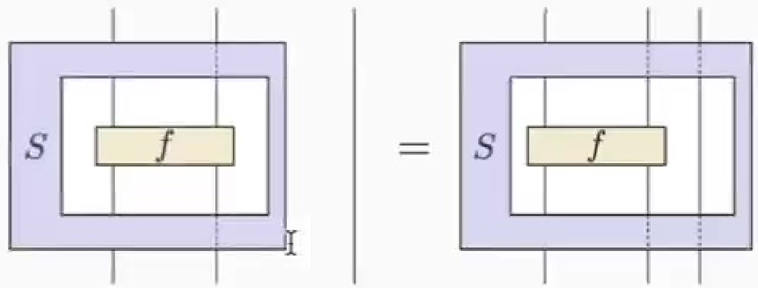
Law 1: Naturality



$$\begin{array}{ccc} \mathcal{C}(a \otimes x, a' \otimes x') & \xrightarrow{\mathcal{C}(1 \otimes g, 1 \otimes h)} & \mathcal{C}(a \otimes y, a' \otimes y') \\ {\scriptstyle S_{xx'}} \downarrow & & \downarrow {\scriptstyle S_{yy'}} \\ \mathcal{C}(b \otimes x, b' \otimes x') & \xrightarrow[\mathcal{C}(1 \otimes g, 1 \otimes h)]{} & \mathcal{C}(b \otimes y, b' \otimes y') \end{array}$$

S is a natural transformation

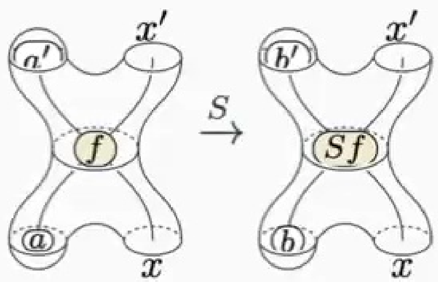
Law 2: Strength

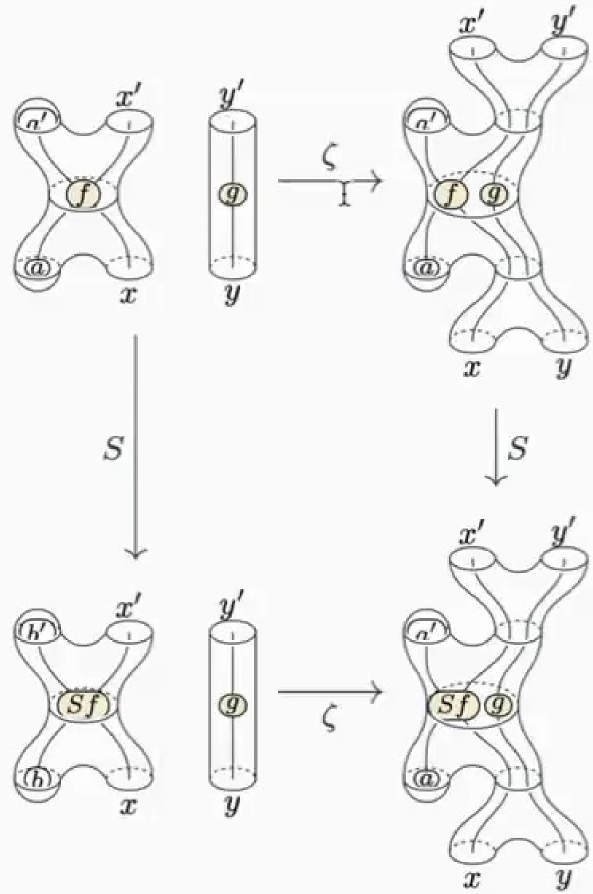


$$\begin{array}{ccc}
\mathcal{C}(a \otimes x, a' \otimes x') \times \mathcal{C}(y, y') & \xrightarrow{\zeta_{xx'yy'}} & \mathcal{C}(a \otimes x \otimes y, a' \otimes x' \otimes y') \\
{\scriptstyle S_{xx'} \times 1}\downarrow & & \downarrow{\scriptstyle S_{x \otimes y, x' \otimes y'}} \\
\mathcal{C}(b \otimes x, b' \otimes x') \times \mathcal{C}(y, y') & \xrightarrow[\zeta_{xx'yy'}]{} & \mathcal{C}(b \otimes x \otimes y, b' \otimes x' \otimes y')
\end{array}$$

S commutes with strengths

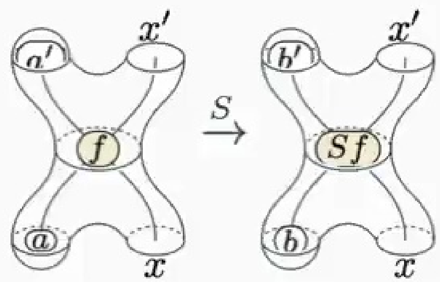
Law 2: Strength





Supermaps

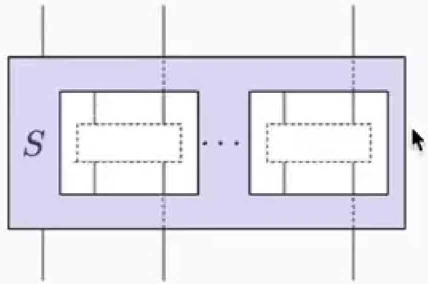
- The single-party supermaps are strong natural transformations

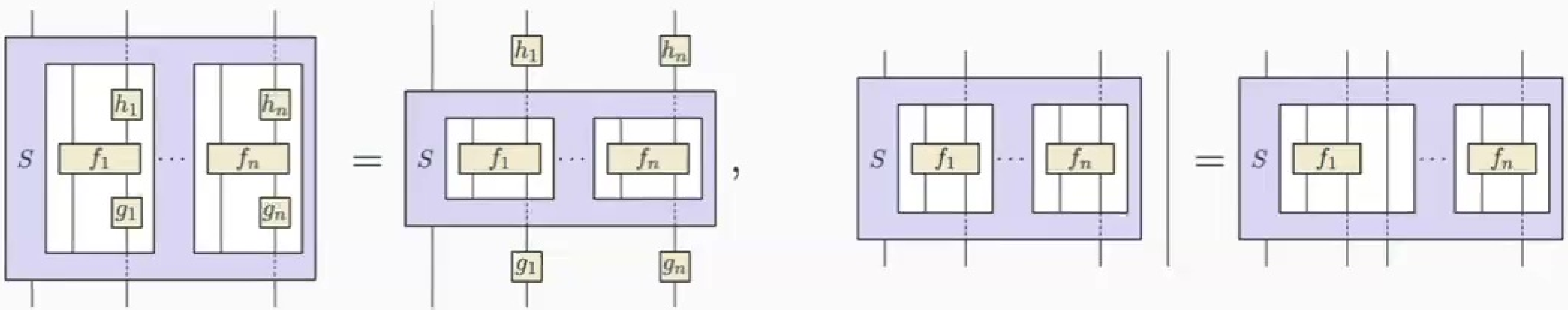


- Recovers the *locally-applicable transformations* of Wilson et al.
- When $\mathcal{C}$ is CPTP these are exactly the quantum supermaps.

2022: Wilson, Chiribella, Kissinger 2024: H, Wilson

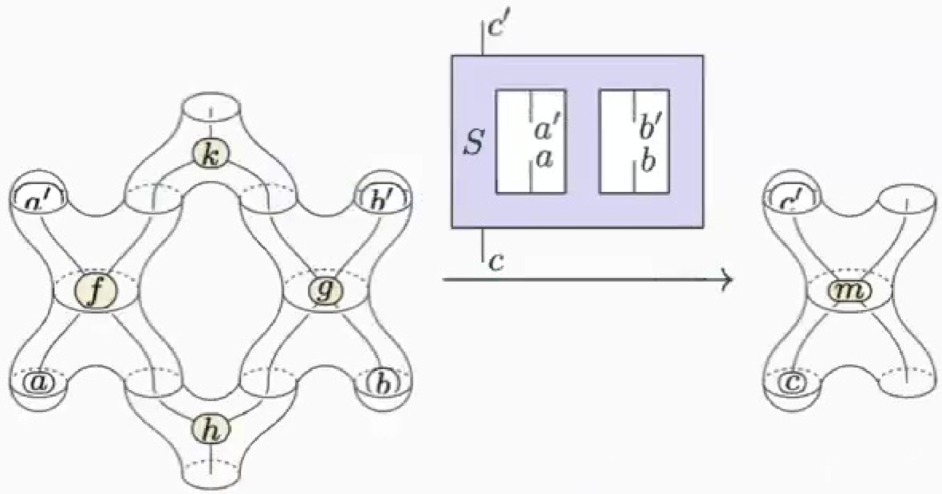
Multi-Partite Supermaps





2022: Wilson, Chiribella, Kissinger; 2022: Wilson, Chiribella

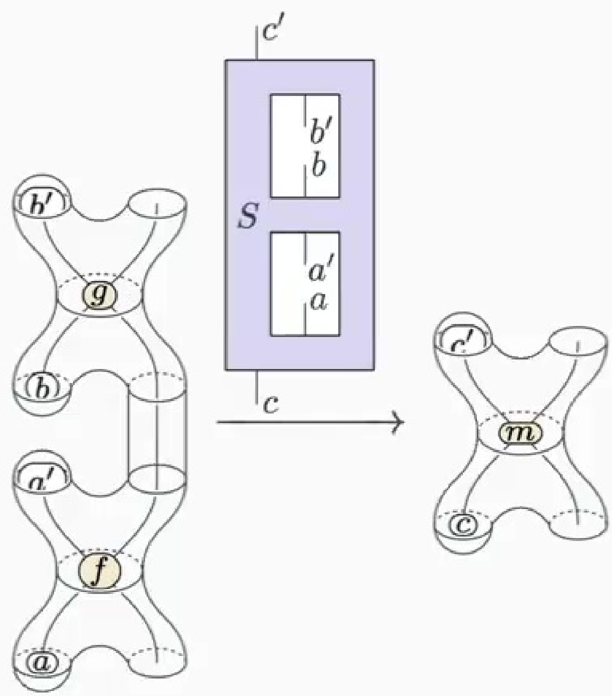
Multi-Partite Supermaps



- The maps $\mathcal{C}_a \otimes \mathcal{C}_b \to \mathcal{C}_c$ are the bipartite supermaps
- Recovers the multi-partite locally applicable transformations
- When $\mathcal{C}$ is CPTP these are precisely the multi-partite quantum supermaps

2022: Wilson, Chiribella, Kissinger 2024: H, Wilson

Multi-Partite Supermaps



- $\mathcal{C}_a \otimes \mathcal{C}_b \to \mathcal{C}_c$ are the bipartite supermaps with a causal ordering on the inputs
- When $\mathcal{C}$ is CPTP these are precisely the 2-holed combs

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Summary so Far

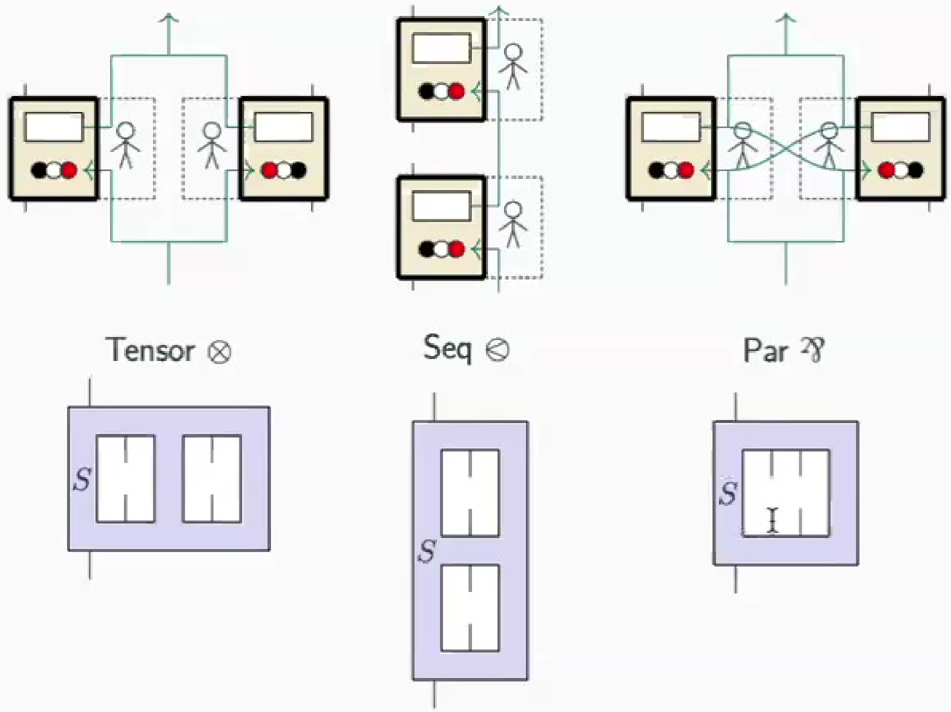
What we have:

- Interventions, contexts and supermaps united in $\mathsf{StProf}(\mathcal{C})$
- $\otimes$ modelling parallel, separable interventions
- $\olessthan$ modelling sequential interventions
- Distributive law between $\otimes$ and $\olessthan$
- $\otimes$ and $\olessthan$ generate the right spaces of supermaps

What we do not have:

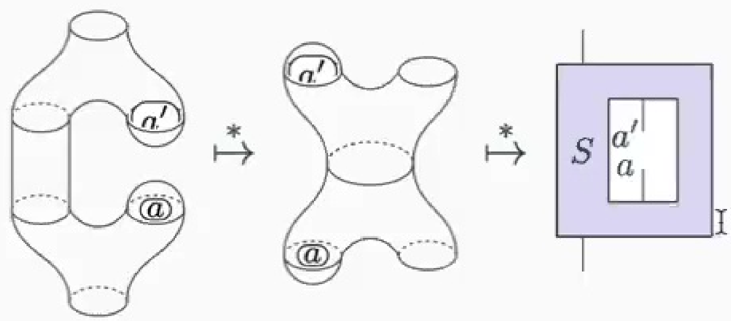
- Duality between interventions and contexts
- A tensor that returns the full space of bipartite processes

Summary so Far



Where is this guy? ↑

Intervention-Context Duality



- $y_{\boldsymbol{a}}^{**}$ is the space of *all* single-party supermaps
- $y_{\boldsymbol{a}}^{**} \cong y_{\boldsymbol{a}}$ iff all single-party supermaps decompose as combs!

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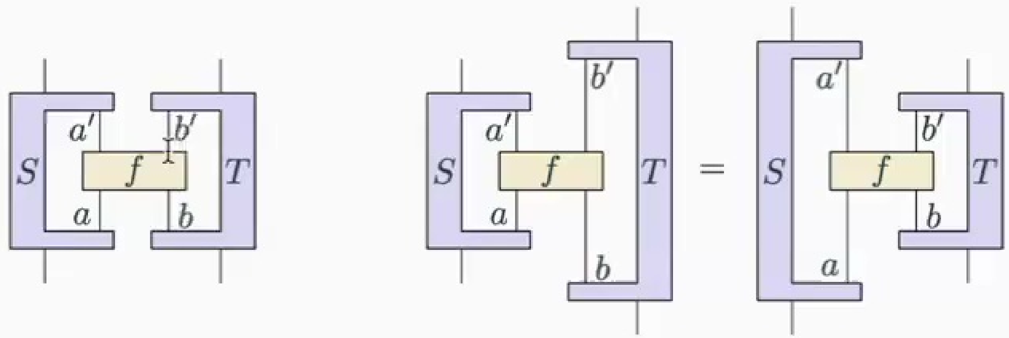
Intervention-Context Duality



- Want $\mathcal{C}_{\boldsymbol{a}} \parr \mathcal{C}_{\boldsymbol{b}} = \mathcal{C}_{\boldsymbol{a} \otimes \boldsymbol{b}}$
- Fails to be a tensor in general: supermaps fail to commute!

2022: Wilson, Chiribella

Adding Duals

$$\mathsf{CSMC}_{\perp} \underset{U}{\overset{\mathfrak{C}\mathrm{hu}}{\underset{\longleftarrow}{\overset{\longrightarrow}{\top}}}} *\mathsf{Aut}$$

- CSMC: Categories with $\otimes$ and weak $(-)^*$ [closed symmetric monoidal categories]
- $*$Aut: Categories with $\otimes, (-)^*, \parr$ [$*$-autonomous categories]
    - Models of multiplicative linear logic

1979: Chu 1997: Pavlović

Adding Duals

Theorem
There is an adjunction,

$$\mathsf{CSNDuo} \underset{U}{\overset{\mathsf{Chu}}{\underset{\longleftarrow}{\overset{\longrightarrow}{\top}}}} \mathsf{BV}$$

- CSNDuo: Categories with $\otimes$, $\lessdot$ and weak $(-)^*$ [closed symmetric normal duoidal categories]
- $*$Aut: Categories with $\otimes, \lessdot, (-)^*, \parr$ [BV-categories]
  - (Possible) models of BV-logic

Can apply to StProf($\mathcal{C}$) to get BV-category StEnv($\mathcal{C}$), the strong Hyland envelope of $\mathcal{C}$

Strong Hyland Envelope

$$\mathcal{C} \xmapsto{D} \mathcal{C} \times \mathcal{C}^{\text{op}} \xmapsto{y} \mathsf{Prof}(\mathcal{C}) = [\mathcal{C}^{\text{op}} \times \mathcal{C}, \mathsf{Set}] \xmapsto{N} \mathsf{StProf}(\mathcal{C}) \xmapsto{\mathsf{Chu}} \mathsf{StEnv}(\mathcal{C})$$

$$\mathsf{SMC} \xrightarrow{D} \mathsf{SProDuo} \xrightarrow{y} \mathsf{CSDuo} \xrightarrow{N} \mathsf{CSNDuo} \xrightarrow{\mathsf{Chu}} \mathsf{BV}$$

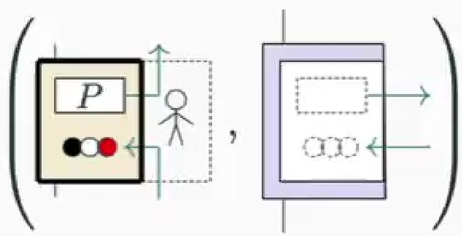
Process theory ($\otimes$) $\rightsquigarrow$ Higher-order process theory ($\otimes, \oplus$) $\rightsquigarrow$ Theory of spacetime events ($\otimes, \lessdot, ⅋, (-)^*$)

We can lift $\otimes$ and (co)freely add $\lessdot$, $(-)^*$ and $⅋$ to get a BV-category!

Spacetime Events via the Strong Hyland Envelope

An object of StEnv($\mathcal{C}$) is $(P, P', P \otimes P' \xrightarrow{\eta} 1)$

- P intervention
- P' context
- η evaluation of context on intervention



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Strong Hyland Envelope

$$\mathcal{C} \xmapsto{D} \mathcal{C} \times \mathcal{C}^{\mathrm{op}} \xmapsto{y} \mathsf{Prof}(\mathcal{C}) = [\mathcal{C}^{\mathrm{op}} \times \mathcal{C}, \mathsf{Set}] \xmapsto{N} \mathsf{StProf}(\mathcal{C}) \xmapsto{\mathsf{Chu}} \mathsf{StEnv}(\mathcal{C})$$

$$\mathsf{SMC} \xrightarrow{D} \mathsf{SProDuo} \xrightarrow{y} \mathsf{CSDuo} \xrightarrow{N} \mathsf{CSNDuo} \xrightarrow{\mathsf{Chu}} \mathsf{BV}$$

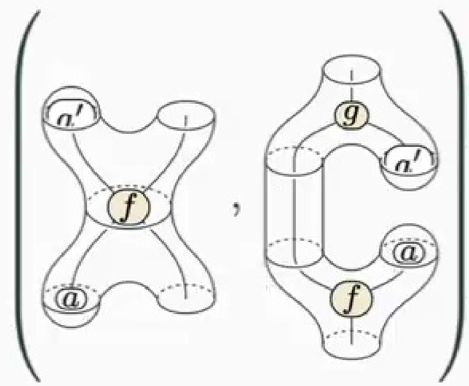
Process theory ($\otimes$) $\rightsquigarrow$ Higher-order process theory ($\otimes, \olessthan$) $\rightsquigarrow$ Theory of spacetime events ($\otimes, \olessthan, \parr, (-)^*$)

We can lift $\otimes$ and (co)freely add $\olessthan$, $(-)^*$ and $\parr$ to get a BV-category!

Spacetime Events via the Strong Hyland Envelope

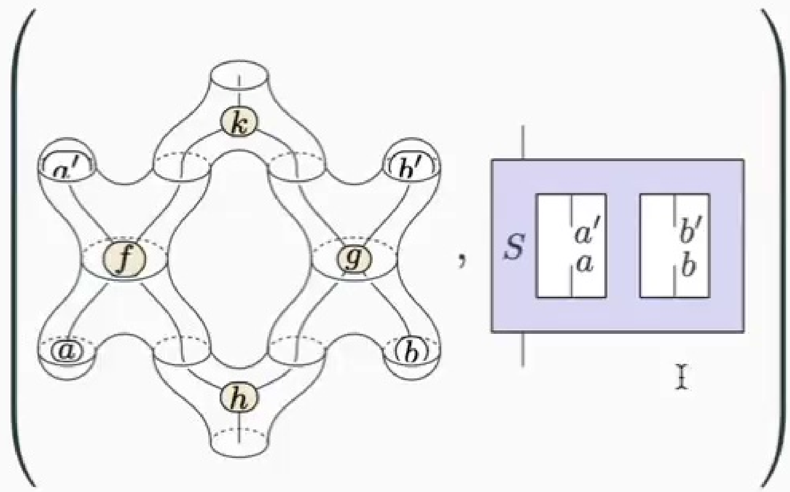
Definition (Event at a)

$$(\mathcal{C}_a, y_a) =$$



Spacetime Events via the Strong Hyland Envelope

$$(\mathcal{C}_{\boldsymbol{a}}, y_{\boldsymbol{a}}) \otimes (\mathcal{C}_{\boldsymbol{b}}, y_{\boldsymbol{b}}) =$$



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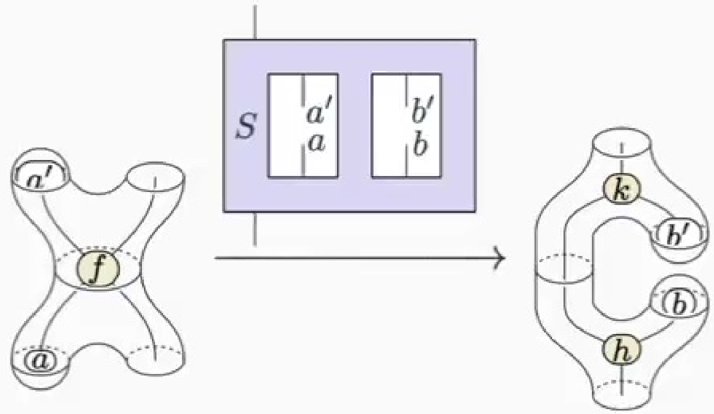
Spacetime Events via the Strong Hyland Envelope

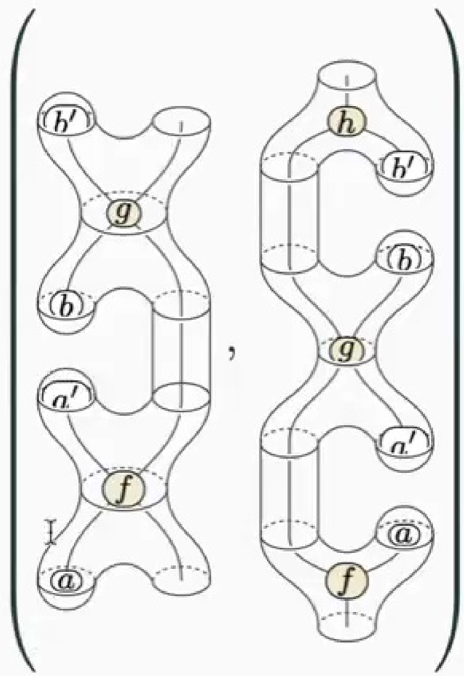


- The higher-order transformations that decompose locally as a comb
- For finite dimensional quantum theory these are exactly the supermaps!

2022: Wilson, Chiribella, Kissinger 2024: H, Wilson

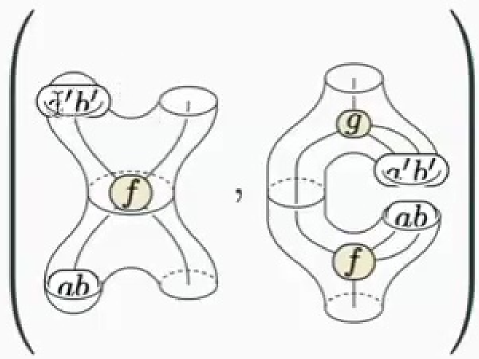
Spacetime Events via the Strong Hyland Envelope

$$(\mathcal{C}_{\boldsymbol{a}}, y_{\boldsymbol{a}}) \olessthan (\mathcal{C}_{\boldsymbol{b}}, y_{\boldsymbol{b}}) =$$

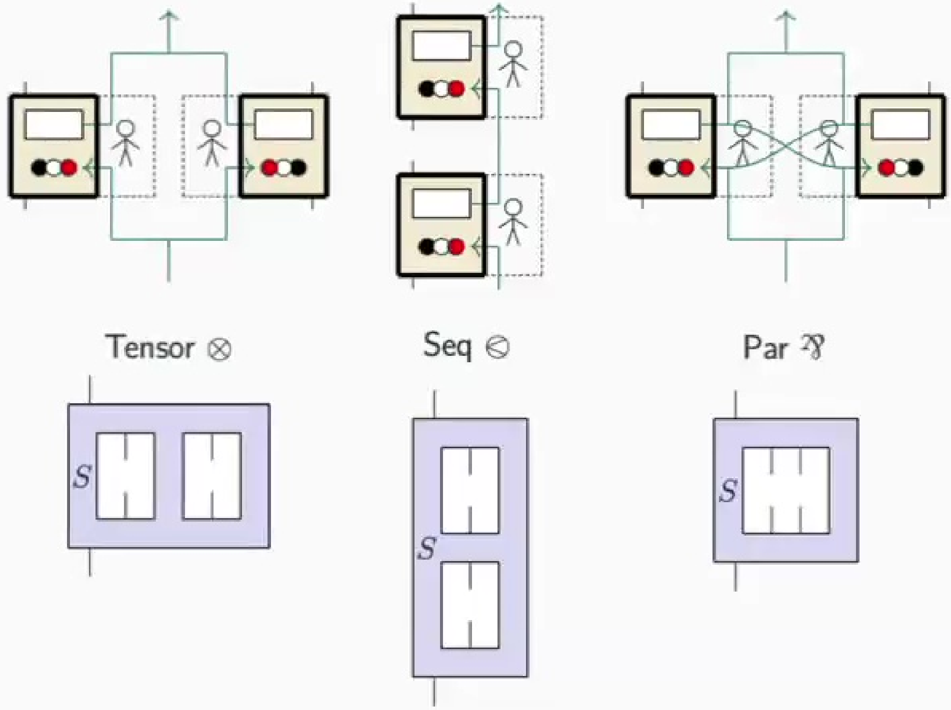


Spacetime Events via the Strong Hyland Envelope

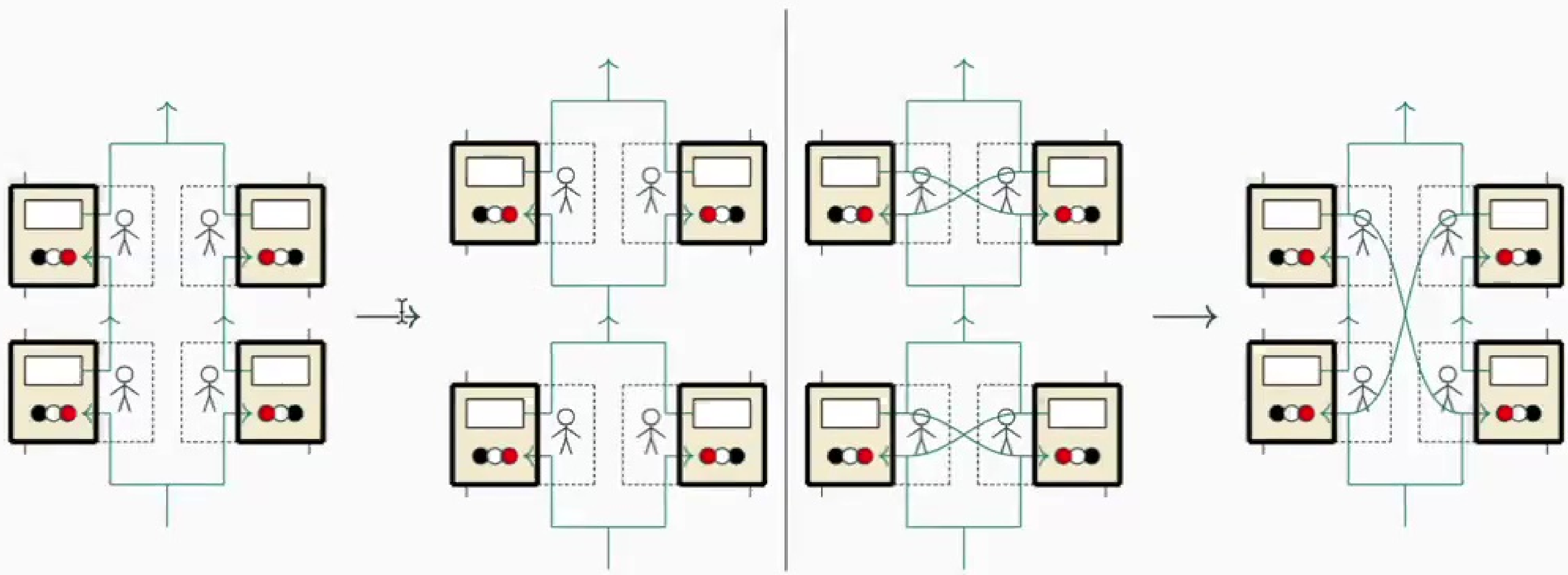
$$(\mathcal{C}_{\boldsymbol{a}}, y_{\boldsymbol{a}}) \parr (\mathcal{C}_{\boldsymbol{b}}, y_{\boldsymbol{b}}) =$$



Spacetime Events via the Strong Hyland Envelope



Spacetime Events via the Strong Hyland Envelope



Spacetime Events via the Strong Hyland Envelope

- $(\mathcal{C}_{\boldsymbol{a}}, y_{\boldsymbol{a}}) \to (\mathcal{C}_{\boldsymbol{b}}, y_{\boldsymbol{b}})$ are combs $\boldsymbol{a} \to \boldsymbol{b}$
- $(\mathcal{C}_{\boldsymbol{a}}, y_{\boldsymbol{a}}) \otimes (\mathcal{C}_{\boldsymbol{b}}, y_{\boldsymbol{b}}) \to (\mathcal{C}_{\boldsymbol{c}}, y_{\boldsymbol{c}})$ are the bipartite supermaps that decompose locally as a comb
- $(\mathcal{C}_{\boldsymbol{a}}, y_{\boldsymbol{a}}) \lessdot (\mathcal{C}_{\boldsymbol{b}}, y_{\boldsymbol{b}}) \to (\mathcal{C}_{\boldsymbol{c}}, y_{\boldsymbol{c}})$ are the causally-ordered supermaps, the two holed combs
- $(\mathcal{C}_{\boldsymbol{a}}, y_{\boldsymbol{a}}) \parr (\mathcal{C}_{\boldsymbol{b}}, y_{\boldsymbol{b}}) \cong (\mathcal{C}_{\boldsymbol{a} \otimes \boldsymbol{b}}, y_{\boldsymbol{a} \otimes \boldsymbol{b}}) \to (\mathcal{C}_{\boldsymbol{c}}, y_{\boldsymbol{c}})$ are combs $\boldsymbol{a} \otimes \boldsymbol{b} \to \boldsymbol{c}$

So, where are we?

- Method for building a theory of spacetime events $\mathsf{StProf}(\mathcal{C})$ over any process theory $\mathcal{C}$
- Objects are intervention-context pairs
- Morphisms are higher-order transformations, the supermaps
- Comes with the connectives of BV-logic:
  - $\otimes$: separable processes
  - $\lhd$: sequenced processes
  - $\parr$: maximal correlations, bipartite processes
  - $(-)^*$: duality between interventions and contexts

Where are we going?

- Can study the supermaps over any process theory. What happens in interesting cases, e.g. infinite dimensional quantum theory?
- Can we develop a general theory of process matrices? Agents communicate by classical data, with their local labs being quantum.
- Can we study inhomogeneous physics? Different labs can experience different local physics.
- Are there additional logical connectives? What about one for indefinite causal order?