Title: Lecture - Quantum Information, PHYS 635

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Collection/Series: Quantum Information (Elective), PHYS 635, February 24 - March 28, 2025

Subject: Quantum Information

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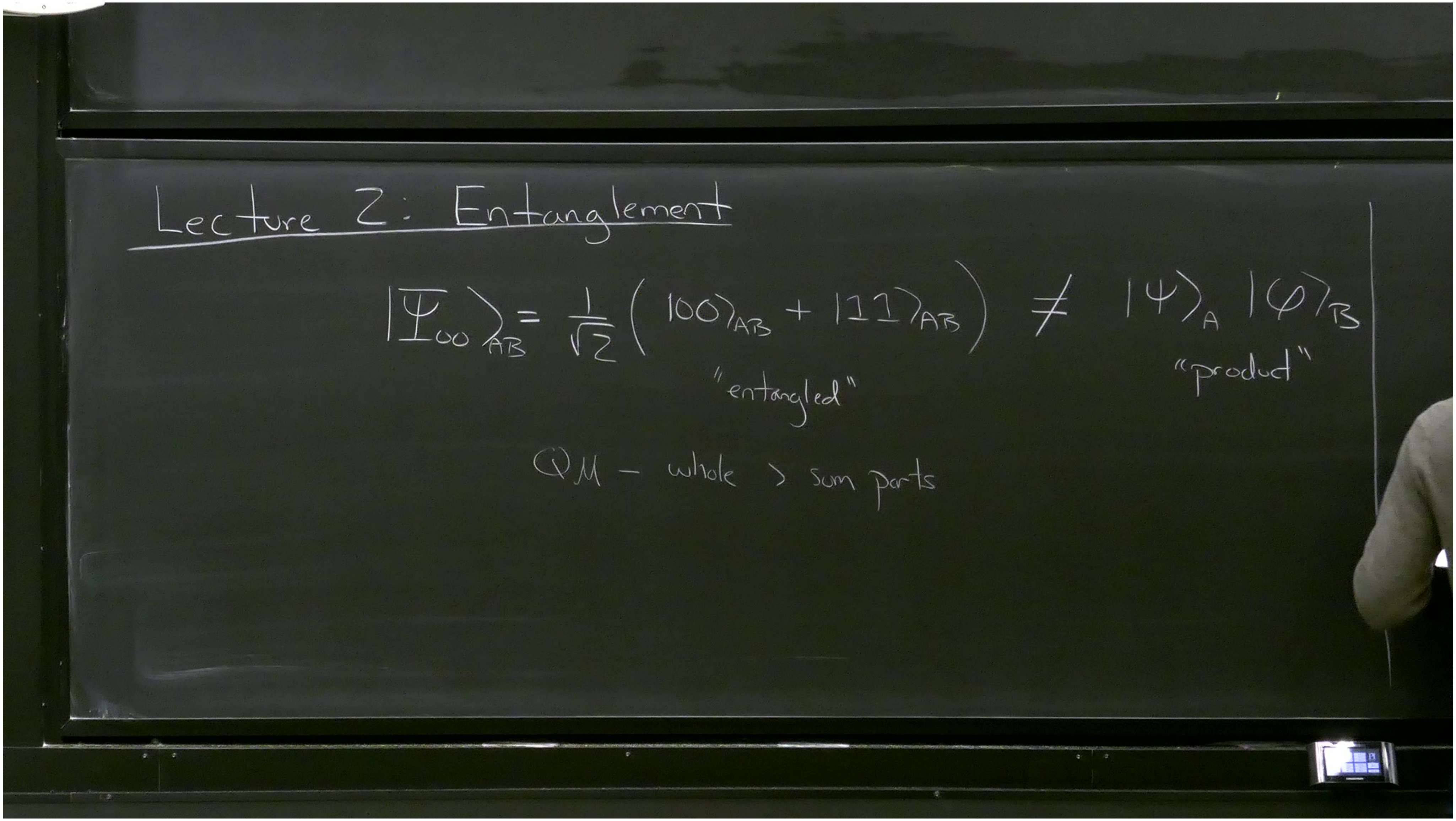

Lecture 2: Entanglement
$|\Psi_{00}\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right) \neq |\psi\rangle_A |\varphi\rangle_B$
"entangled"
"product"
QM – whole > sum parts

$|\varphi\rangle_B$

oduct"

$\Pi_{00} = |00\rangle\langle 00|$ $\quad$ $\Pi_{01} = |01\rangle\langle 01|$

$\Pi_{10} = |10\rangle\langle 10|$ $\quad$ $\Pi_{11} = |11\rangle\langle 11|$

$p_{00} = 1/2$ $\quad$ $p_{01} = 0$

$p_{10} = 0$ $\quad$ $p_{11} = 1/2$

$\Pi_{++} = |++\rangle\langle ++|$ $\quad$ $\Pi_{+-} = |+-\rangle\langle +-|$ $\quad$ $p_{++} =$ $\quad$ $p_{-+} =$

Π_{-+} $\quad$ Π_{--} $\quad$ $p_{+-} =$ $\quad$ $p_{--} =$

$|\varphi\rangle_B$

oduct"

$$\Pi_{00} = |00\rangle\langle 00| \qquad \Pi_{01} = |01\rangle\langle 01|$$

$$\Pi_{10} = |10\rangle\langle 10| \qquad \Pi_{11} = |11\rangle\langle 11|$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \pm |1\rangle\right)$$

$$P_{00} = 1/2 \qquad P_{01} = 0$$

$$P_{10} = 0 \qquad P_{11} = 1/2$$

$$\Pi_{++} = |++\rangle\langle ++| \qquad \Pi_{+-} = |+-\rangle\langle +-| \qquad P_{++} = \qquad P_{-+} =$$

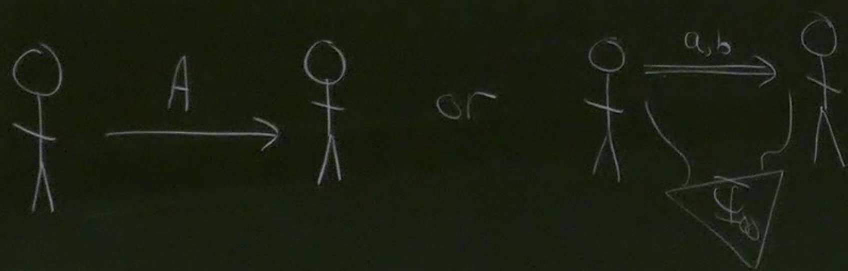
$$\Pi_{-+} \qquad \Pi_{--} \qquad P_{+-} = \qquad P_{--} =$$

philosophy → elbow grease

Teleportation

Alice, A

$$|\psi\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A$$



philosophy $\rightarrow$ 'elbow grease'

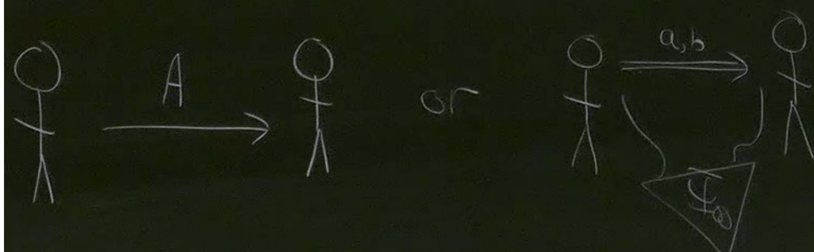
"qubit"

$$|\Psi_{ab}\rangle_{AB} = X_A^a Z_A^b |\Psi_{00}\rangle_{AB}$$

Teleportation

Alice, A

$$|\psi\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A$$



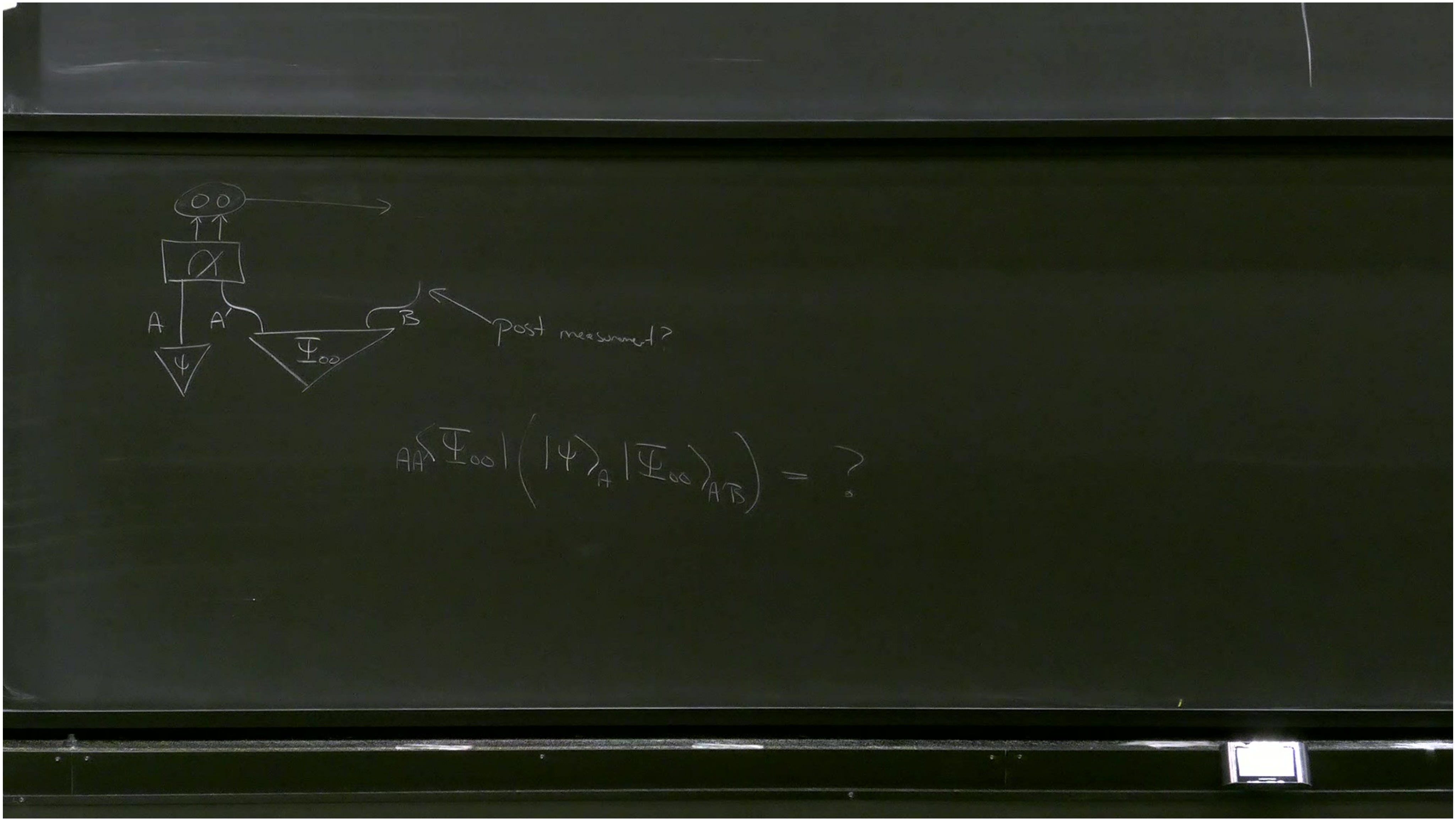
"Bell basis"

$$|\Psi_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\Psi_{01}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$|\Psi_{10}\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right)$$

$$|\Psi_{11}\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle - |01\rangle\right)$$


α
A
A'
B
ψ
Φ_{00}
post measurement?
$_{AA'}\langle\Phi_{00}|\left(|\psi\rangle_A|\Phi_{00}\rangle_{A'B}\right) = ?$

Π_{-+} Π_{--} $p_{+-} = 0$ $p_{--} = 1/2$

$${}_{AA'}\langle \Phi_{00} | \left(|\psi\rangle_A |\Phi_{00}\rangle_{A'B} \right) = \frac{1}{2} \left({}_{AA'}\langle 00| + {}_{AA'}\langle 11| \right) \left(\alpha |0\rangle_A + \beta |1\rangle_A \right) \left(|00\rangle_{A'B} + |11\rangle_{A'B} \right)$$

$$= \frac{1}{2} \left({}_{AA'}\langle 00| + {}_{AA'}\langle 11| \right) \left(\alpha |000\rangle_{AA'B} + \alpha |011\rangle + \beta |100\rangle + \beta |111\rangle \right)$$

$$= \frac{1}{2} \left(\alpha |0\rangle_B + \beta |1\rangle \right) = |\psi\rangle$$

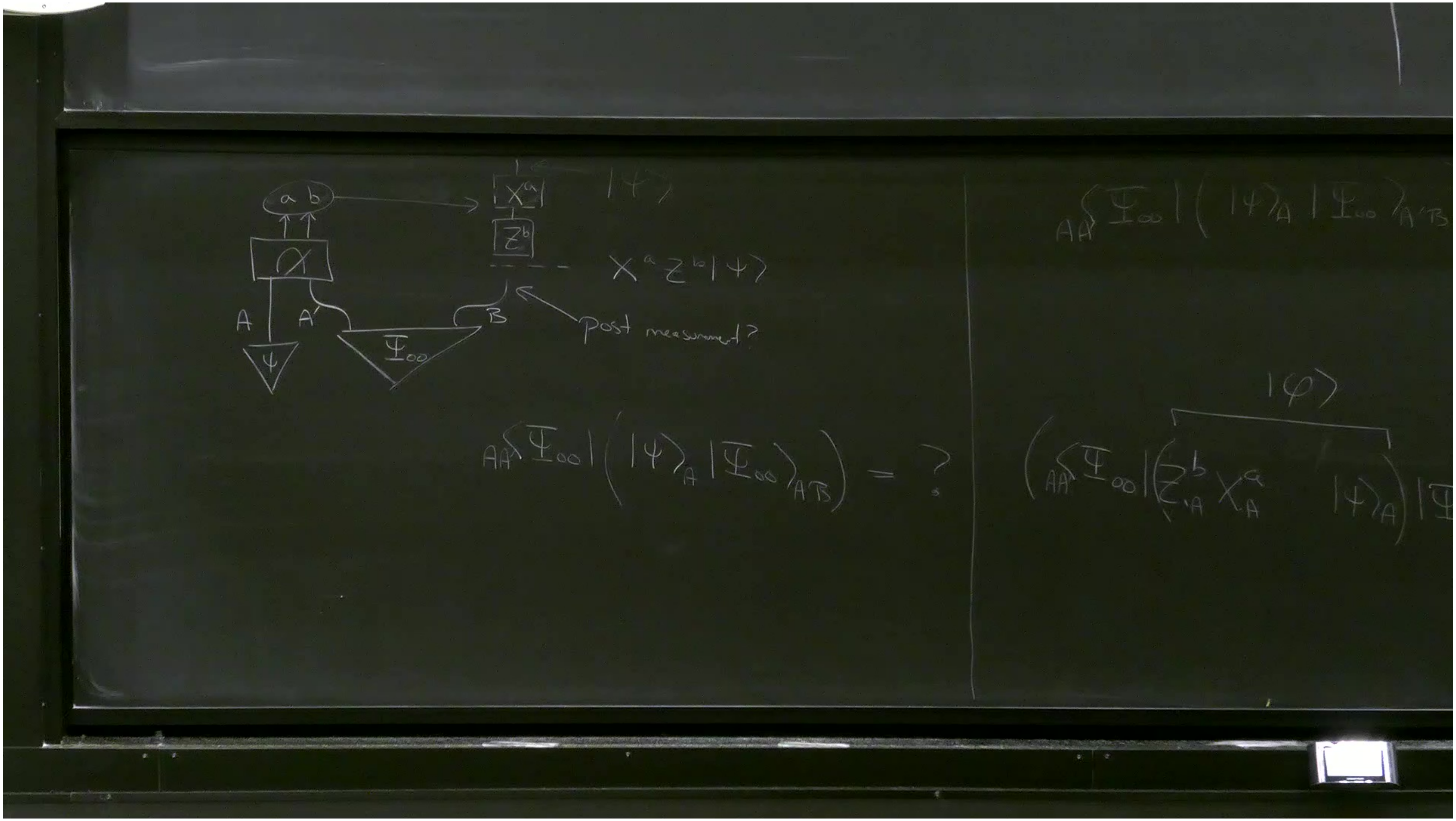
Π_{-+} Π_{--} $p_{+-}=0$ $p_{--}=1/2$

$${}_{AA'}\langle\Phi_{00}|\left(|\psi\rangle_A|\Phi_{00}\rangle_{A'B}\right) = \frac{1}{2}\left({}_{AA'}\langle 00| + {}_{AA'}\langle 11|\right)\left(\alpha|0\rangle_A+\beta|1\rangle_A\right)\left(|00\rangle_{A'B}+|11\rangle_{A'B}\right)$$

$$= \frac{1}{2}\left({}_{AA'}\langle 00| + {}_{AA'}\langle 11|\right)\left(\alpha|000\rangle_{AA'B}+\alpha|011\rangle+\beta|100\rangle+\beta|111\rangle\right)$$

$$= \frac{1}{2}\left(\alpha|0\rangle_B+\beta|1\rangle\right) = |\psi\rangle_B$$

$$\left({}_{AA'}\langle\Phi_{00}|\overbrace{\left(X_A^a Z_A^b\,|\psi\rangle_A\right)}^{|\varphi\rangle}|\Phi_{00}\rangle_{A'B}\right) = |\varphi\rangle_B = X^a Z^b|\psi\rangle$$


a b
X^a
$|\psi\rangle$
Z^b
$X^a Z^b |\psi\rangle$
α
A
A'
B
post measurement?
ψ
Φ_{00}
${}_{AA'}\langle\Phi_{00}|\left(|\psi\rangle_A |\Phi_{00}\rangle_{A'B}\right) = ?$
${}_{AA'}\langle\Phi_{00}|\left(|\psi\rangle_A |\Phi_{00}\rangle_{A'B}\right.$
$|\varphi\rangle$
$\left({}_{AA'}\langle\Phi_{00}|\left(Z^b_A X^a_A \; |\psi\rangle_A\right)|\Phi\right.$

Π_{-+} Π_{--} $P_{+-} = 0$ $P_{--} = 1/2$

$${}_{AA'}\langle \Phi_{00} | \left(|\psi\rangle_A |\Phi_{00}\rangle_{A'B} \right) = \frac{1}{2} \left({}_{AA'}\langle 00| + {}_{AA'}\langle 11| \right) \left(\alpha|0\rangle_A + \beta|1\rangle_A \right) \left(|00\rangle_{A'B} + |11\rangle_{A'B} \right)$$

$$= \frac{1}{2} \left({}_{AA'}\langle 00| + {}_{AA'}\langle 11| \right) \left(\alpha|000\rangle_{AA'B} + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle \right)$$

$$= \frac{1}{2} \left(\alpha|0\rangle_B + \beta|1\rangle \right) = |\psi\rangle_B$$

$$\left({}_{AA'}\langle \Phi_{00} | \underbrace{\left(Z^b_{\cdot A} X^a_{\cdot A} \; |\psi\rangle_A \right)}_{|\varphi\rangle} |\Phi_{00}\rangle_{A'B} \right) = |\varphi\rangle_B = Z^b X^a |\psi\rangle$$

Schmidt decomposition

SVD Every matrix C,

$$C = U \Sigma V$$

U, V unitary

Σ diagonal, non-negative real entries.

Σ diagonal, non-negative real entries.

$$|\Psi\rangle_{AB} = \sum_{ij} c_{ij}\, |i\rangle_A |j\rangle_B$$

(SVD)

$$= \sum_{ij} \left(\sum_k U_{ik}\, \sigma_{kk}\, V_{kj} \right) |i\rangle_A |j\rangle_B$$

$$= \sum_k \sigma_{kk} \underbrace{\left(\sum_i U_{ik} |i\rangle_A \right)}_{|\psi_k\rangle} \underbrace{\left(\sum_j V_{kj} |j\rangle_B \right)}_{|\varphi_k\rangle}$$

$$= \sum_k \sqrt{\lambda_k}\, |\psi_k\rangle_A |\varphi_k\rangle_B$$

schmidt coefficients

Schmidt vectors.

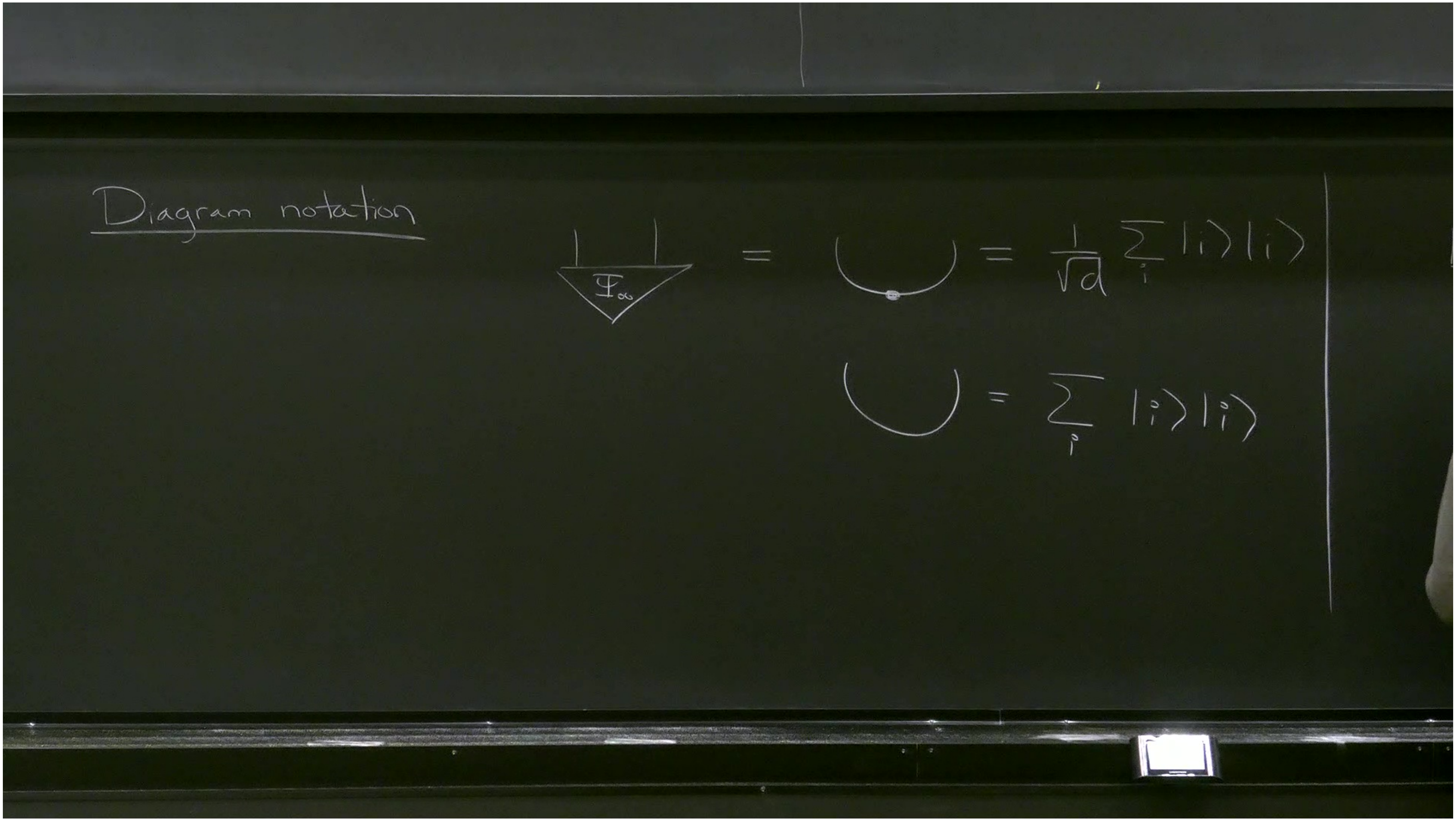
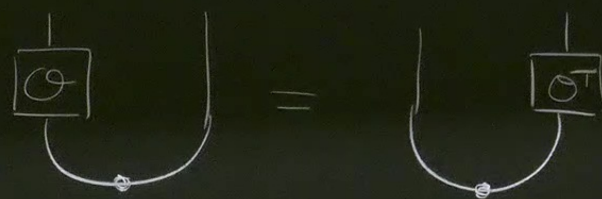


Diagram notation
Ψ_∞
$= \frac{1}{\sqrt{d}} \sum_i |i\rangle|i\rangle$
$= \sum_i |i\rangle|i\rangle$

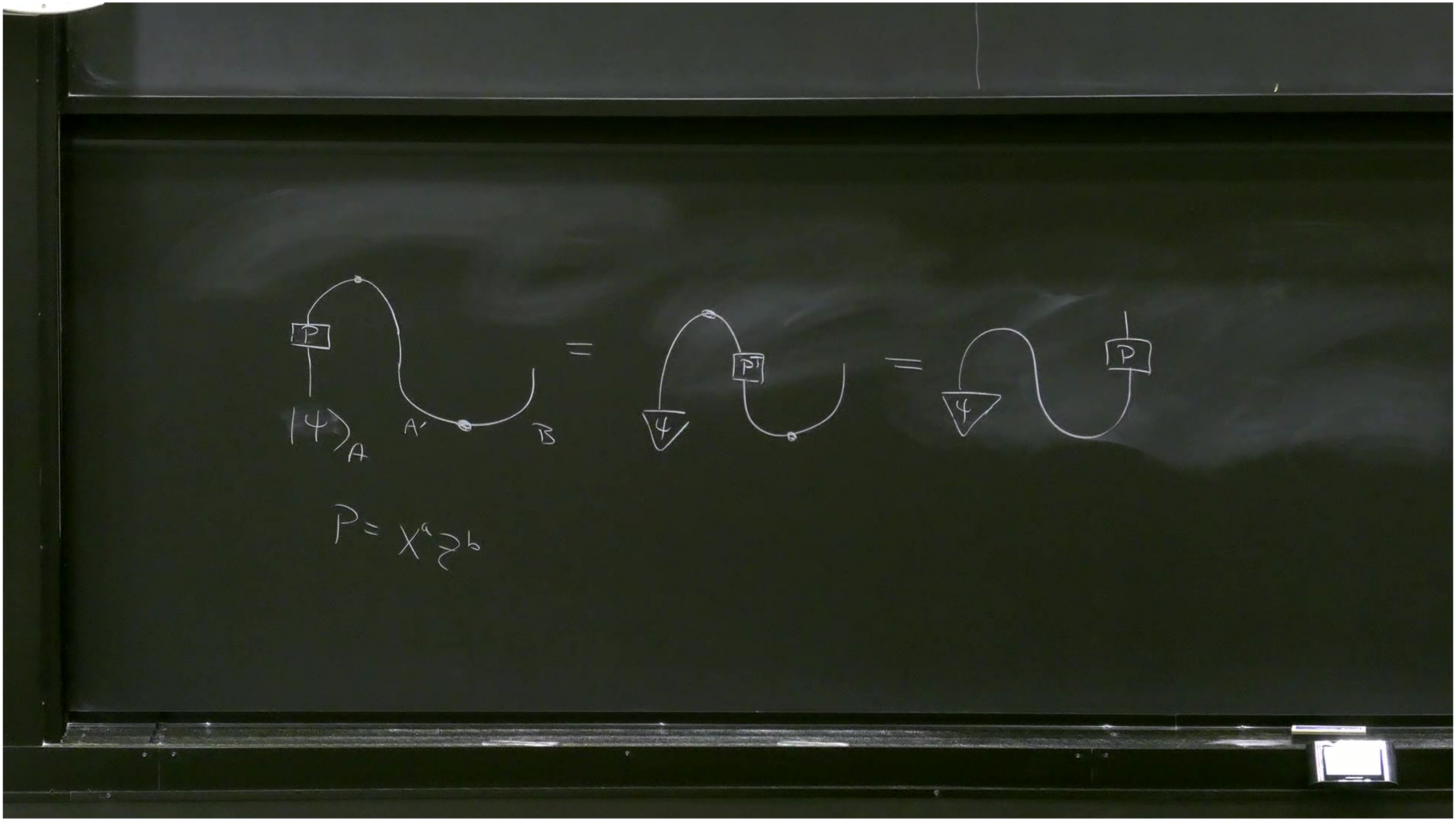
$$\frac{1}{\sqrt{d}}\sum_i |i\rangle|i\rangle$$

$$\sum_i |i\rangle|i\rangle$$



$$\langle j|\langle k|\left(O_A \sum_i |i\rangle|i\rangle\right) = \langle j|O_A|k\rangle$$

$$\langle j|\langle k|\left(O_B^T \sum_i |i\rangle|i\rangle\right) = \langle k|O_B^T|j\rangle = \langle j|O_B|k\rangle$$

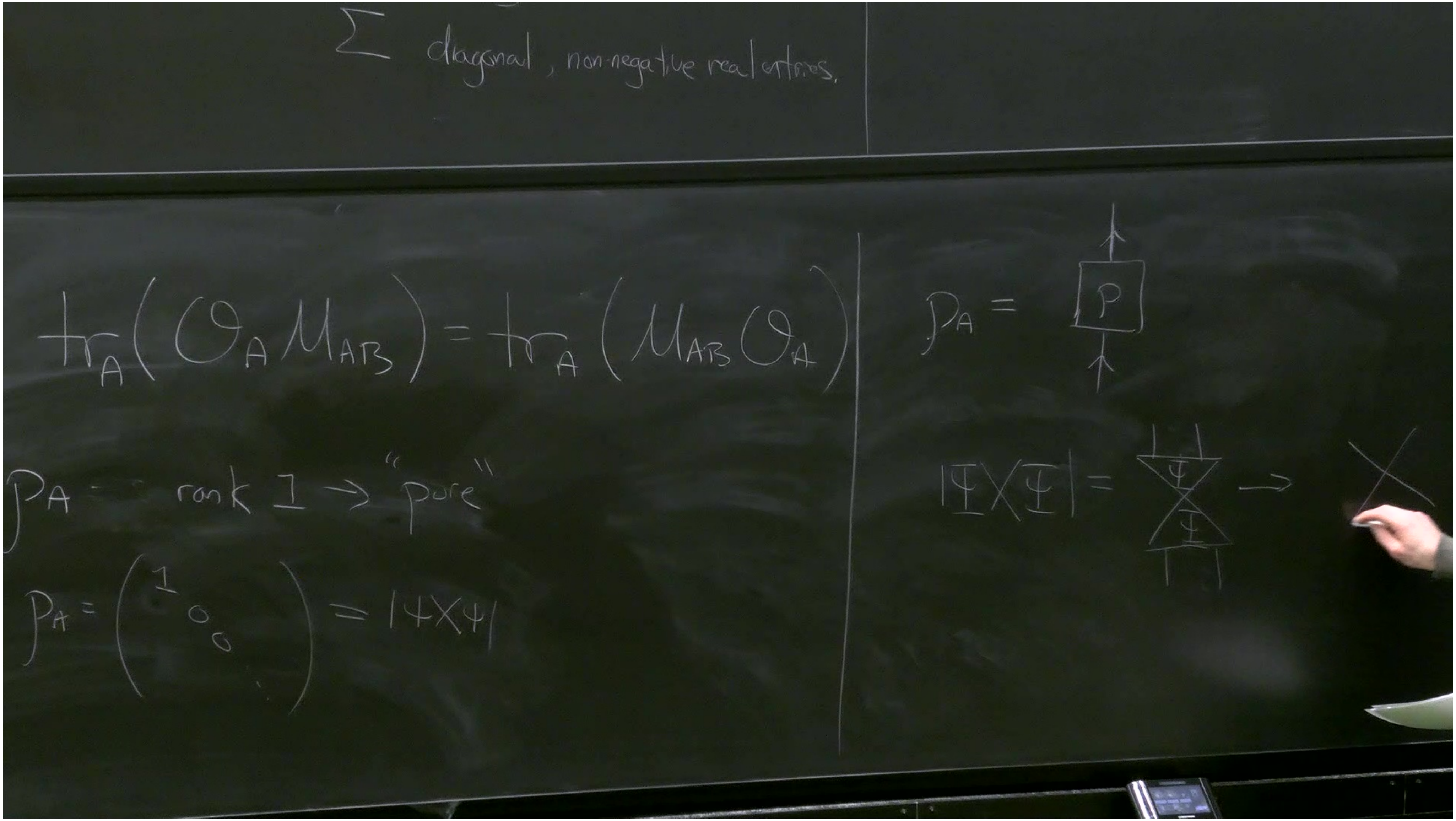

P
|ψ⟩_A
A'
B
=
P^T
ψ
=
ψ
P
P = X^a Z^b

Σ diagonal, non-negative real entries.

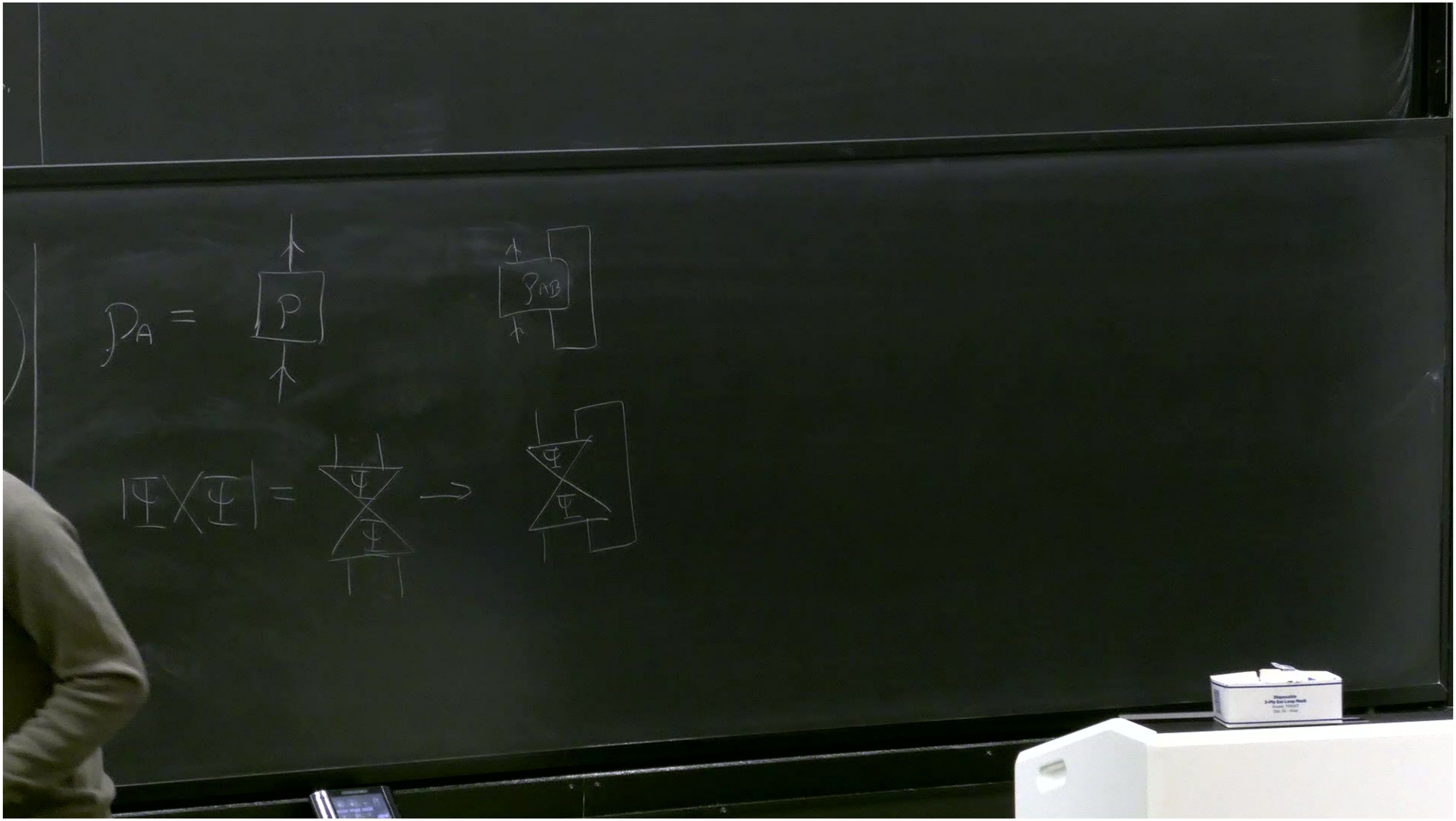
$$\mathrm{tr}_A(\mathcal{O}_A M_{AB}) = \mathrm{tr}_A(M_{AB}\,\mathcal{O}_A)$$

ρ_A – rank 1 $\rightarrow$ "pure"

$$\rho_A = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & \ddots \end{pmatrix}$$



Σ diagonal, non-negative real entries.
$\mathrm{tr}_A(\mathcal{O}_A M_{AB}) = \mathrm{tr}_A(M_{AB}\mathcal{O}_A)$
ρ_A — rank 1 → "pure"
$\rho_A = \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \\ & & & \ddots \end{pmatrix} = |\psi\rangle\langle\psi|$
$\rho_A =$
$|\Psi\rangle\langle\Psi| =$


ρA =
ρ
ρAB
|Ψ⟩⟨Ψ| =
Ψ