Title: Lecture - Quantum Information, PHYS 635

Speakers: Alex May

Collection/Series: Quantum Information (Elective), PHYS 635, February 24 - March 28, 2025

Subject: Quantum Information

Date: February 24, 2025 - 11:30 AM

URL: https://pirsa.org/25020009

Abstract:

Quantum information theory (QIT)

Prof: Alex May
+ Bindiya Arora,
Leo Lessa

Notes: PSI portal
or alexmpmay.com/notes

IT)

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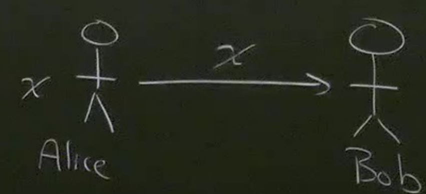
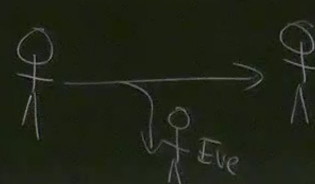
QIT — limits and possibilities of "information processing"

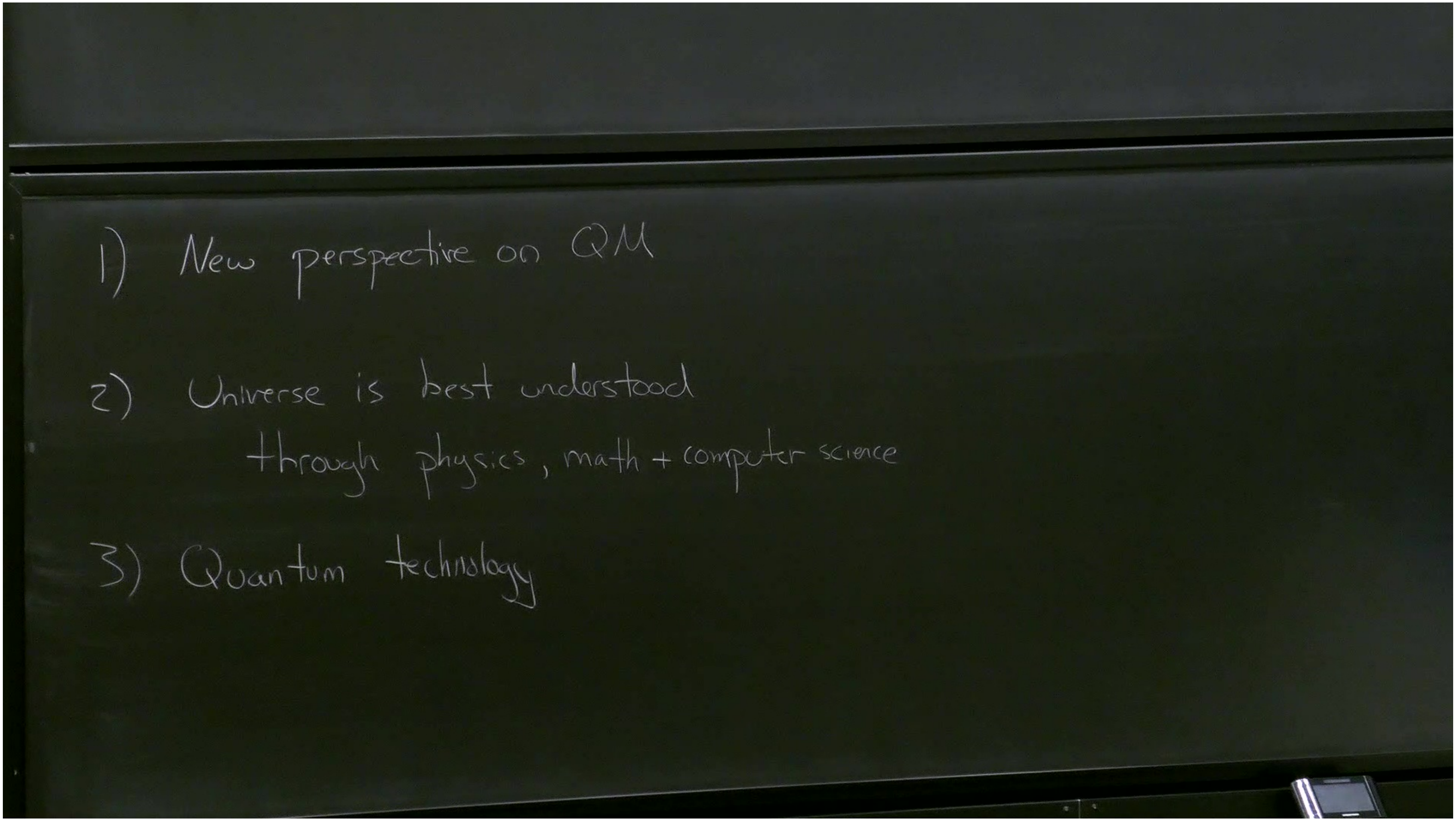
1) Communication



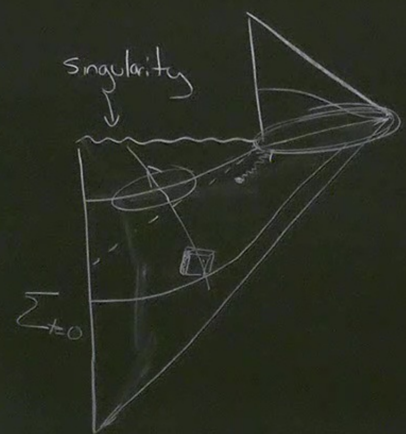
2) Computation:

3) Cryptography

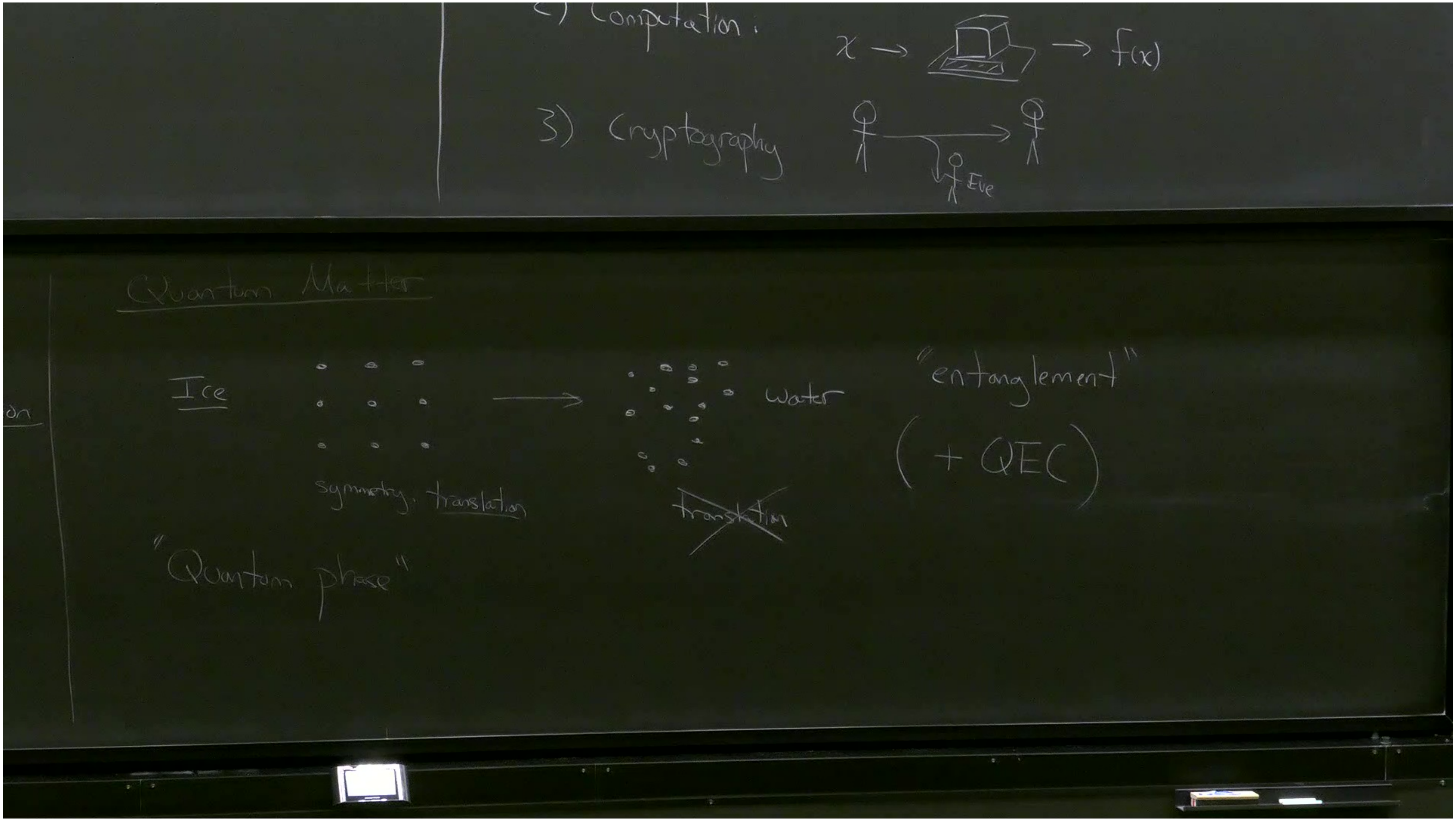


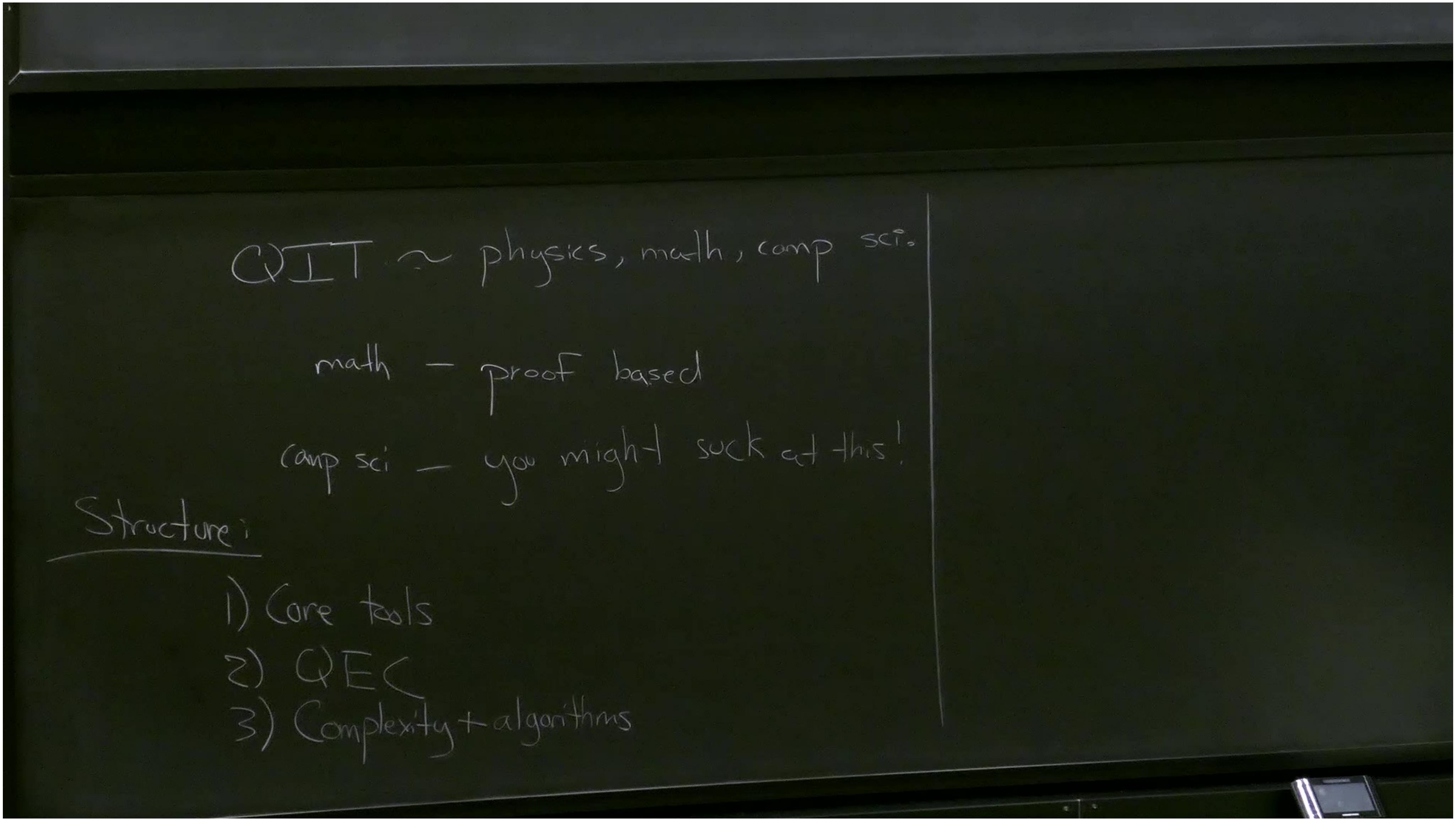

1) New perspective on QM
2) Universe is best understood
through physics, math + computer science
3) Quantum technology

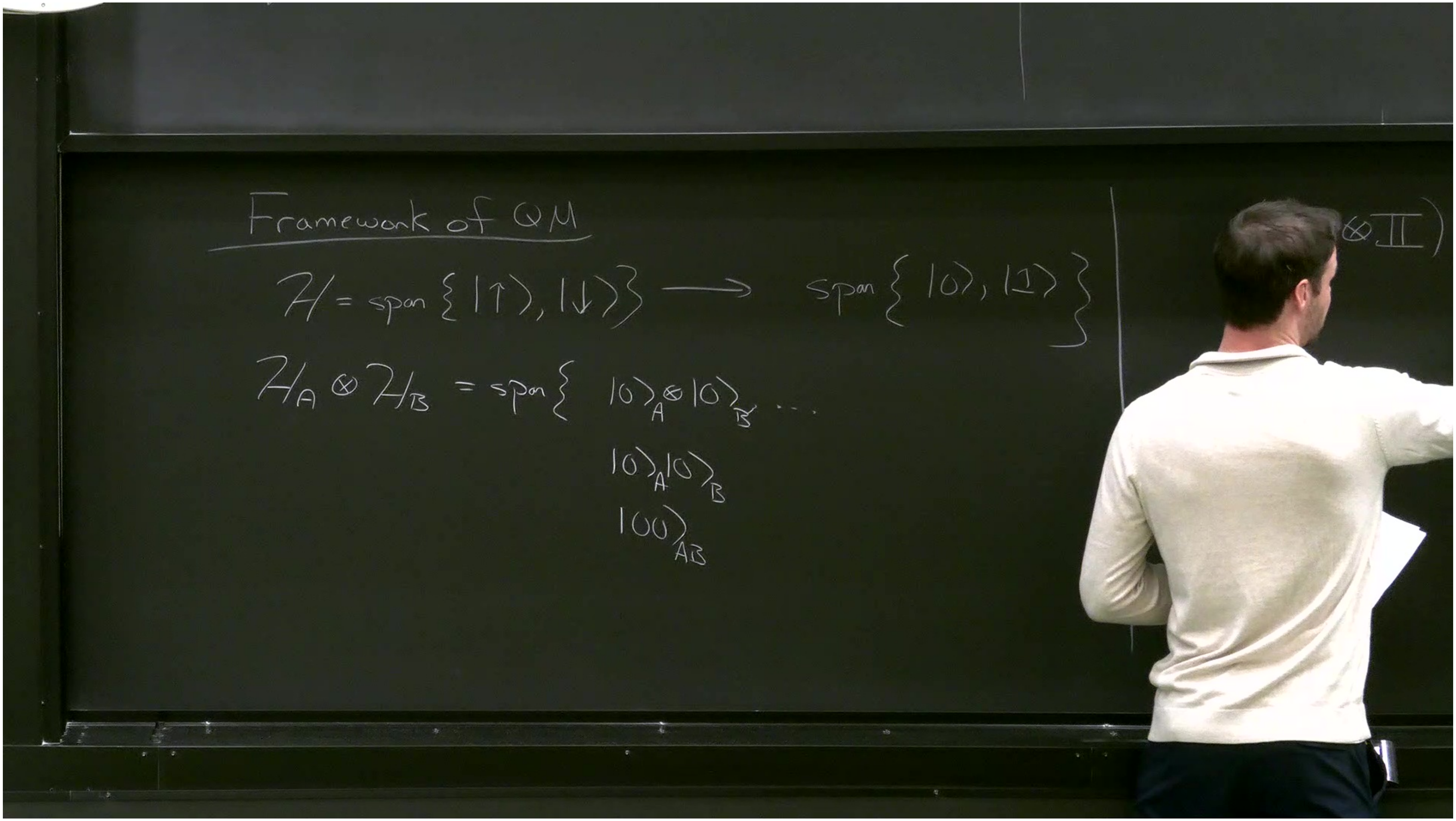
Black hole info problem



Quantum error-correction


2) Computation: $x \to$ [computer] $\to f(x)$
3) Cryptography
Eve
Quantum Matter
Ice
Water
symmetry: translation
~~translation~~
"Quantum phase"
"entanglement"
(+ QEC)


QIT ~ physics, math, comp sci.
math – proof based
comp sci – you might suck at this!
Structure:
1) Core tools
2) QEC
3) Complexity + algorithms


Framework of QM
$\mathcal{H} = \text{span}\{|\uparrow\rangle, |\downarrow\rangle\} \longrightarrow \text{span}\{|0\rangle, |1\rangle\}$
$\mathcal{H}_A \otimes \mathcal{H}_B = \text{span}\{ |0\rangle_A \otimes |0\rangle_B \ldots$
$|0\rangle_A |0\rangle_B$
$|00\rangle_{AB}$
$\otimes \mathbb{I})$

"Quantum phase"

$$(U_A \otimes \mathbb{I})\,|01\rangle_{AB} = U_A\,|01\rangle_{AB}$$

$$\|\,|\psi\rangle\,\|_{\mathcal{H}} = \|\,|\psi\rangle\,\| = \langle\psi|\psi\rangle$$

Time evolution:

$$|\psi\rangle \rightarrow U|\psi\rangle \qquad U = \text{unitary}$$

$$U^\dagger U = \mathbb{I}$$

Pauli operators:

$$\mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

2) QEC

3) Complexity + algorithms

$$COPY_{A\to B} |\psi\rangle_A |0\rangle_B = |\psi\rangle_A |\psi\rangle_B$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$COPY_{A\to B} |0\rangle|0\rangle =$$

$$|1\rangle|0\rangle =$$

$$COPY_{A\to B} |+\rangle_A|0\rangle$$

1) Core tools
2) QEC
3) Complexity + algorithms

$$\mathrm{COPY}_{A\to B}\,|\psi\rangle_A|0\rangle_B = |\psi\rangle_A|\psi\rangle_B$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

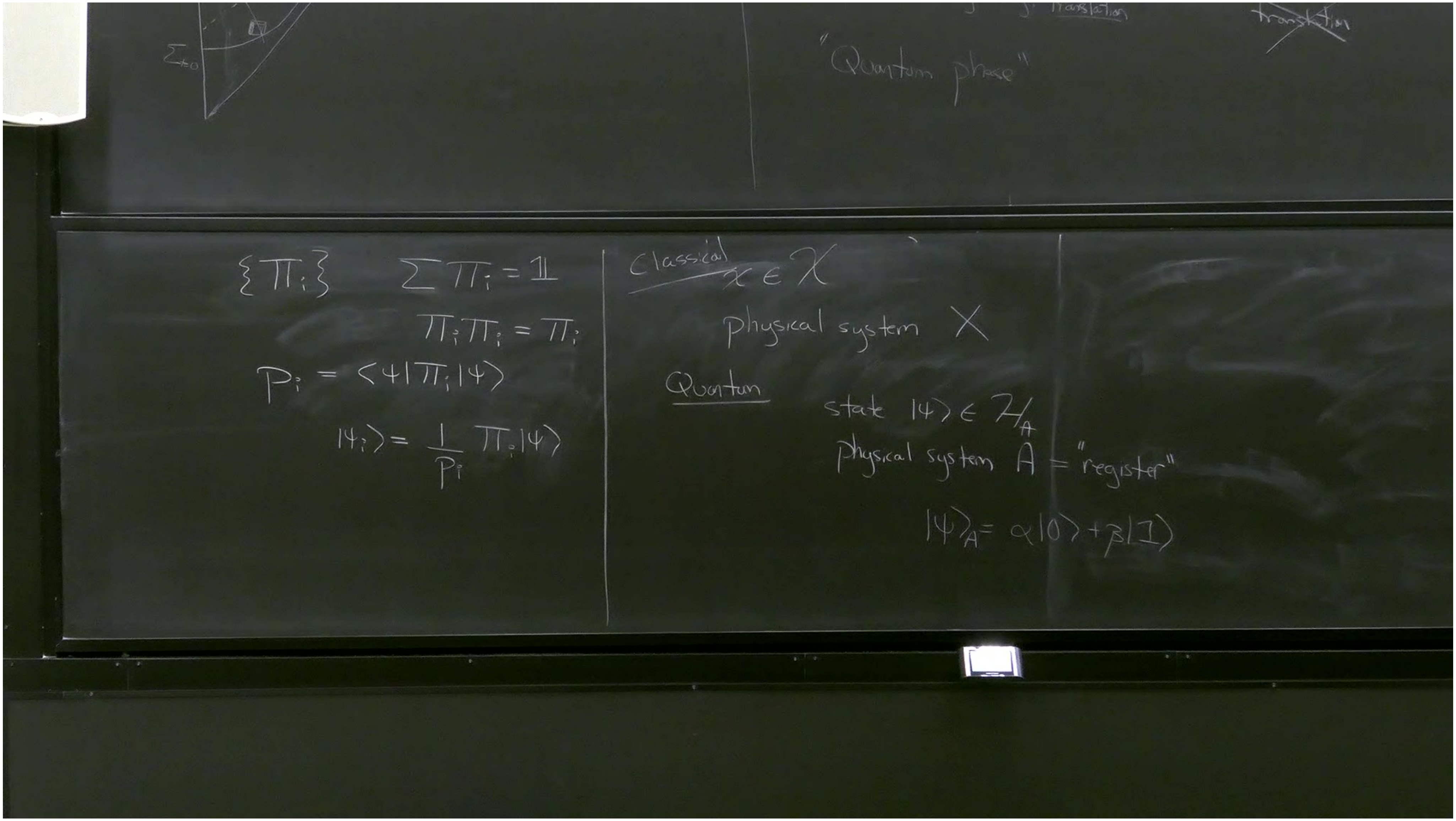
$$\mathrm{COPY}_{A\to B}\,|+\rangle_A|0\rangle_B = |+\rangle_A|+\rangle_B$$

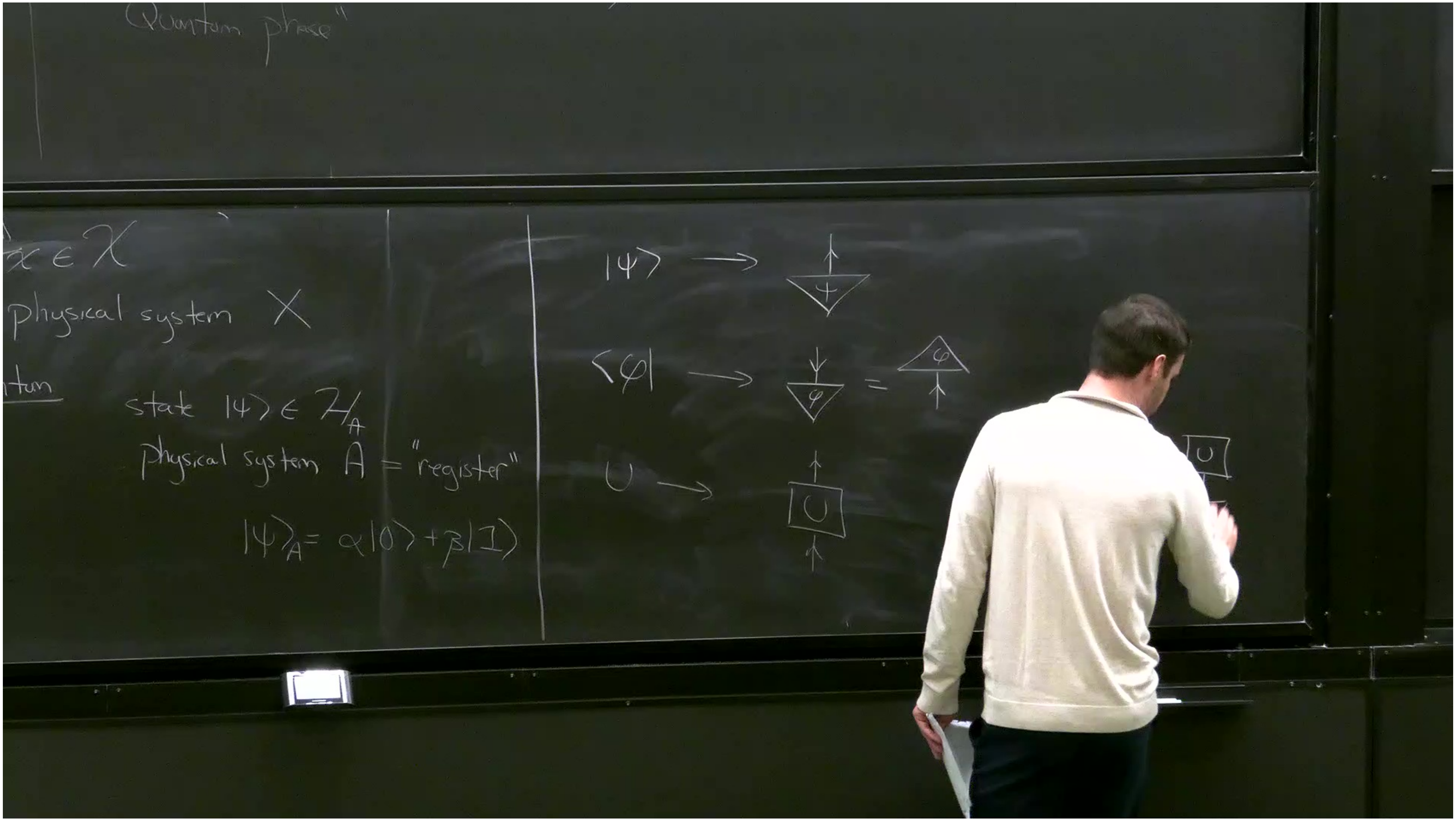
$$|+\rangle|+\rangle \neq \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

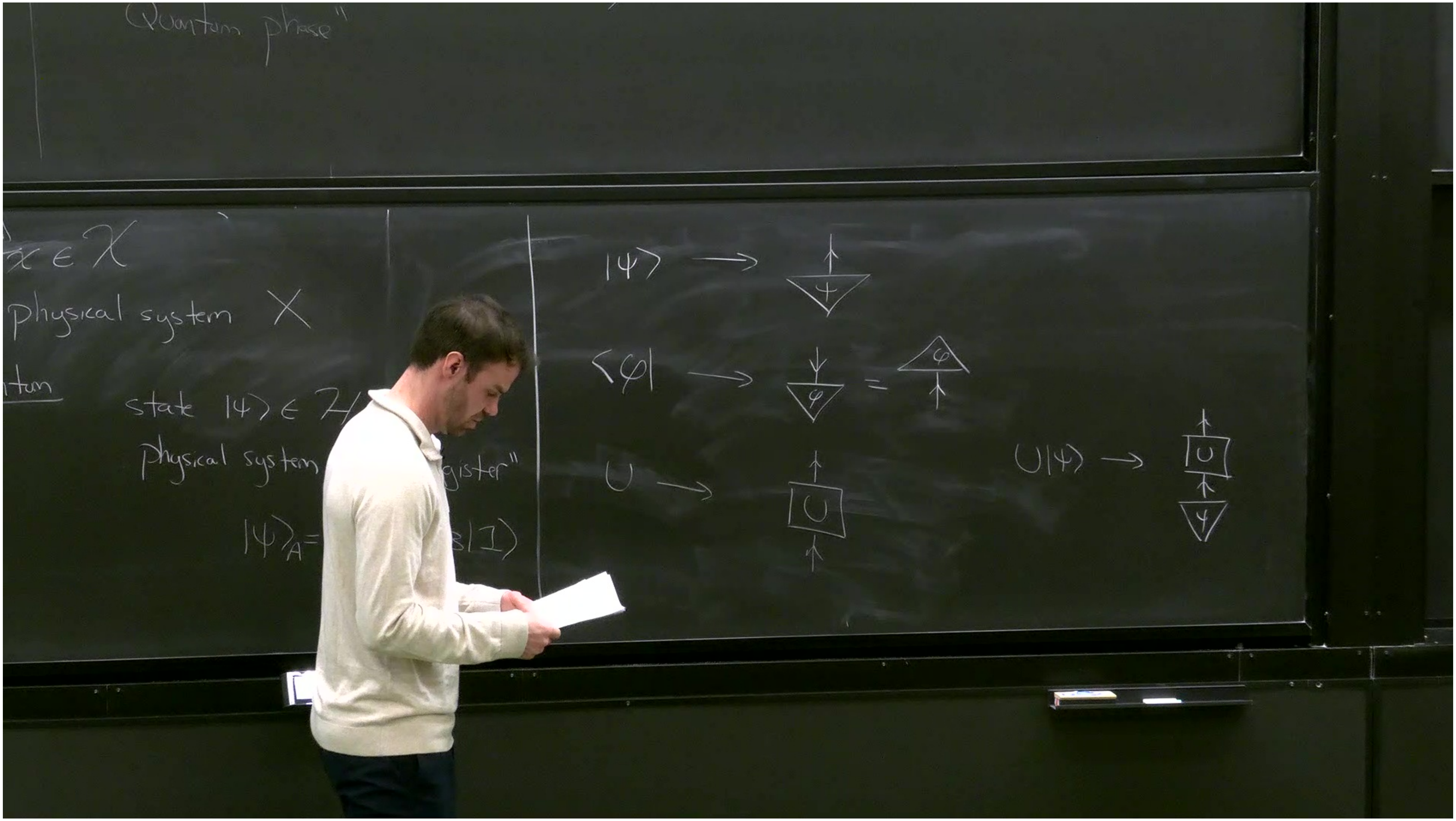
$$\mathrm{COPY}_{A\to B}|+\rangle|0\rangle = \mathrm{COPY}_{A\to B}\left(\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\right)|0\rangle$$

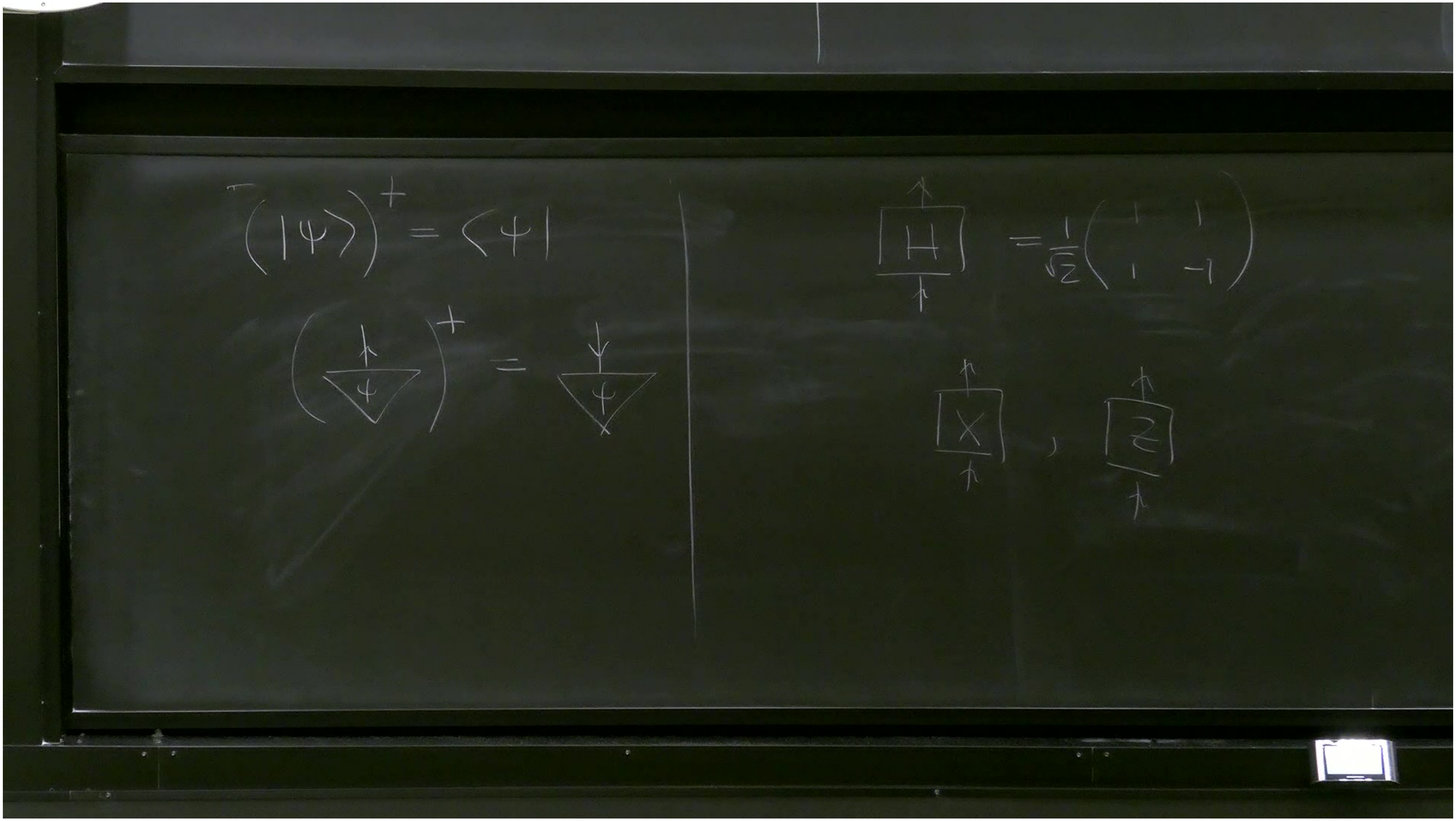
$$= \frac{1}{\sqrt{2}}\mathrm{COPY}_{A\to B}|0\rangle|0\rangle + \frac{\mathrm{COPY}_{A\to B}}{\sqrt{2}}|1\rangle|0\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$


translation
translation
"Quantum phase"
$\{\Pi_i\}$ $\sum \Pi_i = \mathbb{1}$
$\Pi_i \Pi_i = \Pi_i$
$p_i = \langle\psi|\Pi_i|\psi\rangle$
$|\psi_i\rangle = \frac{1}{p_i}\Pi_i|\psi\rangle$
Classical $x \in \mathcal{X}$
physical system X
Quantum
state $|\psi\rangle \in \mathcal{H}_A$
physical system A = "register"
$|\psi\rangle_A = \alpha|0\rangle + \beta|1\rangle$


"Quantum phase"
x ∈ X
physical system X
ntum
state |ψ⟩ ∈ H_A
physical system A = "register"
|ψ⟩_A = α|0⟩ + β|1⟩
|ψ⟩ →
ψ
⟨φ| →
φ
=
φ
U →
U
U


"Quantum phase"
x ∈ X
physical system X
state |ψ⟩ ∈ H
physical system
gister"
|ψ⟩_A =
|ψ⟩ →
⟨φ| →
U →
U|ψ⟩ →


(|ψ⟩)† = ⟨ψ|
ψ
=
ψ
H
= 1/√2 (1 1 ; 1 −1)
X
,
Z