Title: Lecture - Quantum Field Theory III - PHYS 777

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Subject: Quantum Fields and Strings

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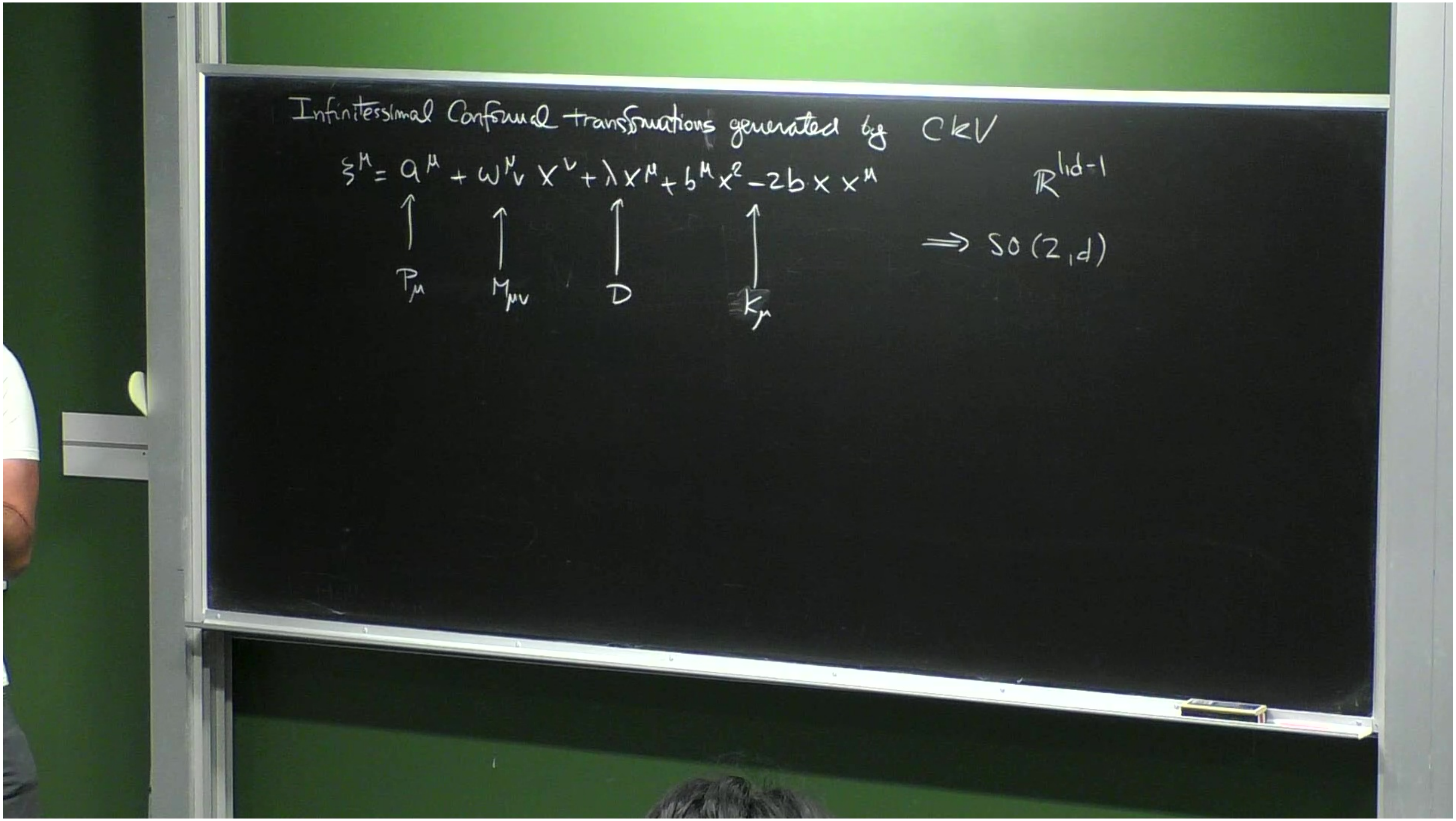
Infinitessimal Conformal transformations generated by CkV
$\xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + b^\mu x^2 - 2b\cdot x\, x^\mu$
P_μ
$M_{\mu\nu}$
D
K_μ
$\mathbb{R}^{1,d-1}$
$\Longrightarrow SO(2,d)$

Infinitessimal Conformal transformations generated by CKV

$$\xi^\mu = \underset{P_\mu}{a^\mu} + \underset{M_{\mu\nu}}{\omega^\mu{}_\nu x^\nu} + \underset{D}{\lambda x^\mu} + \underset{K_\mu}{b^\mu x^2 - 2b\cdot x\, x^\mu}$$

$\mathbb{R}^{1,d-1}$

$\Longrightarrow SO(2,d)$

- CFT has enhancement from $SO(1,d-1) \longrightarrow \underline{SO(2,d)}$

$T_{\mu\nu}$: $\exists$ for any local QFT

1) P_μ: $\partial^\mu T_{\mu\nu} = 0$

2) $M_{\mu\nu}$: $T_{\mu\nu} = T_{\nu\mu}$

3) ?

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3) ?

use $\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{D}(\partial\cdot\xi)\,\eta_{\mu\nu}$

ξ is a CKV

$$\delta S = \frac{1}{2}\int dx\; T_{\mu\nu}\,\delta_\xi h^{\mu\nu} \sim \int dx\; \underbrace{T_{\mu\nu}\,\eta^{\mu\nu}}_{\text{trace of the EM tensor.}}(\partial\cdot\xi)$$

$$\delta_\xi h^{\mu\nu} = (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu)$$

$\boxed{1}$ constraint due to $\exists\, D$: $\quad T^{\mu}_{\mu} = \partial_\mu L^{\mu}$

$\boxed{2}$ " " " K_μ: $\quad T^{\mu}_{\mu} = \partial_\mu \partial_\nu L^{\mu\nu}$

j_μ is a conserved current, you can always improvement transf.

$$\partial^\mu j_\mu = 0$$

$$\hat{j}_\mu = j_\mu + \partial_\nu \theta^{[\mu\nu]} \qquad \boxed{\partial^\mu \hat{j}_\mu = 0} \qquad \int j_\mu A^\mu + \int F_{\mu\nu} \theta^{\mu\nu}$$

$$Q = \int j_0 = \int \tilde{j}_0$$

j_μ is a conserved current, you can always improvement transf.

$\partial^\mu j_\mu = 0$

$\tilde{j}_\mu = j_\mu + \partial_\nu \Theta^{[\mu\nu]}$ $\boxed{\partial^\mu \tilde{j}_\mu = 0}$ $\int j_\mu A^\mu + \int F_{\mu\nu} \Theta^{\mu\nu}$

$Q = \int j_0 = \int \tilde{j}_0$

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in a CFT, improve $T_{\mu\nu} \longrightarrow \tilde{T}_{\mu\nu}$ such that $\boxed{\tilde{T}^\mu_\mu = 0}$

$\int R_{\mu\nu\rho\sigma} \Theta^{\mu\nu\rho\sigma}$ scalar QFT $\tilde{T}^\mu_\mu =$

Local operators in a CFT

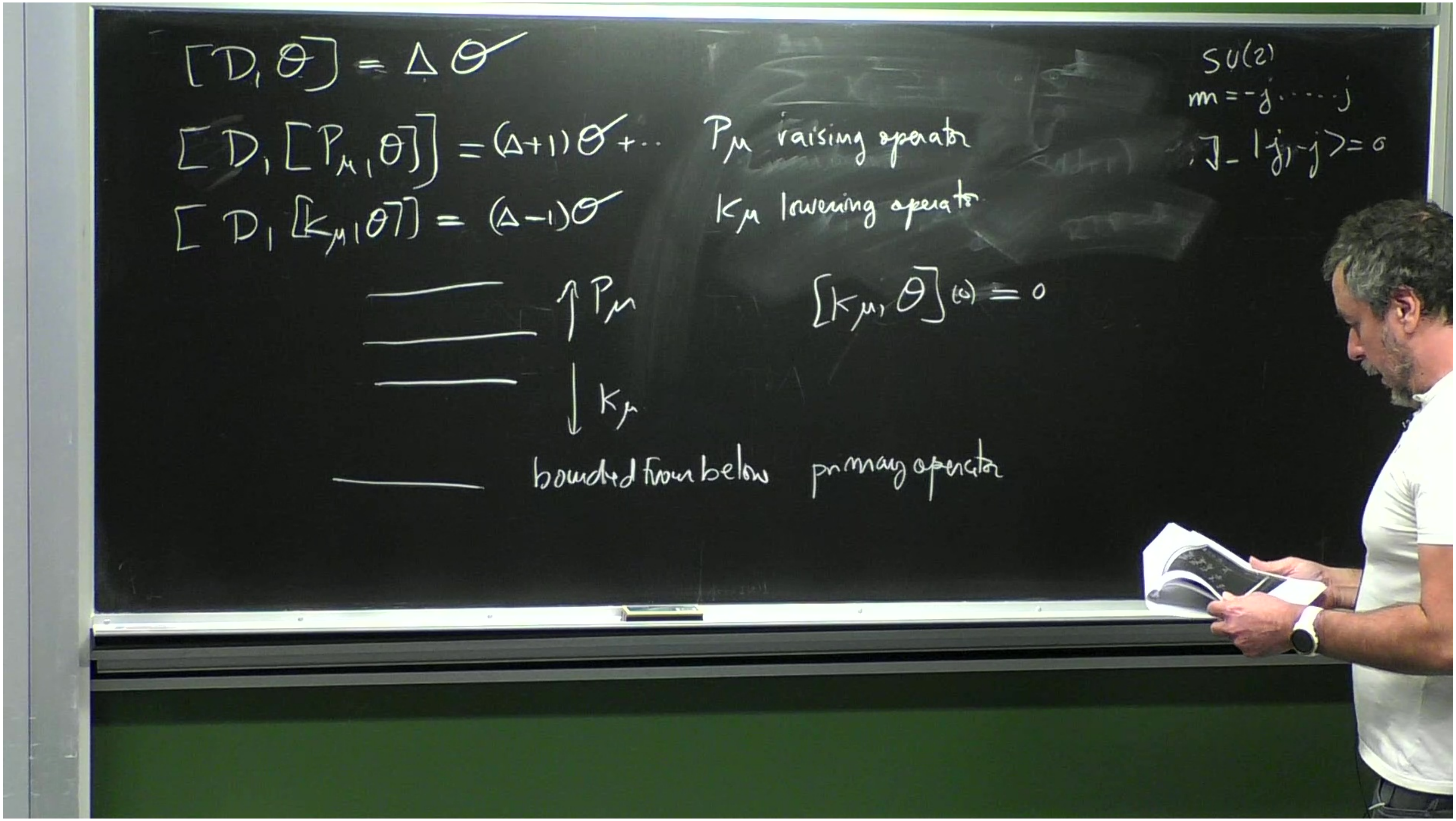
- Realize conformal algebra on operators. What are the quantum number of local ops in a CFT

{ - Lorentz spin: (s) $M_{\mu\nu}$
- "scaling dimension: Δ D.

[1] primary

[2] descendant

$$e^{iP\cdot x}\,\mathcal{O}(0)\,e^{-iP\cdot x} = \mathcal{O}(x)$$

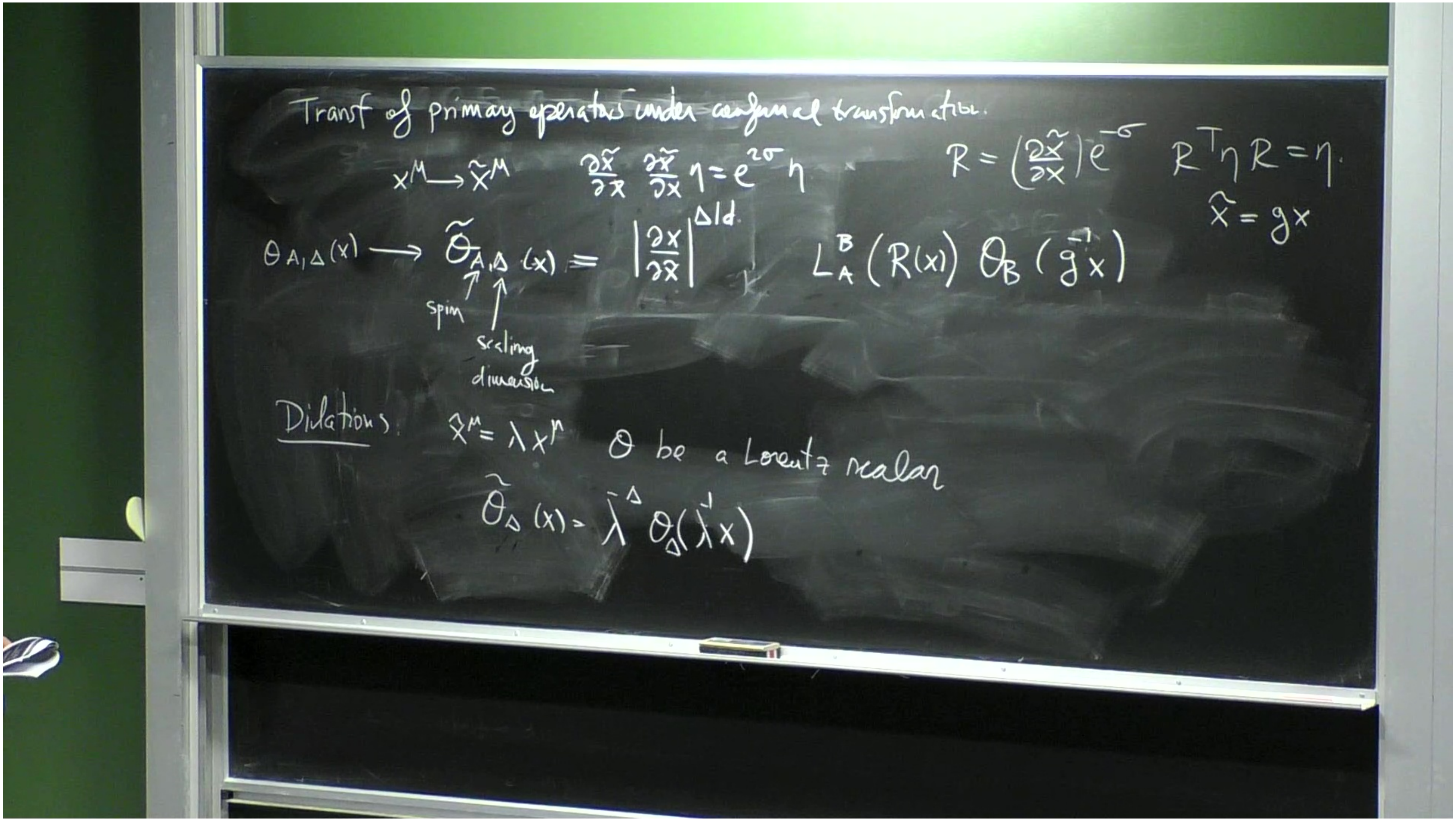

$[D, \mathcal{O}] = \Delta \mathcal{O}$
$[D, [P_\mu, \mathcal{O}]] = (\Delta+1)\mathcal{O} + \dots$
P_μ raising operator
$[D, [K_\mu, \mathcal{O}]] = (\Delta-1)\mathcal{O}$
K_μ lowering operator
P_μ
K_μ
$[K_\mu, \mathcal{O}](0) = 0$
bounded from below
primary operator
SU(2)
$m = -j \dots j$
$J_- |j, -j\rangle = 0$

Representations $\quad \mathcal{O} \quad \Delta \quad$ primary

$P_\mu \mathcal{O} \quad \Delta+1$

$P_{\mu_1} P_{\mu_2} \mathcal{O} \quad \Delta+2$ descendants

∞'al dimensional


Transf of primary operators under conformal transformation.
$x^\mu \to \tilde{x}^\mu$
$\frac{\partial \tilde{x}}{\partial x} \frac{\partial \tilde{x}}{\partial x} \eta = e^{2\sigma} \eta$
$R = \left(\frac{\partial \tilde{x}}{\partial x}\right) e^{-\sigma}$
$R^T \eta R = \eta$.
$\hat{x} = g x$
$O_{A,\Delta}(x) \longrightarrow \tilde{O}_{A,\Delta}(x) = \left|\frac{\partial x}{\partial \tilde{x}}\right|^{\Delta/d} L_A^B(R(x))\, O_B(g^{-1}x)$
spin
scaling dimension
Dilations:
$\tilde{x}^\mu = \lambda x^\mu$
O be a Lorentz scalar
$\tilde{O}_\Delta(x) = \lambda^{-\Delta} O_\Delta(\lambda^{-1}x)$

Dilations: $\tilde{x}^\mu = \lambda x^\mu$ $\quad \Theta$ be a Lorentz scalar

$$\tilde{\Theta}_\Delta(x) = \lambda^{-\Delta}\,\Theta_\Delta(\lambda^{-1}x)$$

Under infinitessimal $x'^\mu = x^\mu + \xi^\mu$

$$\delta\Theta_{A,\Delta}(x) = \tilde{\Theta}_{A,\Delta}(x) - \Theta_{A,\Delta}(x) =$$

$$= -\underbrace{\xi^\mu\partial_\mu\Theta_{A,\Delta}(x)}_{\text{orbital}} + i\,\Omega_{\mu\nu}(x)\underbrace{\left(M_R^{\mu\nu}\right)^B{}_A}_{\text{Representation of Lorentz}}\Theta_{B,\Delta}(x) - \underbrace{\Delta}_{\text{scaling dimension}}\omega(x)\;\Theta_{A,\Delta}(x)$$

$$\Omega_{\mu\nu}(x) = \omega_{\mu\nu} - 2(x_\mu b_\nu - x_\nu b_\mu)$$

$$\omega(x) = \lambda - 2x\cdot b$$

ξ_1, ξ_2 CKV's

$$[\xi_1, \xi_2] = \xi_3$$

$\uparrow$ CKV

$$[\delta_{\xi_1}, \delta_{\xi_2}]\mathcal{O} = \delta_{[\xi_1,\xi_2]}\mathcal{O}$$ if satisfied realized conformal algebra on $\mathcal{O}$

$$\langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_n(x_n) \rangle$$

$$\langle \delta(\mathcal{O}_1(x_1) \cdots \mathcal{O}_n(x_n)) \rangle = 0$$ Ward identities

$$= \delta\mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) \cdots \mathcal{O}_n(x_n) + \mathcal{O}_1(x_1)\, \delta\mathcal{O}_2(x_2) \cdots \mathcal{O}_n(x_n) + \cdots$$

$$\delta\mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\cdots\mathcal{O}_n(x_n)+\mathcal{O}_1(x_1)\,\delta\mathcal{O}_2(x_2)\cdots\mathcal{O}_n(x_n)+\cdots$$

$$x^\mu \to \frac{x^\mu+b x^\mu}{(1+b^\mu x_\mu+b^2x^2)}$$

2 and 3 points (position dependence is completely fixed)

in contrast to Poincaré QFT

$$\langle \mathcal{O}_\Delta(x)\,\mathcal{O}_\Delta(y)\rangle = f(|x-y|) \overset{\text{CFT}}{=} \frac{C}{|x_1-x_2|^{2\Delta}}$$

x_1 x_2 K_μ

$x_1 \quad x_2 \quad x_3$

$$\longrightarrow (x_1', \infty, x_3')$$

$$\Big\downarrow P_\mu$$

$$(0, \infty, x_3'')$$

$$\delta O_1(x_1)\, O_2(x_2) \cdots O_n(x_n) + O_1(x_1)\, \delta O_2(x_2) \cdots O_n(x_n) + \cdots$$

$$x^\mu \to \frac{x^\mu + b x^\mu}{(1 + b^\mu x_\mu + b^2 x^2)}$$

2 and 3 points (position dependence is completely fixed)

in contrast to Poincaré QFT

$$\langle O_{\Delta_1}(x)\, O_{\Delta_2}(y) \rangle = f(|x-y|) \overset{\text{CFT}}{=} \frac{C}{|x_1 - x_2|^{(\Delta_1+\Delta_2)}}$$

x_2 K_μ

x_1 $\longrightarrow (x_1', \infty, x_3')$

x_1 x_2 x_3

$\downarrow P_\mu$

$(0, \infty, x_3'')$