Title: Lecture - Quantum Field Theory III - PHYS 777

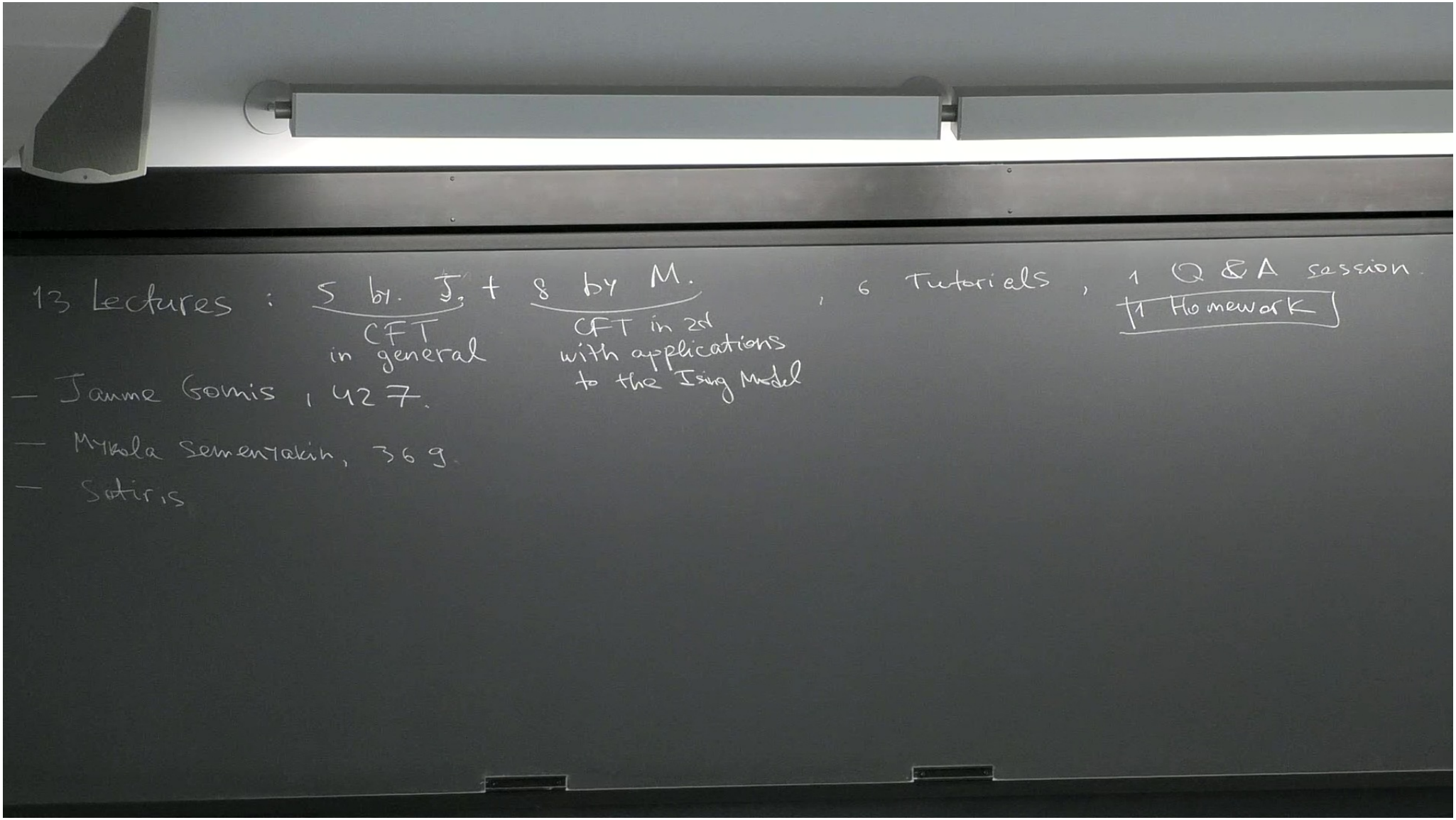
Speakers: Jaume Gomis

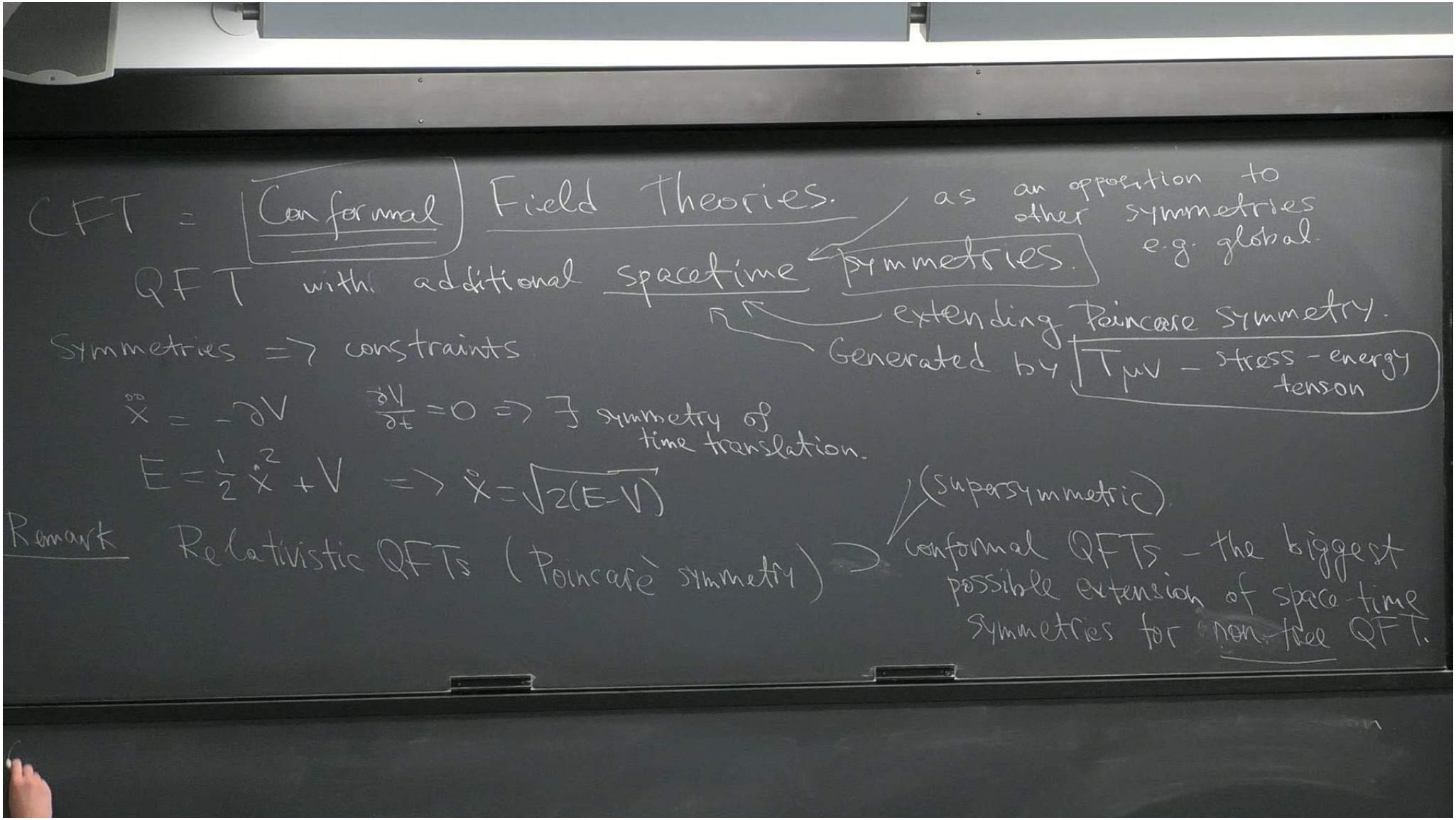
Collection/Series: Quantum Field Theory III, PHYS 777-, February 24 - March 28, 2025

Subject: Quantum Fields and Strings

Date: February 24, 2025 - 10:15 AM

URL: https://pirsa.org/25020006


13 Lectures : 5 by J. + 8 by M. , 6 Tutorials , 1 Q&A session.
CFT in general
CFT in 2d with applications to the Ising Model
11 Homework
– Jaume Gomis , 427.
– Mykola Semenyakin , 369.
– Satiris


CFT = Conformal Field Theories.
QFT with additional spacetime symmetries.
as an opposition to other symmetries e.g. global.
extending Poincare symmetry.
Generated by $T_{\mu\nu}$ - stress-energy tenson
Symmetries => constraints
$\ddot{x} = -\partial V$
$\frac{\partial V}{\partial t} = 0 \Rightarrow \exists$ symmetry of time translation.
$E = \frac{1}{2}\dot{x}^2 + V \Rightarrow \dot{x} = \sqrt{2(E-V)}$
Remark Relativistic QFTs (Poincaré symmetry) (supersymmetric)
conformal QFTs - the biggest possible extension of space-time symmetries for non-free QFT.

(Poincaré symmetry)

possible extension of space-time symmetries for non-free QFT.

Conformal transformations: $\vec{x} \to \lambda \vec{x}$, $t \to \lambda^z t$, z – dynamical critical exponent

- $z = 1$ relativistic QFT
- $z = 2$ Free Schrödinger equation $\quad i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m}\left(-i\hbar \frac{\partial}{\partial x}\right)^2 \psi$

Remark Theories with scale are non-conformal e.g massive ones

Remark Length-scales are very important in physics, in particular we have to pick a scale to have any reasonable considerations

water: hydrodynamics ⇝ Schrödinger eq ⇝ QCD

effective field theory

Classically conformal theories

Spin 0 1) $\Box\phi + \lambda\phi^3 = 0$

Spin 1/2 2) $\not{D}\psi = 0$

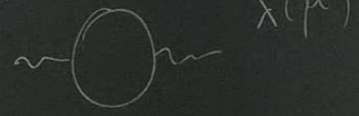
Spin 1 3) $D_\mu F^{\mu\nu} = 0$

$d=4$: $\phi \to \frac{1}{\sigma}\phi$ $\quad x^\mu \to \sigma x^\mu$

↘ Yukawa $\mathcal{L}(\phi,\psi) \sim \bar{\psi}\psi\phi$

Theories that are conformal on a classical level are not (often) on quantum

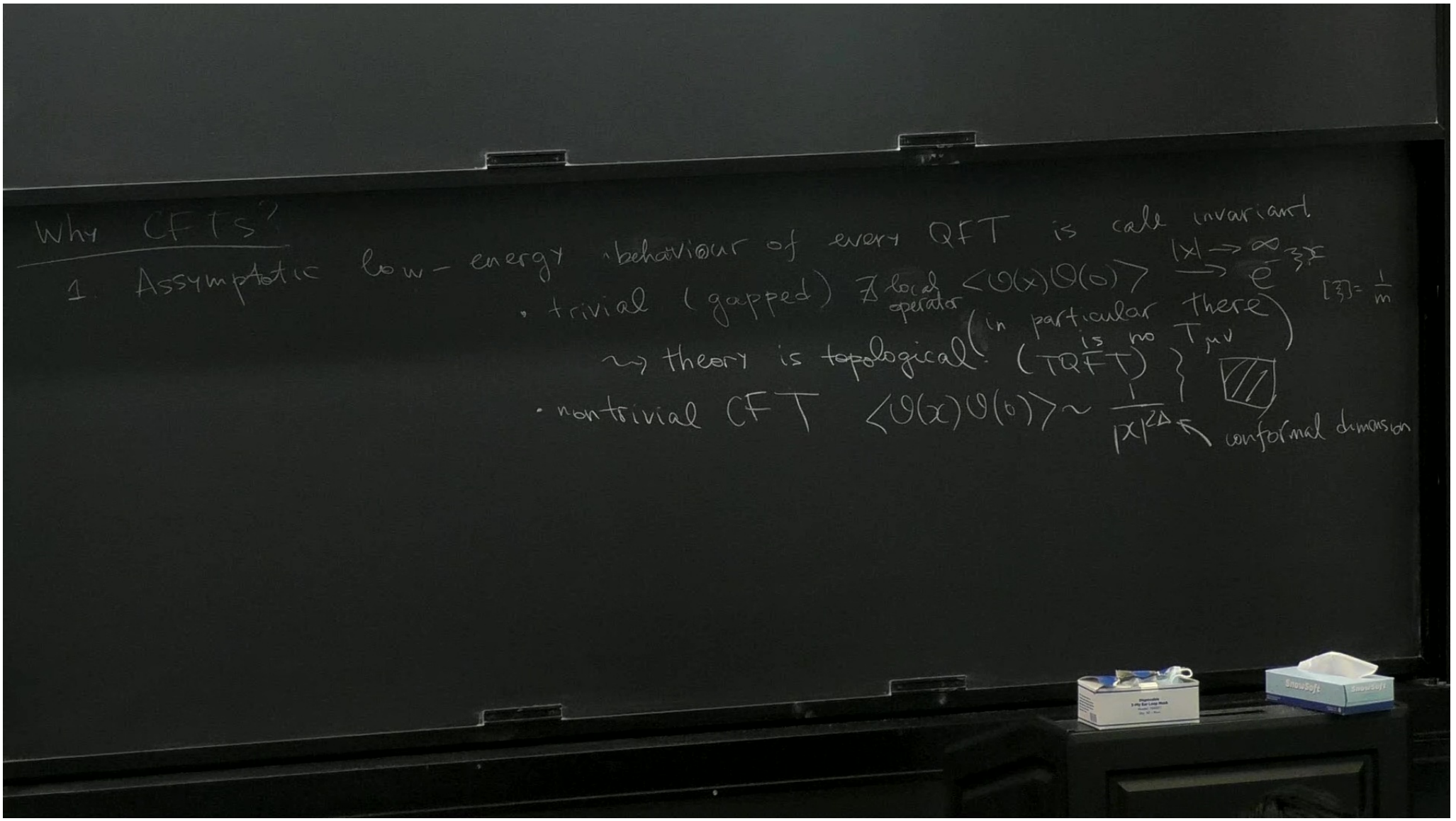
energy scale ↓ μ $\quad \mu\frac{d\lambda}{d\mu} = \beta(\lambda) \neq 0$



Why CFTs?

1. Assymptotic low-energy behaviour of every QFT is scale invariant

- trivial (gapped) $\nexists$ local operator $\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \xrightarrow{|x|\to\infty} e^{-\frac{|x|}{\xi}}$ $[\xi] = \frac{1}{m}$
  $\rightsquigarrow$ theory is topological (in particular there is no $T_{\mu\nu}$) (TQFT)
- nontrivial CFT $\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \sim \frac{1}{|x|^{2\Delta}}$ ← conformal dimension

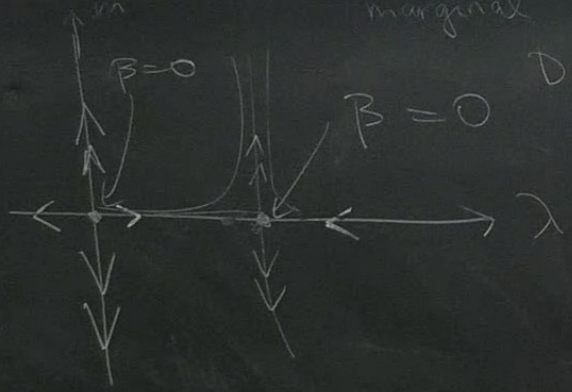


effective field theory.

Any massive QFT is a deformation of CFT.

$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 + \lambda\phi^4 + \frac{1}{2}m^2\phi^2$

λ: marginal, m^2: relevant



$D = 4 - \epsilon$

$\text{CFT} + \int \lambda \mathcal{O}_\Delta d^D x$

1) $\Delta > D$: irrelevant operators

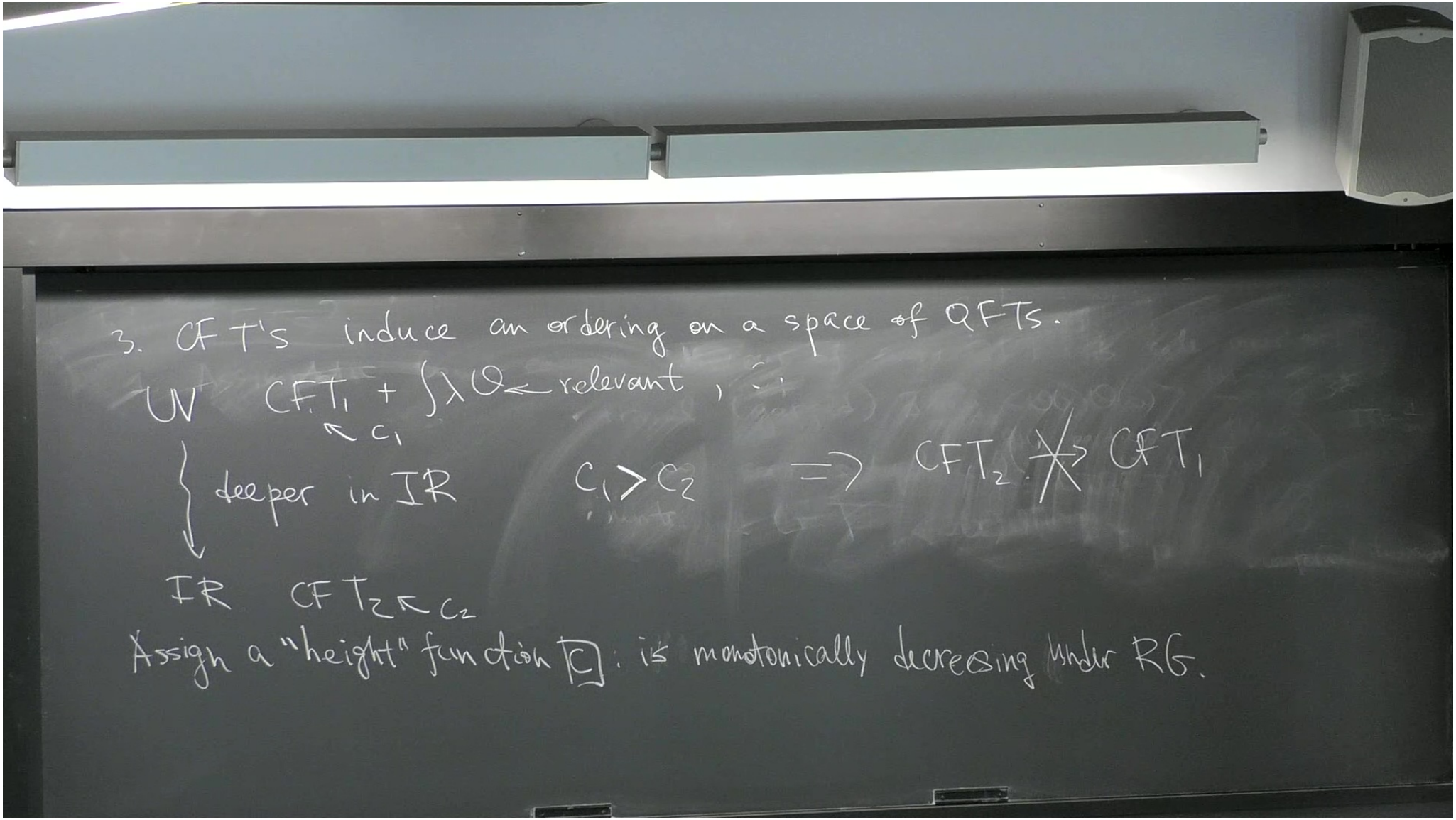
λE^α, $\alpha > 0$

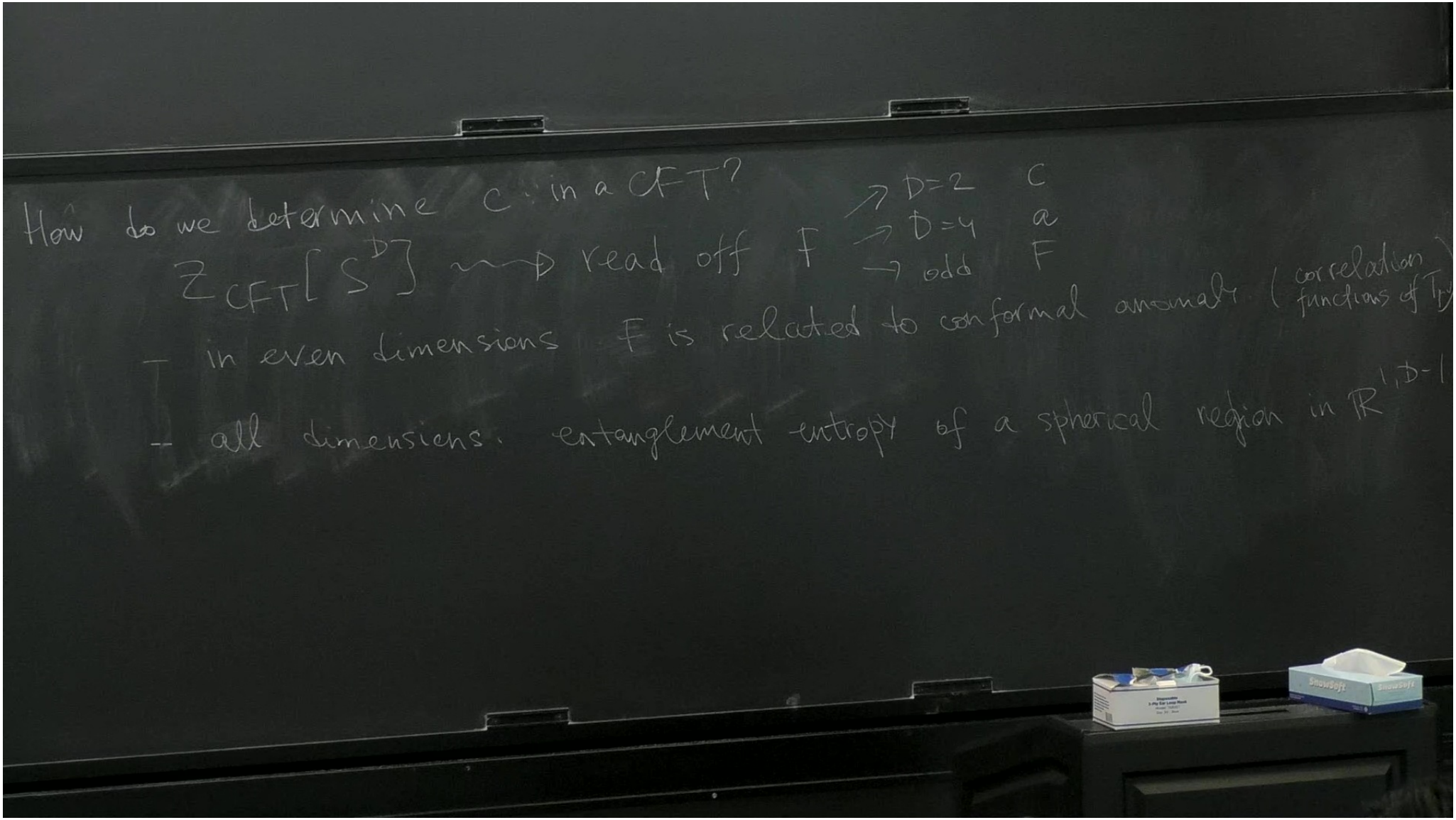
$E \to 0$ $\to 0$

2) $\Delta = D$: marginal operator.

(quantum corrections are essent.)

3) $\Delta < D$: relevant operators in IR


3. CFT's induce an ordering on a space of QFTs.
UV CFT$_1$ + $\int \lambda \mathcal{O}$ ← relevant, ∴
c_1
deeper in IR
$c_1 > c_2$ ⇒ CFT$_2$ ↛ CFT$_1$
IR CFT$_2$ c_2
Assign a "height" function c: is monotonically decreasing under RG.


How do we determine c in a CFT?
$Z_{CFT}[S^D]$ ⇝ read off F
→ D=2 C
→ D=4 a
→ odd F
- in even dimensions F is related to conformal anomaly (correlation functions of $T_{\mu\nu}$)
-- all dimensions: entanglement entropy of a spherical region in $\mathbb{R}^{1,D-1}$

relevant operators in IR

4. CFTs are relevant in Nature ⇒ phase transitions.

- Driven by thermal fluctuations.

3d Ising model

$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j + B \sum \sigma_i$

$\mathbb{Z}_2$ is broken

conformal symmetry

$\mathbb{Z}_2$ is unbroken

$\langle \sigma_i \sigma_j \rangle \propto e^{-\frac{|i-j|}{\xi}}$

$\xi \to \infty$

Other: AdS/CFT

CFT

AdS

- String theory

$\mathbb{R}^{D-1,1}$

$\mathbb{R}^{D-1,1}$ Maxwell — Driven by quantum fluctuations

: Einstein-Hilb

2d world sheet theory

$T = 0$

$\beta = 0$

$\lambda \to \infty$

$\Delta \to 0$