Title: Lecture - Quantum Foundations, PHYS 639

Speakers: Lucien Hardy

Collection/Series: Quantum Foundations (Elective), PHYS 639, January 6 - February 5, 2025

Subject: Quantum Foundations

Date: February 03, 2025 - 11:30 AM

URL: https://pirsa.org/25020004

Generalized noncontextuality:
what parts of quantum theory are genuinely nonclassical?

David Schmid

We all believe that quantum theory is weird and can’t be explained by “any classical theory”.

Teleportation?

Remote steering?

No-cloning?

Entanglement?

Wave-particle duality?

Nonlocality?

Quantum interference?

Coherent superposition?

Uncertainty relations?

Classically explainable!

-noncommutativity
-complementarity
-interference
-no-cloning
-teleportation
-dense coding
-entanglement
-remote steering
-quantum eraser
-mmts must disturb
-ambiguity of mixtures
-no perfect state discr.
…

(Spekkens toy theory)

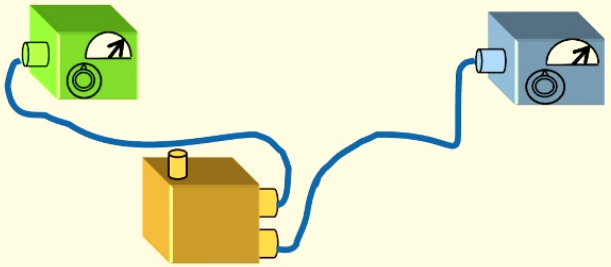
Genuinely nonclassical

-contextuality
-computational speedups
-nonlocality

We need a principled way of dividing phenomena into
those which can be “explained classically”, and
those which are rigorous proofs of nonclassicality.

What is needed to witness nonlocality

particular causal structure
two or more systems
entanglement
incompatible mmts
freedom of choice
highly efficient detectors
space-like separation



What is needed to witness contextuality

none of the above are needed

What we want in a notion of nonclassicality

	Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios
Nonlocality	✓	✓	✗
Contextuality		✓	✓

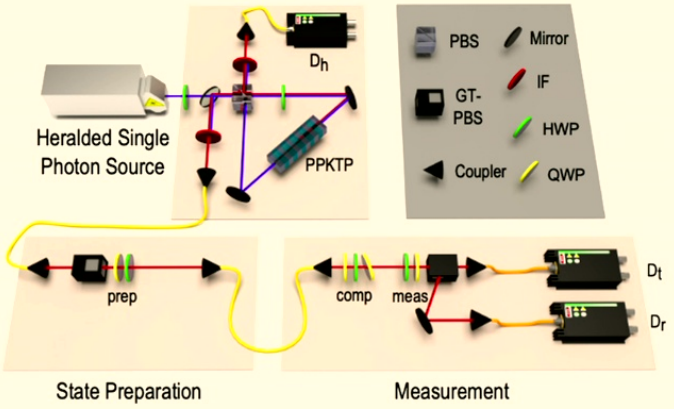
computation
communication
cryptography

metrology
cloning
state discrimination

arxiv:1505.06244

Don't need:

any particular causal structure
multiple systems
entanglement
incompatible measurements
freedom of choice
highly efficient detectors
space-like separation



Like in a Bell test, one doesn't need:

validity of quantum theory
determinism
pure states
projective mmts

Observed facts

Operational theory

$$p(X|M,P)$$

Ontological model of an operational theory

Hypothesized explanations

$\lambda \in \Lambda$ Ontic state space

The ontic state fully describes the properties possessed by a system at a given time:
-its position, momentum, mass, charge, color…

Outcomes of measurements just reveal something about these properties
ƛ causally mediates between P and M

Observed facts

Operational theory

$$p(X|M,P)$$

Ontological model of an operational theory

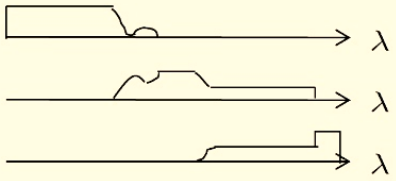
Hypothesized explanations

$\lambda \in \Lambda$ Ontic state space

$\mathsf{P} \leftrightarrow \mu(\lambda|\mathsf{P})$

$\mathsf{M} \leftrightarrow \xi(X|\mathsf{M},\lambda)$

epistemic state



response functions

$$p(X|M,P) = \sum_\lambda \xi(X|M,\lambda)\,\mu(\lambda|P)$$

arXiv:0706.2661

16

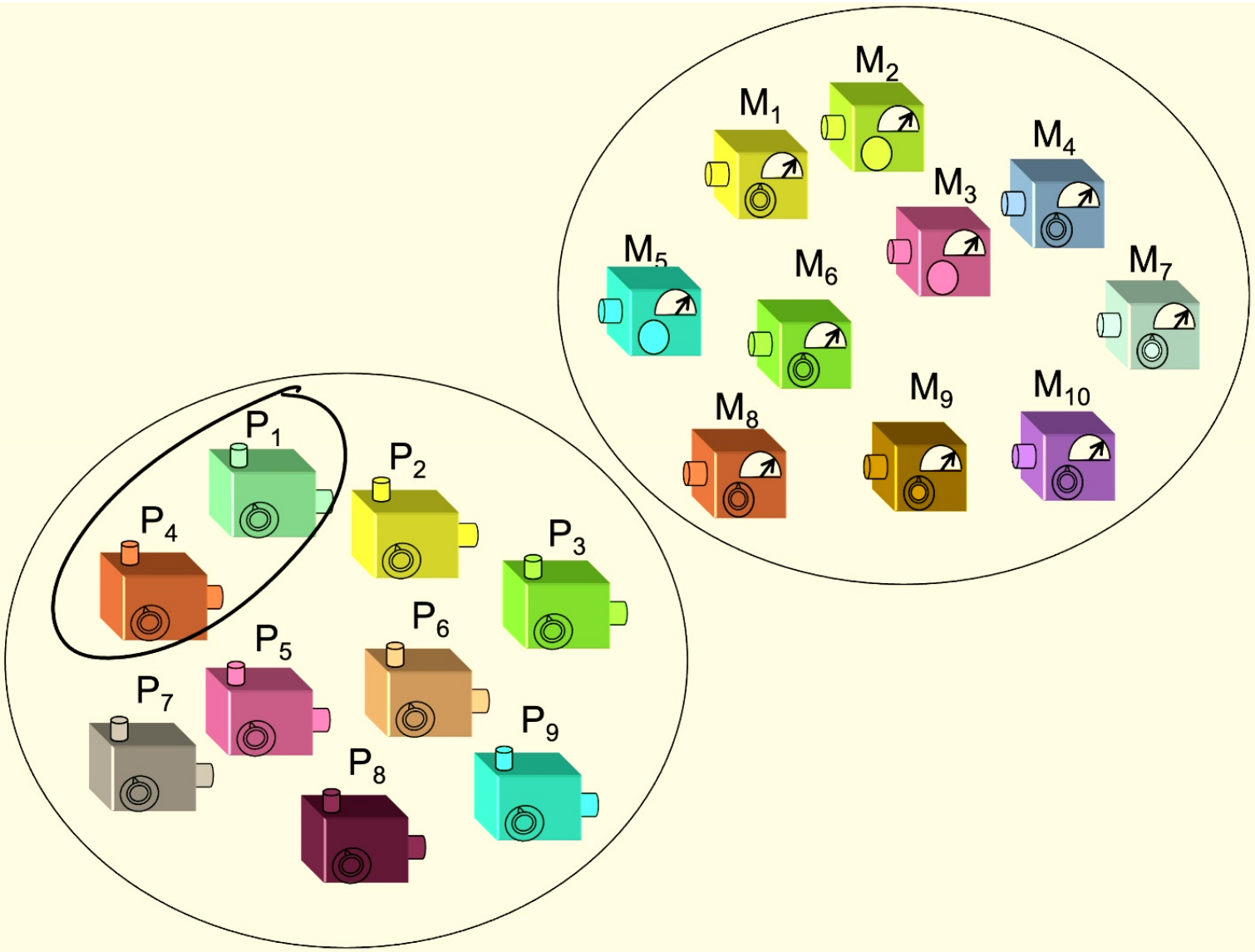
An ontological model of an operational theory is noncontextual if

experimental procedures which always lead to the same observational data → identical representations in the ontological model

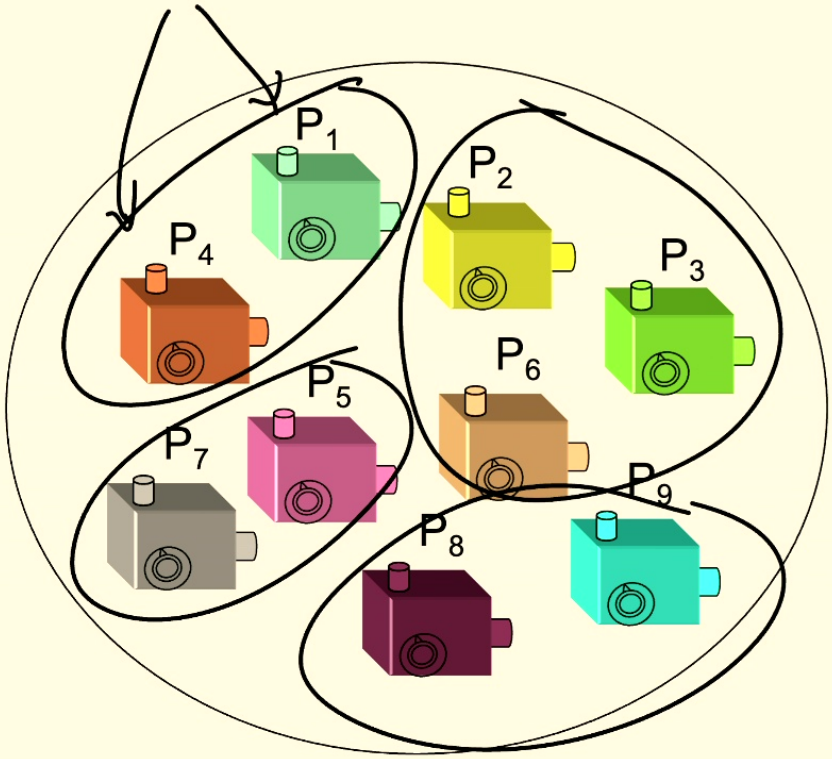
arXiv:0406166

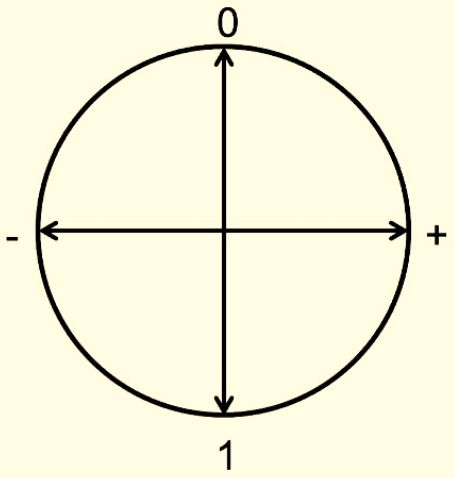
17

Noncontextuality for preparations


M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9

Quantum example



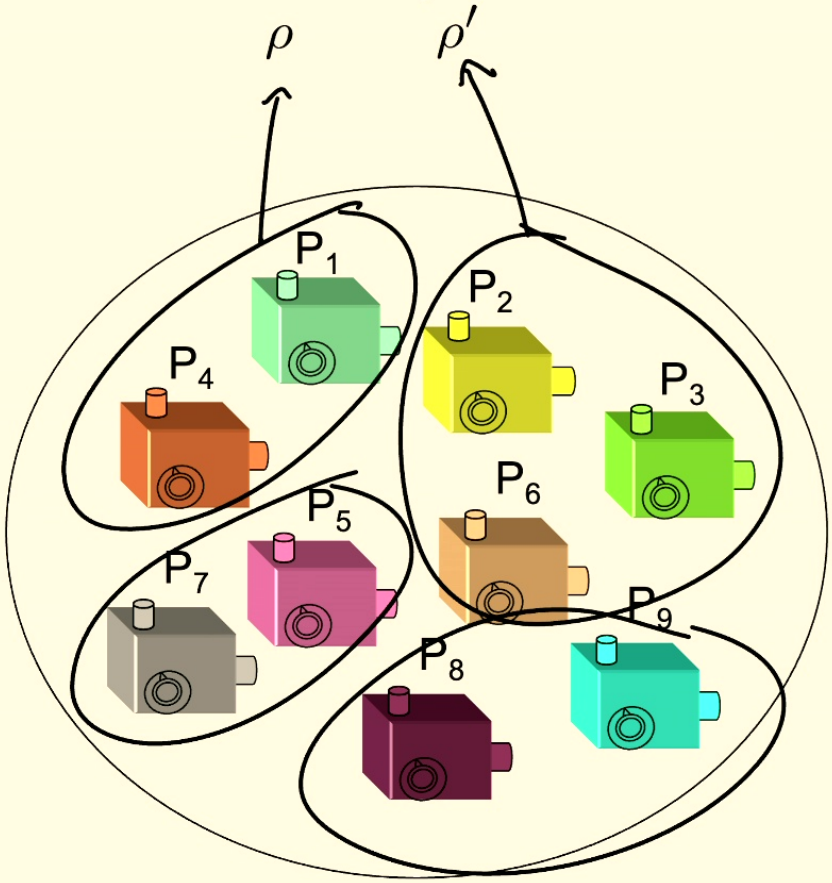


$$\frac{1}{2}I = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

$$= \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$

Quantum example

Different density operators



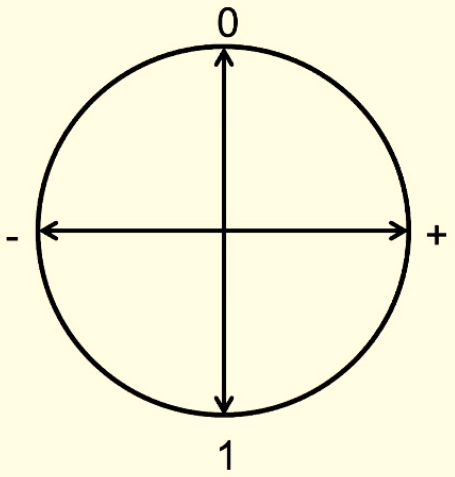
operational equivalence

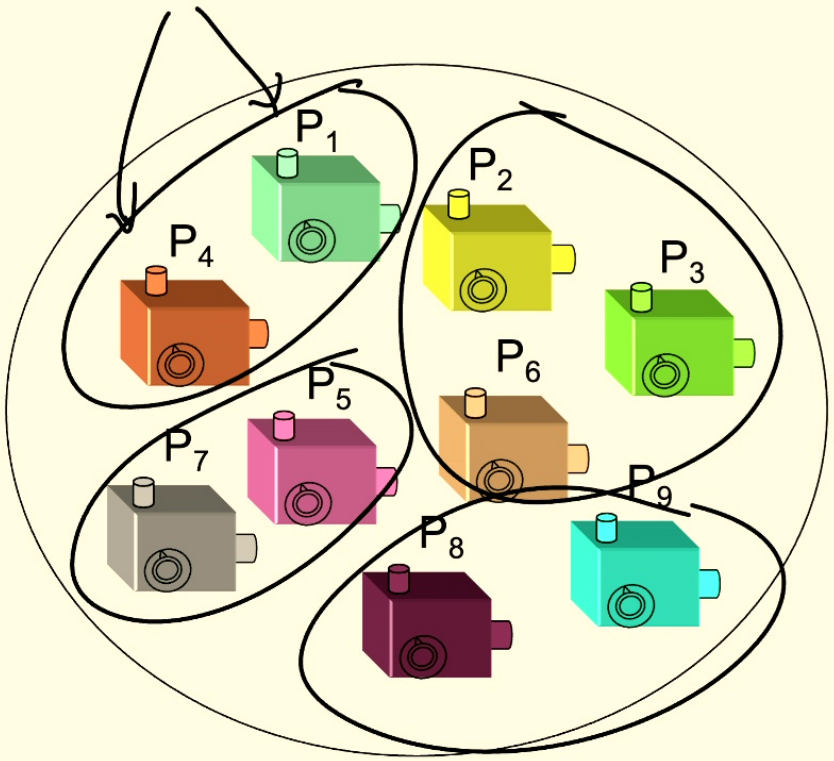
$\Leftrightarrow$

same density operator

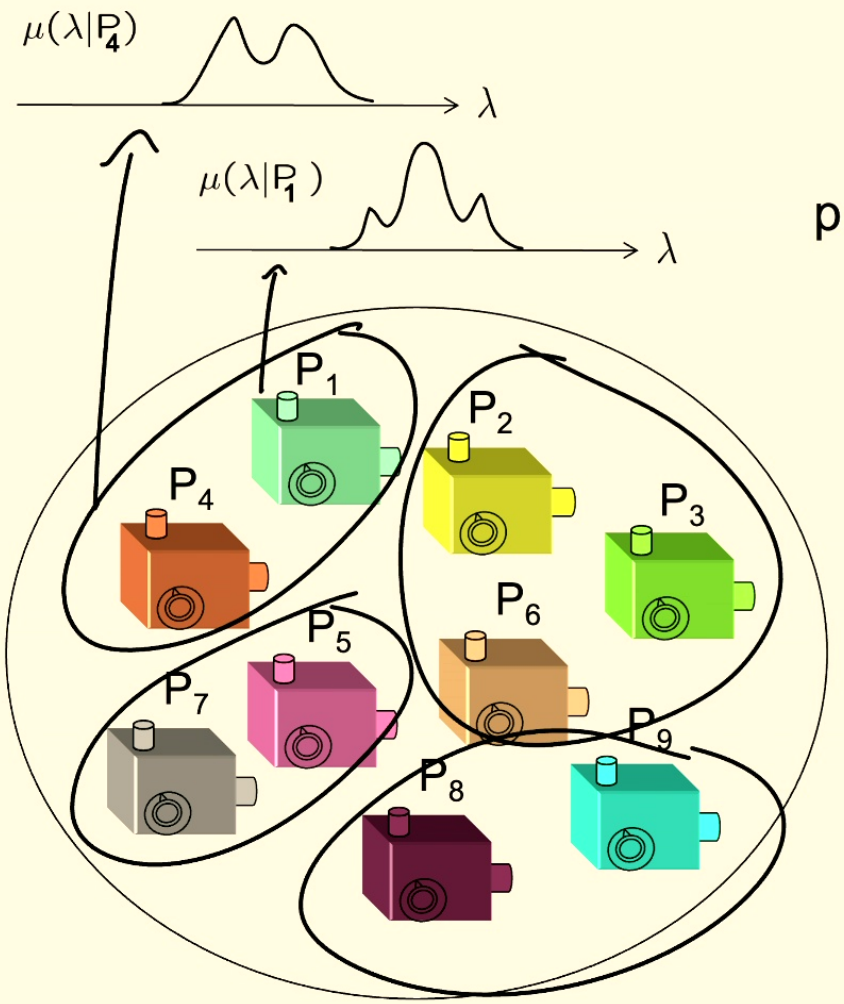
22

Quantum example



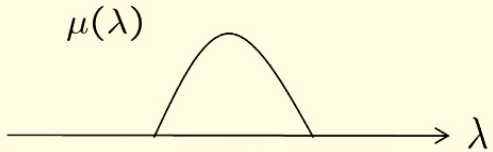


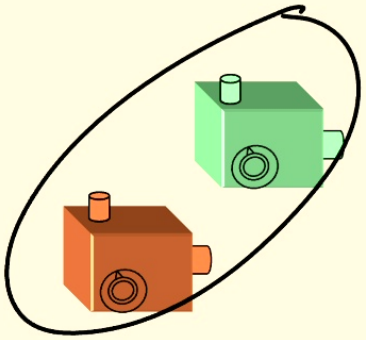
Preparation contextual ontological model



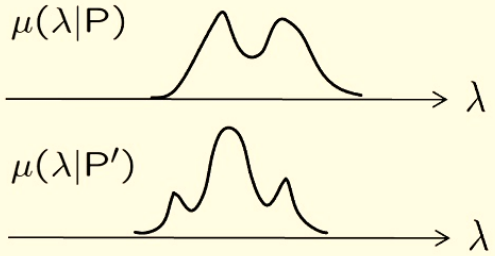
representation of each preparation **does** depend on the context

Preparation noncontextual model

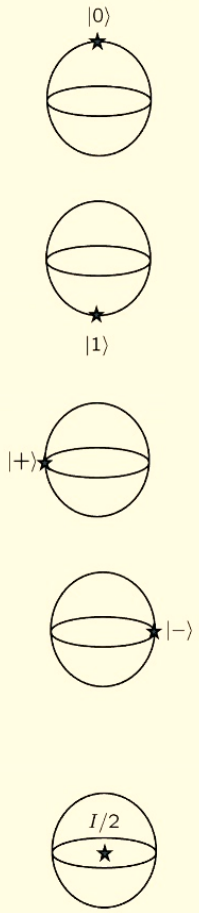




Preparation contextual model

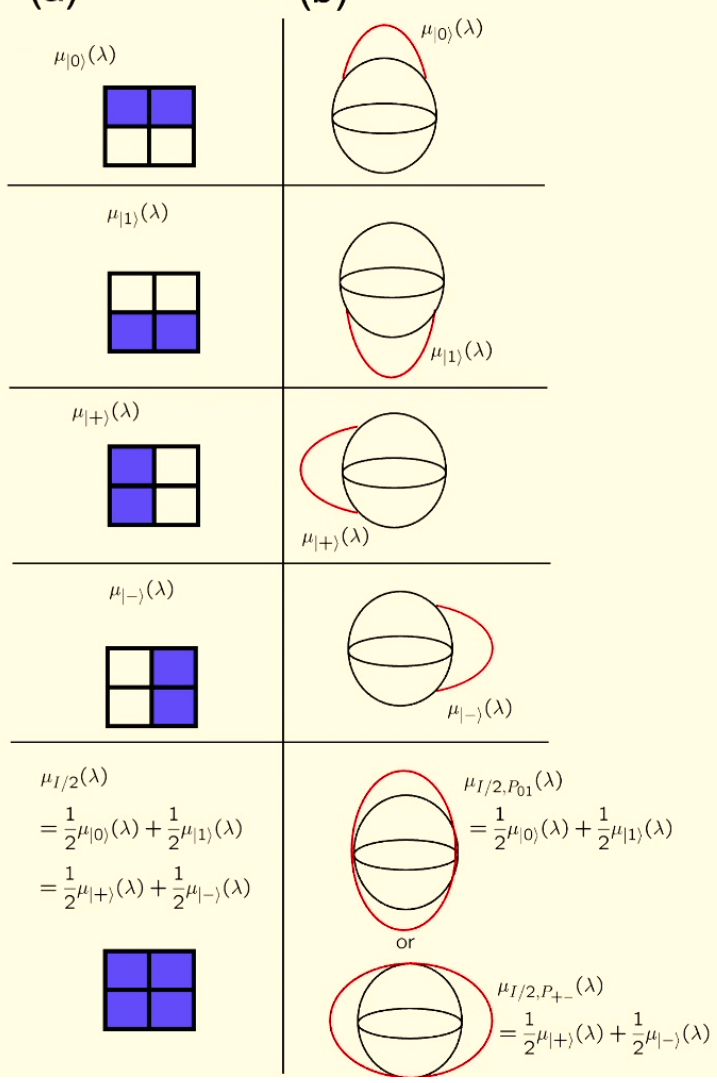


Five quantum states of a qubit

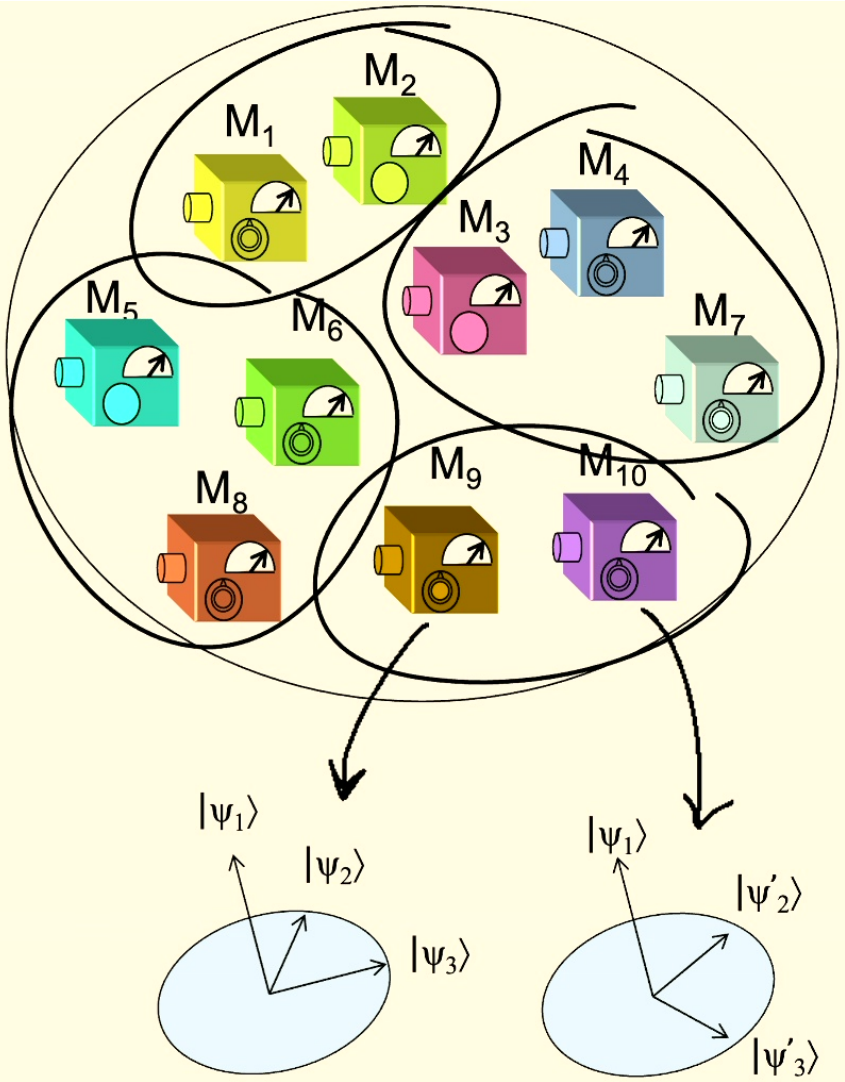


(a) A preparation noncontextual model of these (Spekkens toy theory)

(b) A preparation contextual model of these (Kochen-Specker model)

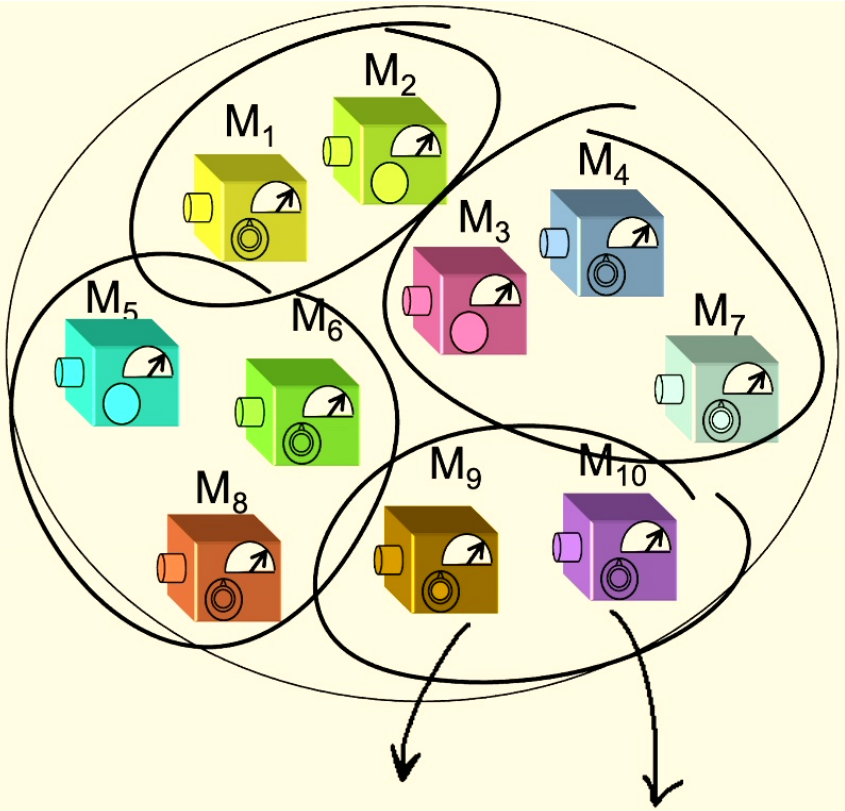


Example from quantum theory



$$|\psi_2\rangle\langle\psi_2| + |\psi_3\rangle\langle\psi_3| = |\psi_2'\rangle\langle\psi_2'| + |\psi_3'\rangle\langle\psi_3'|$$

$$\{|\psi_1\rangle\langle\psi_1|, I - |\psi_1\rangle\langle\psi_1|\}$$



Measurement
noncontextual model

Observed facts ⟶ **Hypothesized explanations**

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M}) \xrightarrow{\text{noncontextuality}} \mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

$$\mathsf{M} \simeq \mathsf{M}'$$

$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}') \xrightarrow{\text{noncontextuality}} \xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$

34

This is a rigorous way of dividing phenomena into
those which can be “explained classically”
and
those which are rigorous proofs of nonclassicality:

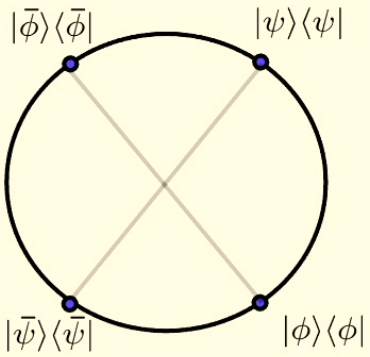
An experiment/theory/phenomena is classically-explainable if one can reproduce the operational predictions in some NCOM.

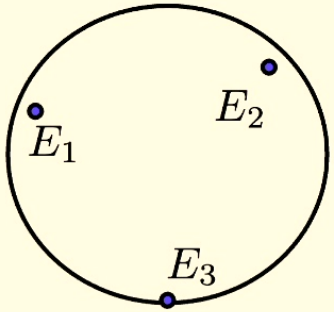
35

Typical construction of a
noncontextuality no-go theorem

consider a specific circuit

…and the states/effects/transformations on it



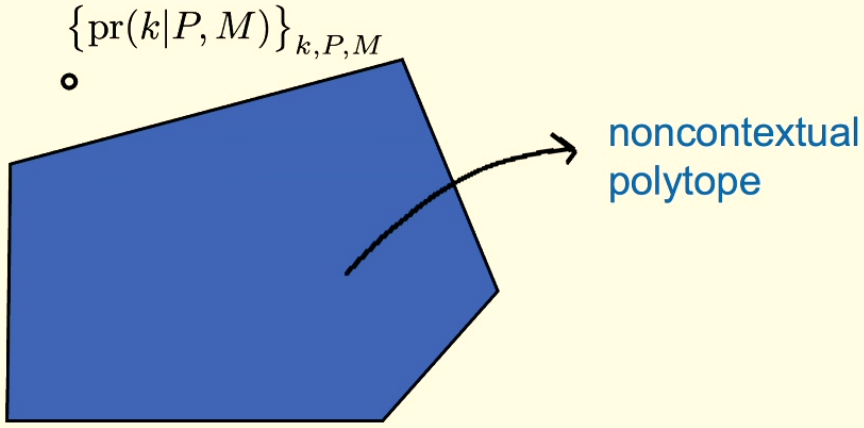


Find the operational equivalences these satisfy:

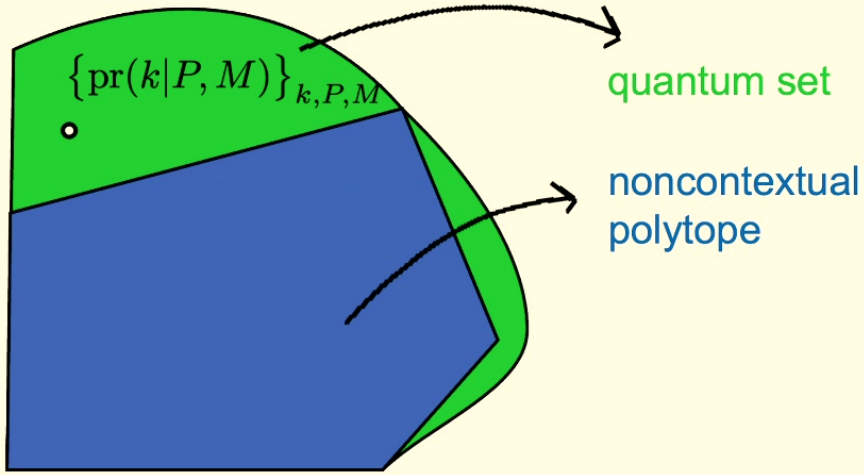
$$\frac{1}{2}|\phi\rangle\langle\phi| + \frac{1}{2}|\bar{\phi}\rangle\langle\bar{\phi}| = \frac{1}{2}|\psi\rangle\langle\psi| + \frac{1}{2}|\bar{\psi}\rangle\langle\bar{\psi}|$$

These imply constraints on any NC representation:

$$\frac{1}{2}\mu_{\phi}(\lambda) + \frac{1}{2}\mu_{\bar{\phi}}(\lambda) = \frac{1}{2}\mu_{\psi}(\lambda) + \frac{1}{2}\mu_{\bar{\psi}}(\lambda)$$



Observing data which violates any noncontextuality inequality is a proof of nonclassicality.



Observing data which violates any noncontextuality inequality is a proof of nonclassicality.

Such proofs don't rely on the correctness of quantum theory

Why is noncontextuality required for a good explanation?

41

Leibniz's principle of the identity of indiscernibles—
if a difference in set-up is not distinguished in the observable phenomena then it should not be distinguished in the ontological picture either

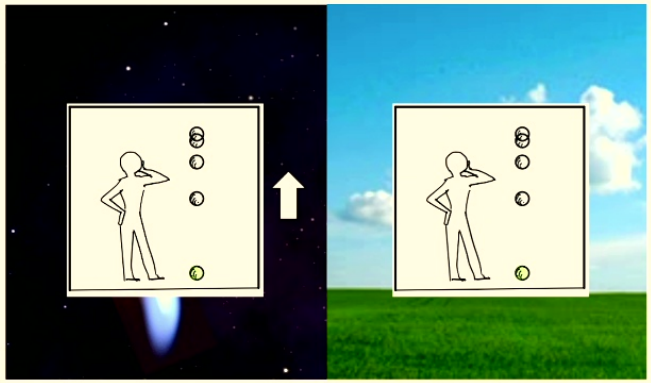
This is a methodological principle which guides us in constructing good physical theories

arxiv:1909.04628

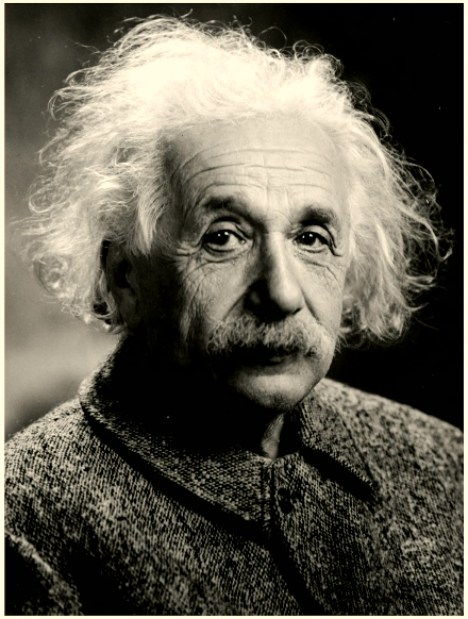
42

Leibniz's principle in action

Einstein's arguments against the ether



Einstein's strong equivalence principle



arxiv:1909.04628

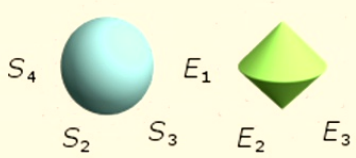
43

Classicality in the framework of Generalized Probabilistic Theories

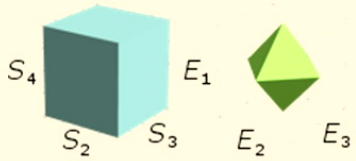
Different theories are defined by their:

1. Convex geometry

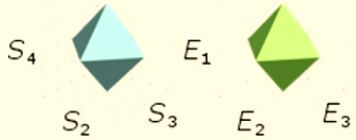
qubit



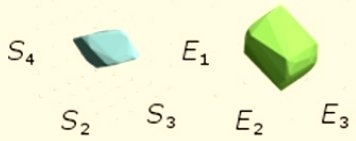
Boxworld (3d)



Spekkens toy theory



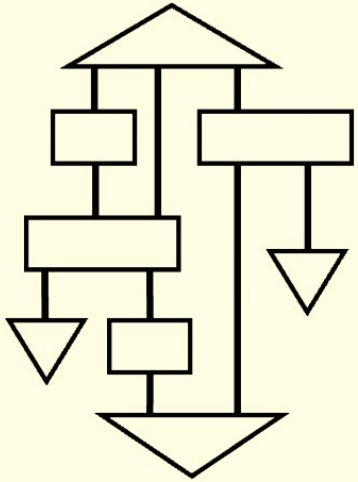
random GPT



probabilities = inner products

2. Compositional structure

-multipartite states
-multipartite effects
-$T_1(T_2)=T_3$
-etc

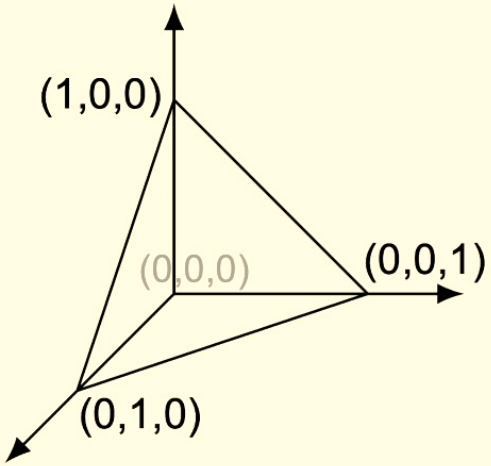


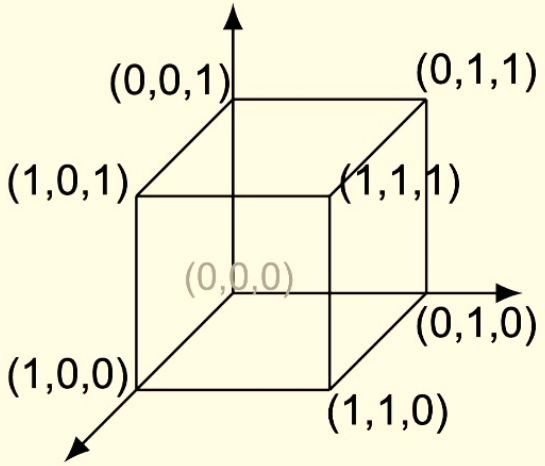
Traditionally, a GPT has been considered "classical" if it is *simplicial*:

state space: simplex

effect space: dual of simplex

d=3

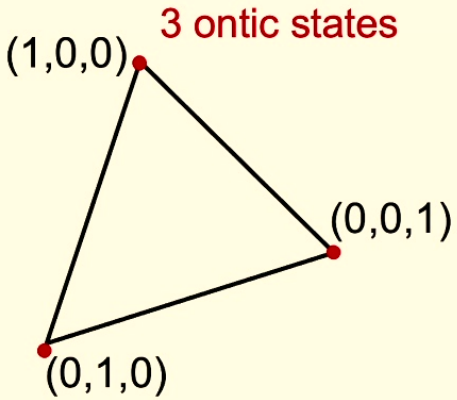


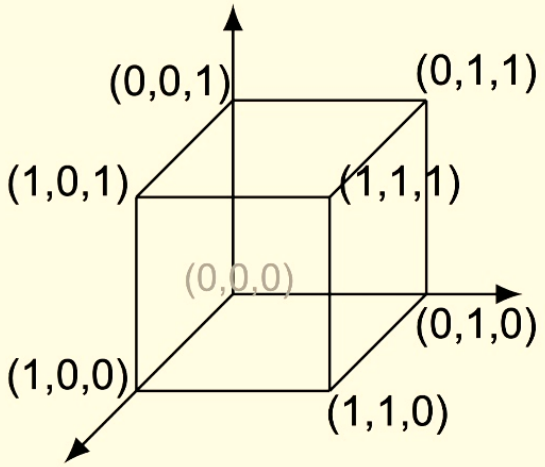


normalized states

effects

d=3



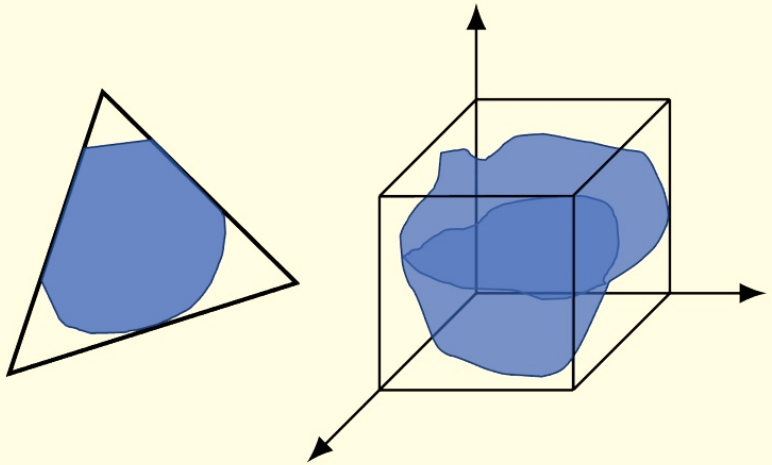


Classical statistical theory: probability distributions over a set of classical states

49

simplicial $\Rightarrow$ classical

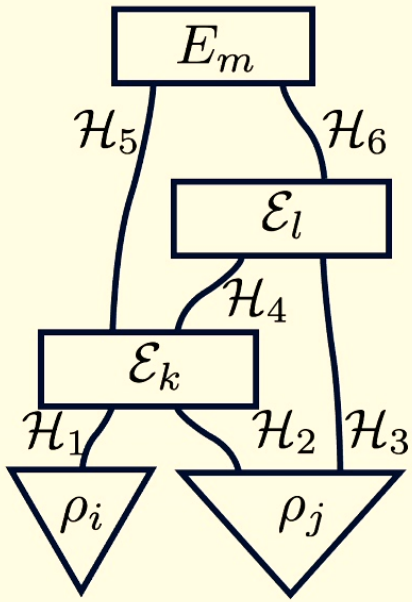
But what about subtheories of a simplicial theory?



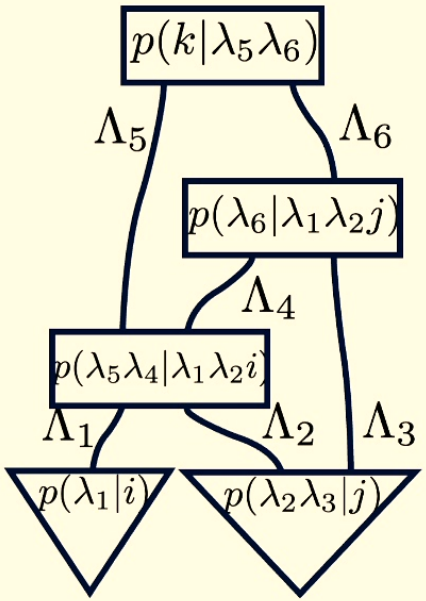
simplicial $\Rightarrow$ classical

simplex-embeddable $\Leftrightarrow$ classically explainable

The simplest view of a noncontextual explanation:



Linear Map



quantum gates
quantum systems

(sub)stochastic maps
random variables

https://arxiv.org/pdf/1911.10386v2.pdf

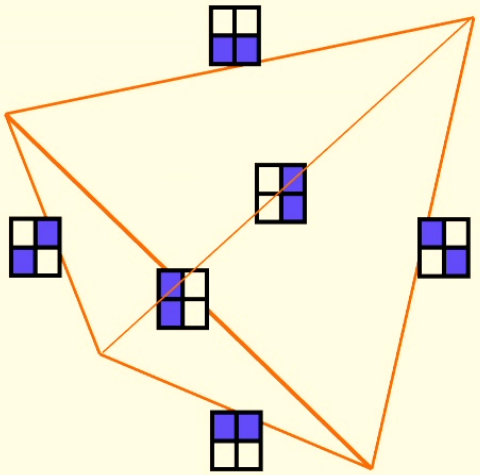
Examples of nonclassical phenomena

Minimum error state discrimination

In quantum theory there is no error-free discrimination of non-orthogonal states.

Some have claimed this is evidence of nonclassicality.

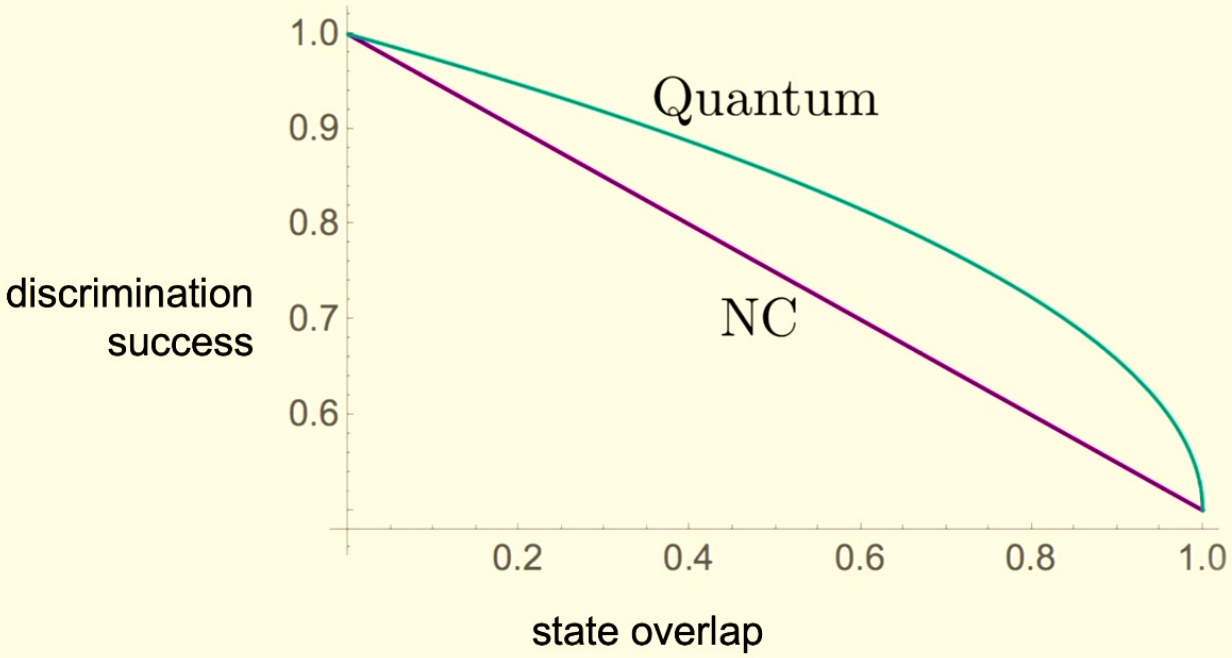
But this is easy to explain classically!



https://arxiv.org/abs/1706.04588

56

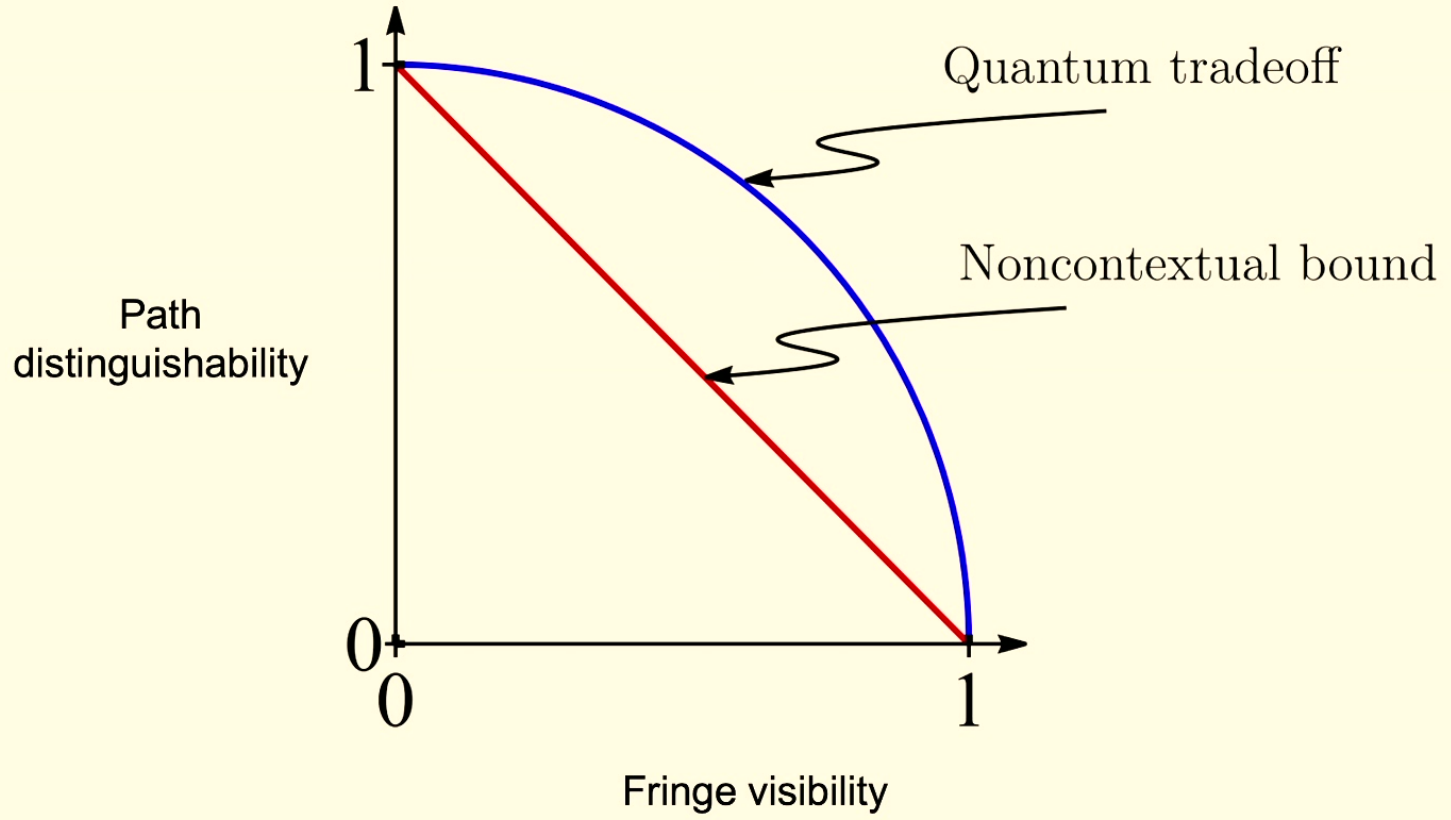
Minimum error state discrimination



https://arxiv.org/abs/1706.04588

57

Interference



https://arxiv.org/pdf/2211.09850

Suggested references:

Basic definition of generalized noncontextuality:
https://arxiv.org/abs/quant-ph/0406166

Noncontextuality in the GPT framework:
https://arxiv.org/pdf/1911.10386v2.pdf

NC beyond prepare and measure scenarios:
https://arxiv.org/pdf/2005.07161.pdf

Deriving all the noncontextuality inequalities:
https://arxiv.org/pdf/1710.08434.pdf

A linear program for testing simplex-embeddability:
https://arxiv.org/pdf/2204.11905

Experimental tests of noncontextuality:
https://arxiv.org/abs/1710.05948

Going beyond the ontological models framework:
https://arxiv.org/pdf/2009.03297.pdf

GPT shadows:
https://arxiv.org/abs/2112.09719

Thanks for your attention!

62