Title: Lecture - Gravitational Physics, PHYS 636

Speakers: Ruth Gregory

Collection/Series: Gravitational Physics (Elective), PHYS 636, January 6 - February 5, 2025

Subject: Cosmology, Strong Gravity

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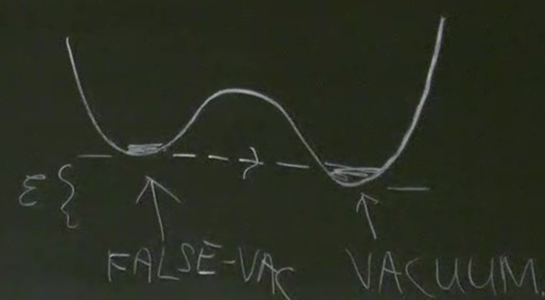
LECTURE 13 Gravitational Instantons

Tur

Coleman "Fate of the false vacuum"

PRD <u>15</u> 2929 (77)

Coleman de Luccia PRD <u>21</u> 3305 (80)



ational Instantons

"false vacuum"

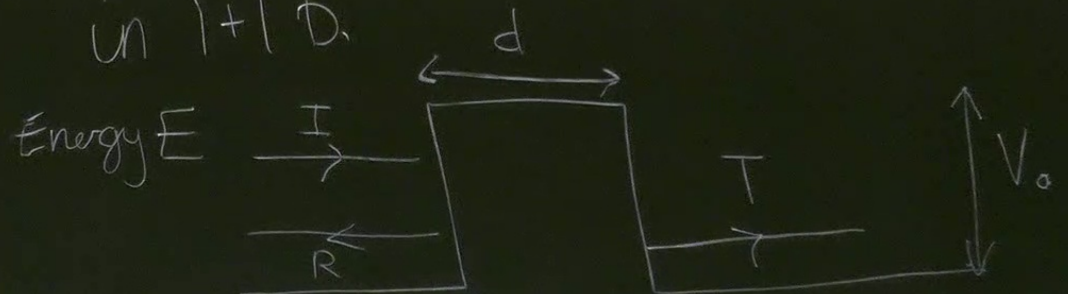
2929 (77)

21 3305 (80)

VACUUM.

Tunnelling is perhaps the most startling phenomenon in quantum physics.

- Recall Schrödinger tunneling in 1+1 D.



...ps the

...omenon

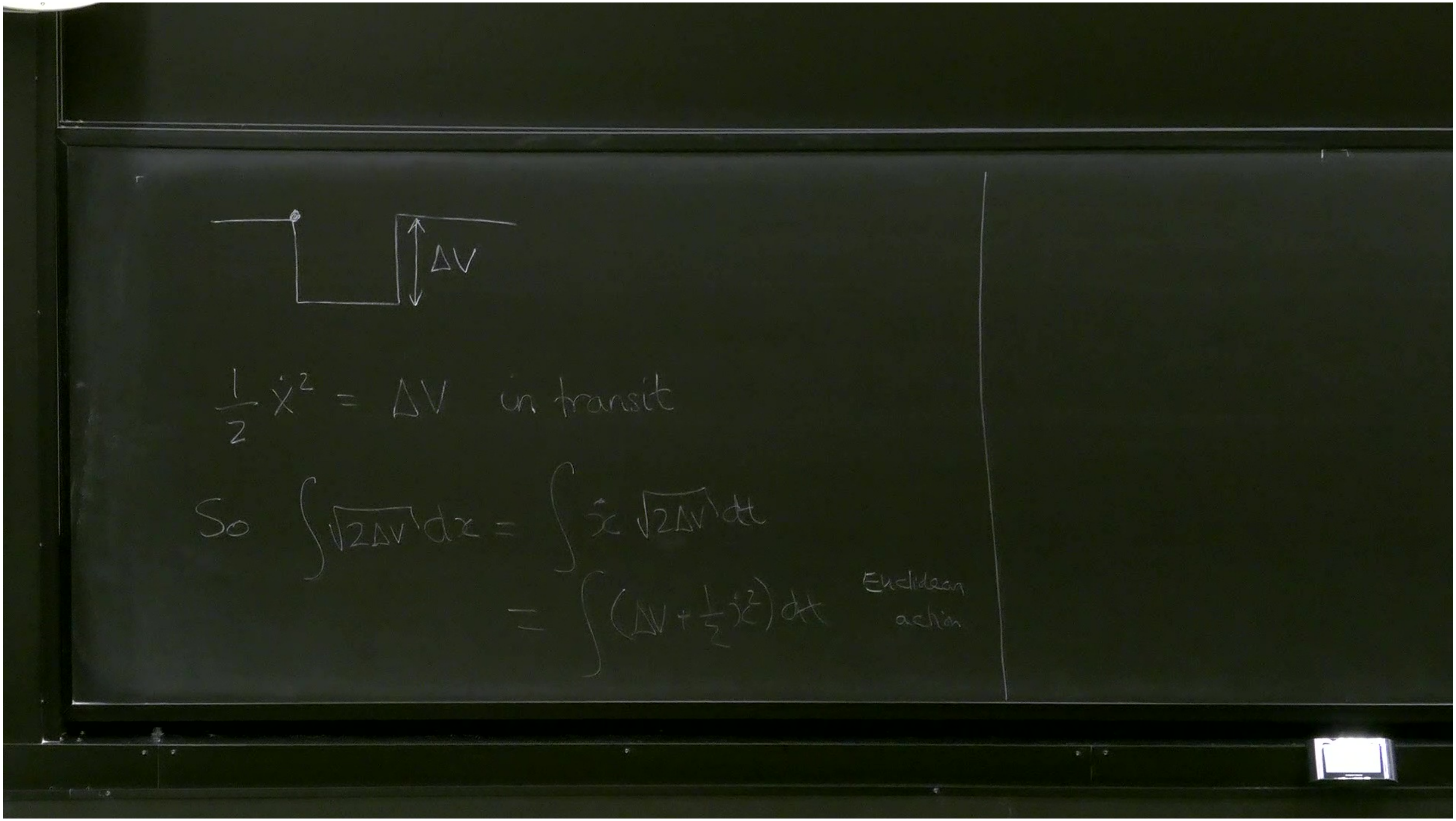
...r tunneling

Find $|T|^2 = \dfrac{1}{1 + \dfrac{V_0^2 \sinh^2 \Omega d}{4E(V_0 - E)}} \sim e^{-2\Omega d}$

where $\Omega^2 = \dfrac{2m}{\hbar^2}(V_0 - E) \sim 2\Delta V$

$$\Omega d = \int_0^d \sqrt{2\Delta V}\, dx$$

Consider a different problem – classical particle


ΔV
$\frac{1}{2}\dot{x}^2 = \Delta V$ in transit
So $\int \sqrt{2\Delta V}\, dx = \int \dot{x} \sqrt{2\Delta V}\, dt$
$= \int \left(\Delta V + \frac{1}{2}\dot{x}^2\right) dt$
Euclidean action

uclidean
action

In general, $P \propto e^{-S_E}$ where S_E is the action of a Euclidean trajectory from initial state to emergence (+ back again - the bounce)

ere

lidean

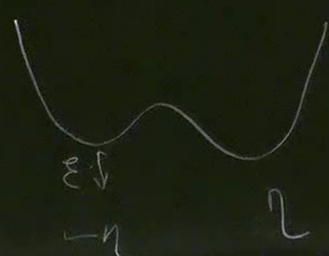
o

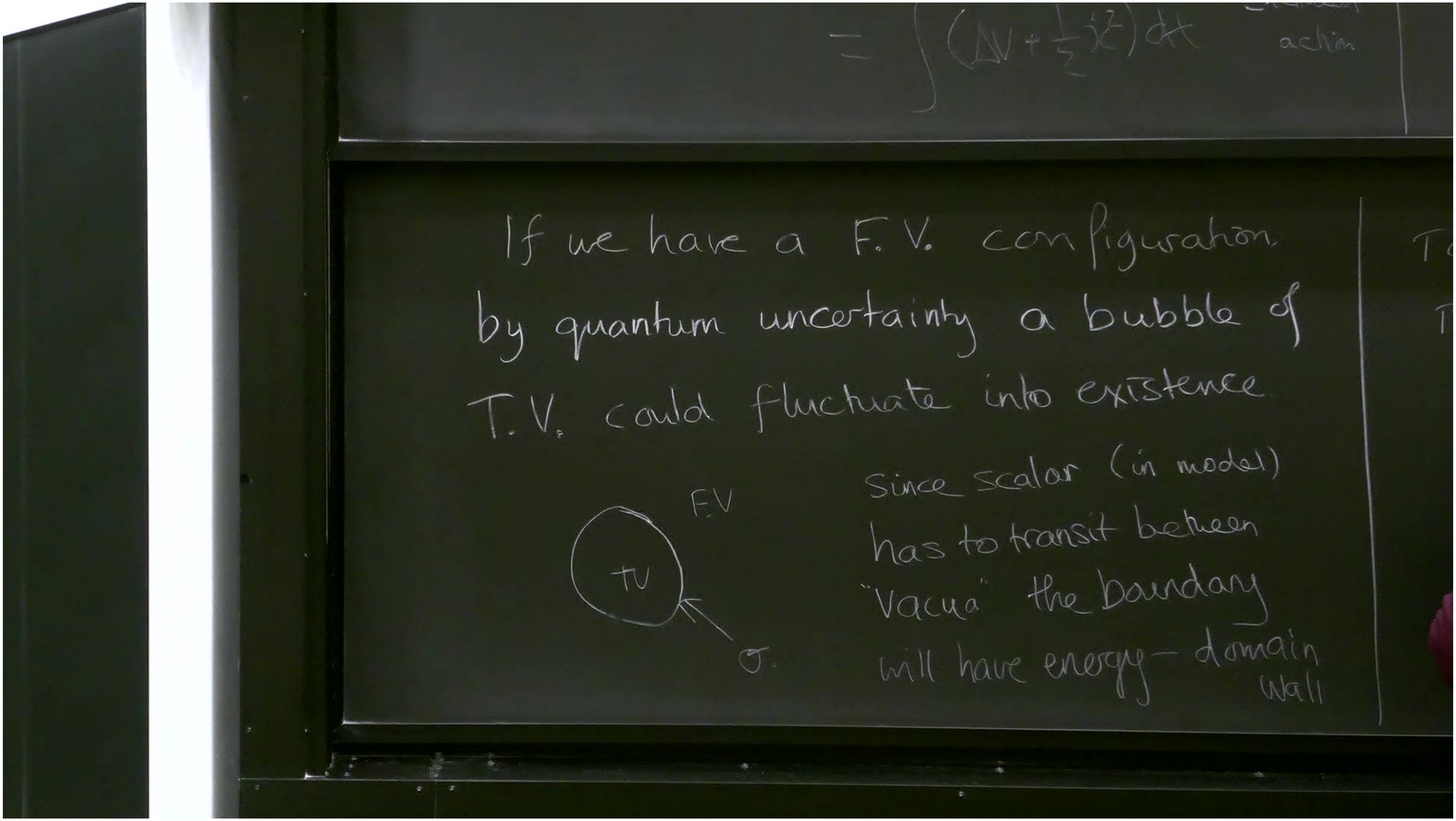
bounce)

Translating to field theory, are looking for Euclidean solns that interpolate (in some sense) between false & true vacua.

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V = \frac{\lambda}{2}(\phi^2-\eta^2)^2 - \frac{\epsilon}{2\eta}(\phi-\eta)$$





$= \int \left(\Delta V + \frac{1}{2}\dot{x}^2\right) dt$
action
If we have a F. V. configuration
by quantum uncertainty a bubble of
T. V. could fluctuate into existence
F.V
TV
σ.
Since scalar (in model)
has to transit between
"vacua" the boundary
will have energy — domain
wall

...nfiguration
...a bubble of
...o existence
...lar (in model)
...ansit between
...the boundary
...energy – domain wall

TOO SMALL – contracts

TOO BIG – larger "cost"

In Euclidean space

Energy in wall: $2\pi^2 R^3 \sigma$

Difference in vacuum energy: $\frac{\pi^2}{2} \varepsilon R^4$

$\delta E = 2\pi^2 \ldots$

$\frac{d}{dR} \delta E = 6\pi \ldots$

$$\delta E = 2\pi^2 R^3 \sigma - \frac{\pi^2}{2} \varepsilon R^4$$

$$\frac{d}{dR}\delta E = 6\pi^2 R^2 \sigma - 2\pi^2 R^4 \varepsilon = 0 \text{ at } R_0 = \frac{3\sigma}{\varepsilon}$$

Scalar model : $\phi'' + \frac{3\phi'}{\rho} \simeq 2\lambda\phi(\phi^2 - \eta^2)$

$$\phi \simeq \eta \tanh[\sqrt{\lambda}\eta(\rho - R_0)]$$

$$\sigma = \frac{4}{3}\sqrt{\lambda}\eta^3$$

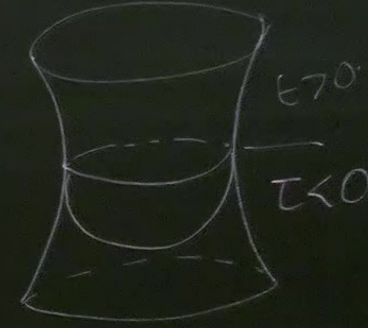
Action: $B = \frac{\pi^2 R_0^3}{2}(4\sigma - \varepsilon R_0)$

$$= \frac{27\pi^2}{2}\,\frac{\sigma^4}{\varepsilon^3}$$

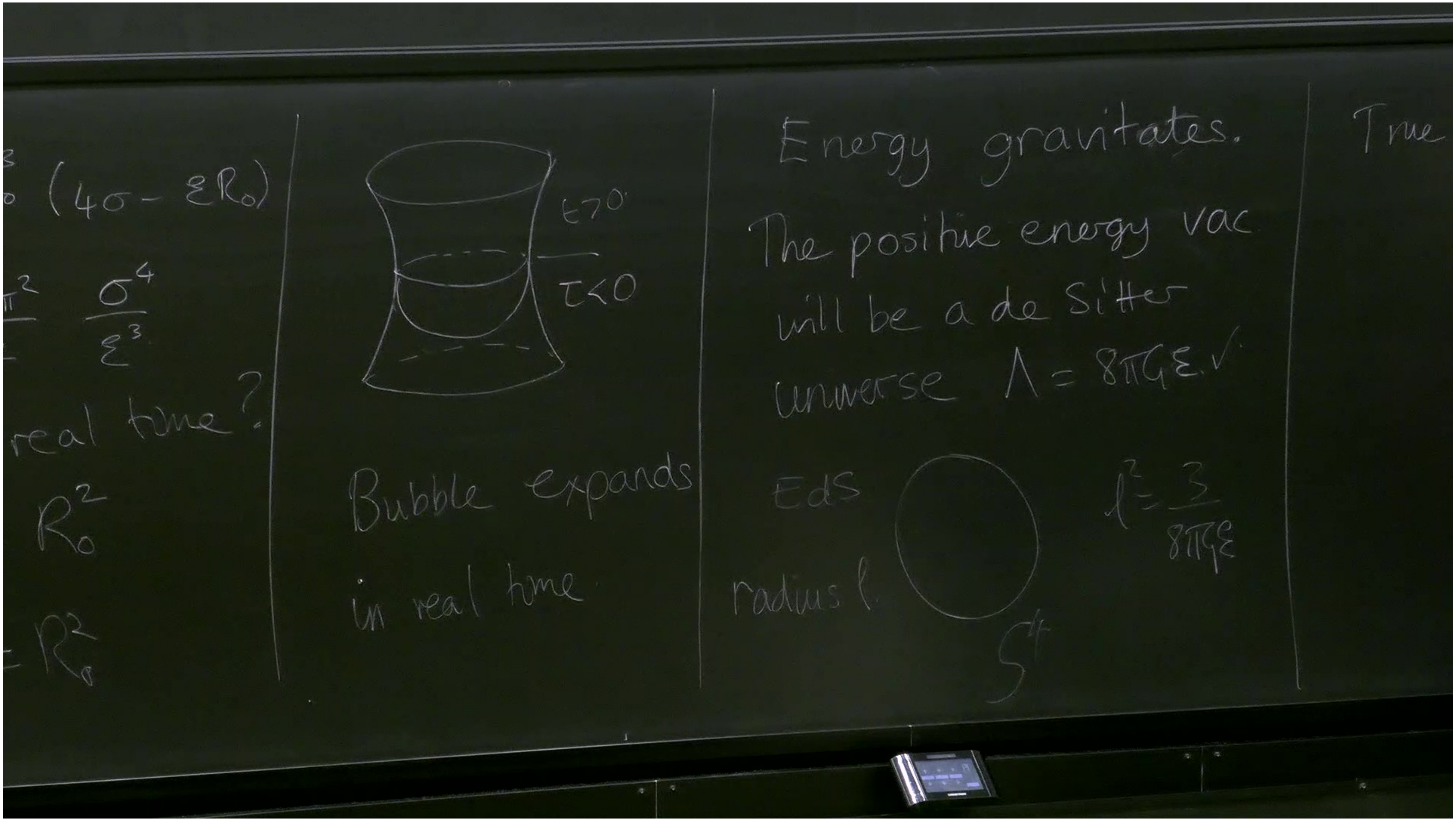
What happens in real time?

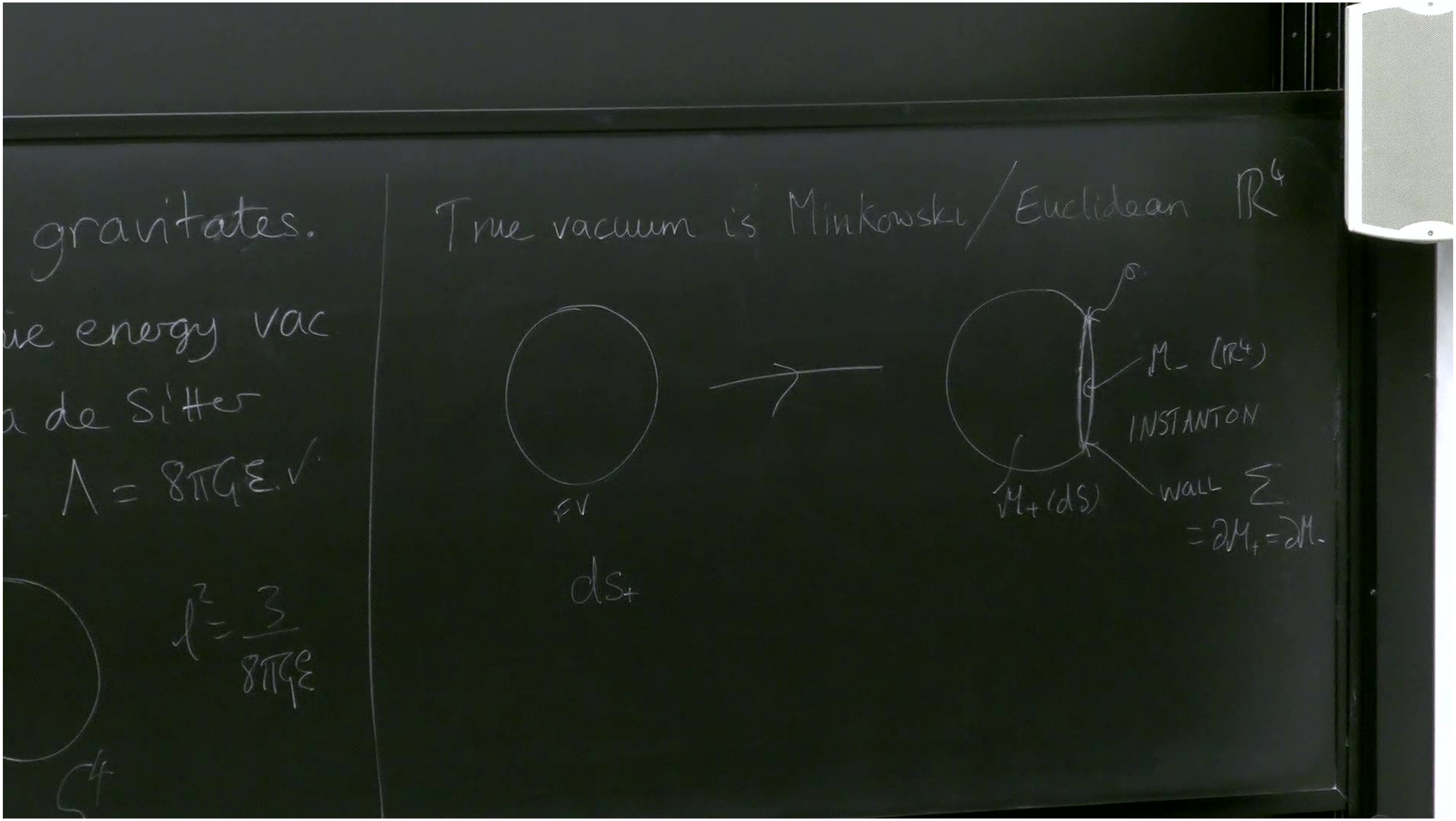
$$r^2 + \tau^2 = R_0^2$$

$$\Rightarrow r^2 - t^2 = R_0^2$$



Bubble expands in real time


$(4\sigma - \varepsilon R_0)$
$\frac{\sigma^4}{\varepsilon^3}$
real time?
R_0^2
εR_0^2
$t > 0$
$\tau < 0$
Bubble expands
in real time
Energy gravitates.
The positive energy vac
will be a de Sitter
universe $\Lambda = 8\pi G \varepsilon_V$
EdS
radius l.
$l^2 = \frac{3}{8\pi G \varepsilon}$
S^4
True


gravitates.
ue energy vac
a de Sitter
Λ = 8πGε.
l² = 3 / 8πGε
S⁴
True vacuum is Minkowski / Euclidean ℝ⁴
FV
dS+
σ
M− (ℝ⁴)
INSTANTON
M+ (dS)
WALL Σ
= ∂M+ = ∂M−

gravitates.

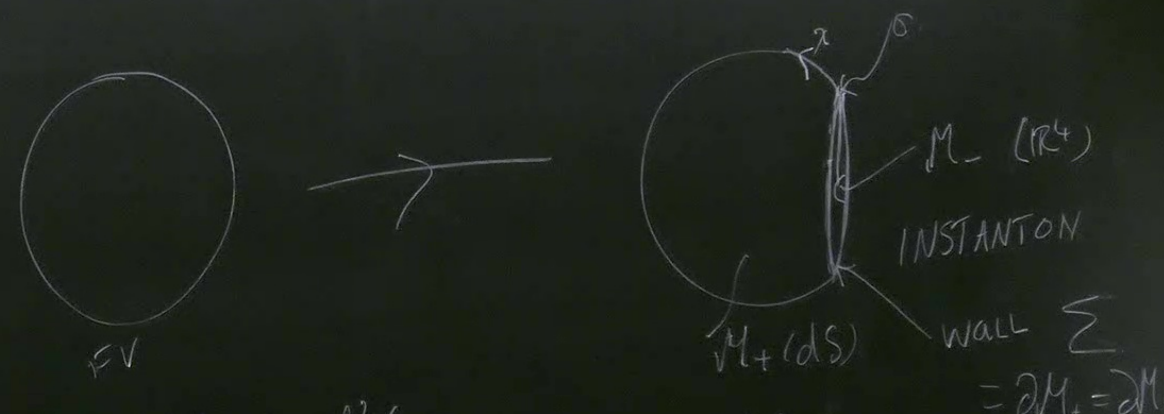
...ue energy vac

...a de Sitter

$\Lambda = 8\pi G \mathcal{E}$ ✓

$\ell^2 = \frac{3}{8\pi G \mathcal{E}}$

True vacuum is Minkowski / Euclidean $\mathbb{R}^4$



(dS) $$ds_+^2 = \ell^2\left(d\chi^2 + \sin^2\chi \, d\Omega_{III}^2\right)$$

($\mathbb{R}^4$) $$ds_-^2 = d\rho^2 + \rho^2 d\Omega_{III}^2$$

Match at $\rho_0 = R_0 = \ell \sin\chi_0$ (Σ)

$$n_+ = \frac{1}{\ell}\frac{\partial}{\partial\chi} \qquad n_- = \frac{\partial}{\partial\rho}$$

$$K_{+\alpha\beta} = \frac{1}{\ell}\Gamma^{\chi}_{\alpha\beta} = -\frac{1}{2\ell} g^{\chi\chi} g_{\alpha\beta,\chi} = -\frac{\cot\chi_0}{\ell} g_{\alpha\beta}$$

$$K_{-\alpha\beta} = \Gamma^{\rho}_{\alpha\beta} = -\frac{1}{\rho_0} g_{\alpha\beta}$$

$\Delta K_{\alpha\beta} - \Delta K\, h_{\alpha}$

(Σ)

$$\Delta K_{\alpha\beta} - \Delta K\, h_{\alpha\beta} = -\frac{2 h_{\alpha\beta}}{\ell \sin\chi_0}(1-\cos\chi_0) = -8\pi G \sigma h_{\alpha\beta}.$$

$g_{\alpha\beta}.$

$$\Rightarrow \frac{(1-\cos\chi_0)}{\ell\sin\chi_0} = \frac{\left(1-\sqrt{1-R^2/\ell^2}\right)}{R} = 4\pi G \sigma.$$

$$B = I_{INST} - I_{BACKGRD}$$

$$= \frac{1}{16\pi G}\int_{\chi<\chi_0} \sqrt{g}\,(\underbrace{R-2\Lambda}_{2\Lambda}) - \frac{1}{8\pi G}\int_{\Sigma} \Delta K \sqrt{h}$$

$$\Delta K_{\alpha\beta} - \Delta K\, h_{\alpha\beta} = -\frac{2 h_{\alpha\beta}}{\ell \sin\chi_0}(1-\cos\chi_0) = -8\pi G\sigma h_{\alpha\beta}.$$

$$\Rightarrow \frac{(1-\cos\chi_0)}{\ell\sin\chi_0} = \frac{\left(1-\sqrt{1-R^2/\ell^2}\right)}{R} = 4\pi G\sigma.$$

$$B = I_{INST} - I_{BACKGRD}$$

$$= \frac{1}{16\pi G}\int_{\chi<\chi_0}\sqrt{g}\,\underbrace{(R-2\Lambda)}_{2\Lambda} - \frac{1}{8\pi G}\int_{\Sigma}\Delta K\sqrt{h} + \int_{\Sigma}\sigma\sqrt{h}$$

with $\Delta K\sqrt{h} \to 12\pi G\sigma$

$$= \frac{3}{8\pi G\ell^2}\cdot 2\pi^2\ell^4\int_0^{\chi_0}\sin^2\chi\, d\chi - \frac{\sigma}{2}\cdot 2\pi^2 R^3$$

$$= \frac{3\pi\ell^2}{4G}\left[\frac{2}{3} - \cos\chi_0 + \frac{1}{3}\cos^3\chi_0\right]$$

$$-\frac{\pi^2 R^{2/3}}{4\pi G}\,\frac{(1-\cos\chi_0)}{\cancel{R}}$$

$$= \frac{\pi\ell^2}{4G}(1-\cos\chi_0)\left[2-\cos\chi_0-\cos^2\chi_0 - \sin^2\chi_0\right]$$

$$\Delta K_{\alpha\beta} - \Delta K\, h_{\alpha\beta} = -\frac{2 h_{\alpha\beta}}{\ell \sin\chi_0}(1-\cos\chi_0) = -8\pi G \sigma h_{\alpha\beta}.$$

$$\Rightarrow \frac{(1-\cos\chi_0)}{\ell \sin\chi_0} = \frac{\left(1-\sqrt{1-R^2/\ell^2}\right)}{R} = 4\pi G\sigma.$$

$$B = I_{INST} - I_{BACKGRD}$$

$$= \frac{1}{16\pi G}\int_{\chi<\chi_0} \sqrt{g}\,\underbrace{(R-2\Lambda)}_{2\Lambda} - \frac{1}{8\pi G}\int_{\Sigma} \overset{12\pi G\sigma}{\Delta K}\sqrt{h} + \oint_{\Sigma}\sigma\sqrt{h}$$

$$= \frac{3}{8\pi G \ell^2}\cdot 2\pi^2 \ell^4 \int_0^{\chi_0} \sin^3\chi\, d\chi - \frac{\sigma}{2}\cdot 2\pi^2 R^3$$

$$= \frac{3\pi \ell^2}{4G}\left[\frac{2}{3} - \cos\chi_0 + \frac{1}{3}\cos^3\chi_0\right]$$

$$-\frac{\pi^2 R^{2/3}}{4\pi G}\frac{(1-\cos\chi_0)}{\cancel{R}}$$

$$= \frac{\pi\ell^2}{4G}(1-\cos\chi_0)\left[\frac{2-\cos\chi_0-\cos^2\chi_0}{\sin^2\chi_0}\right]$$

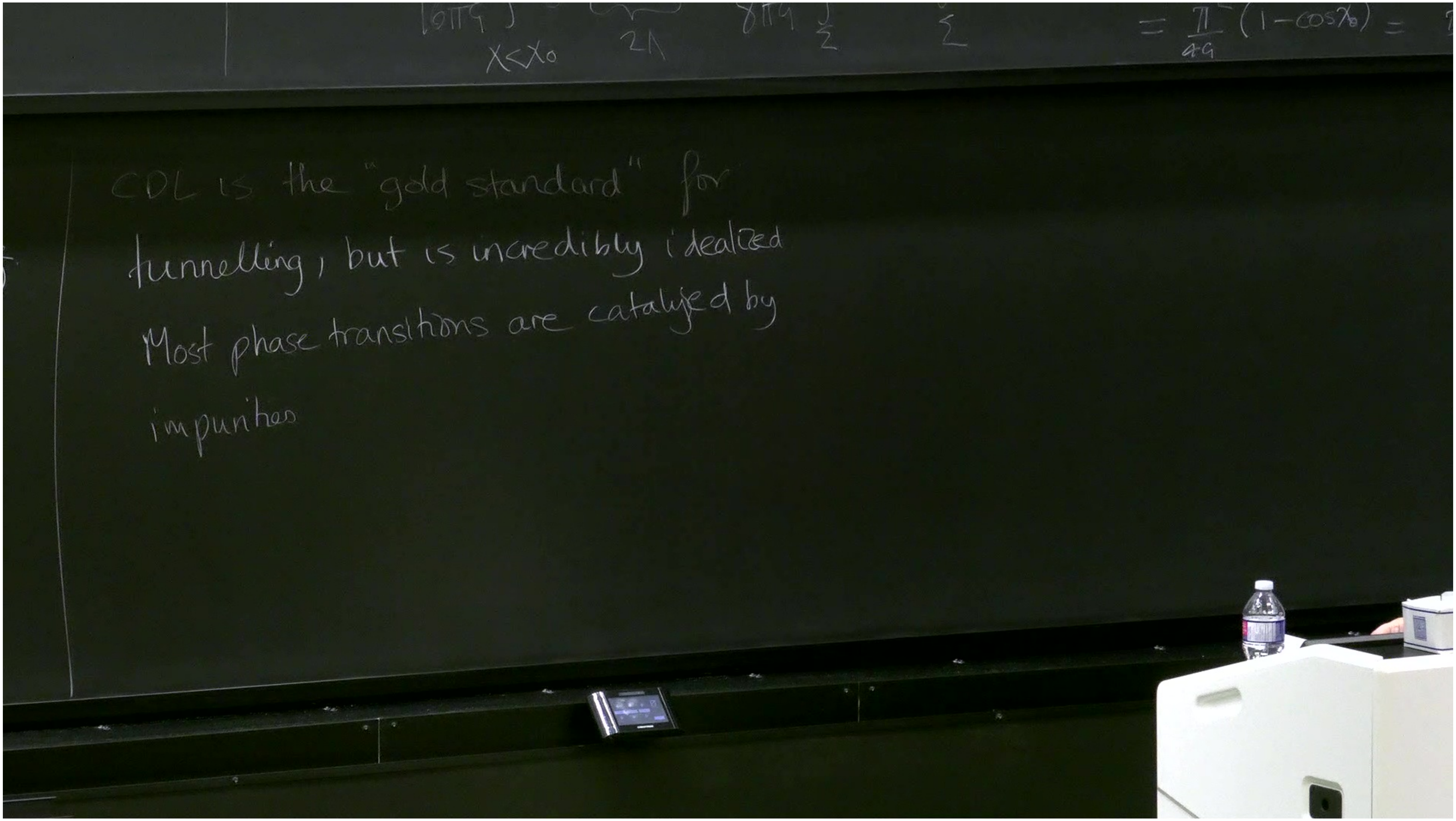
$$= \frac{\pi^2}{4G}(1-\cos\chi_0)^2 = \frac{\pi\ell^2}{4G}(4\pi G\sigma R)^2.$$

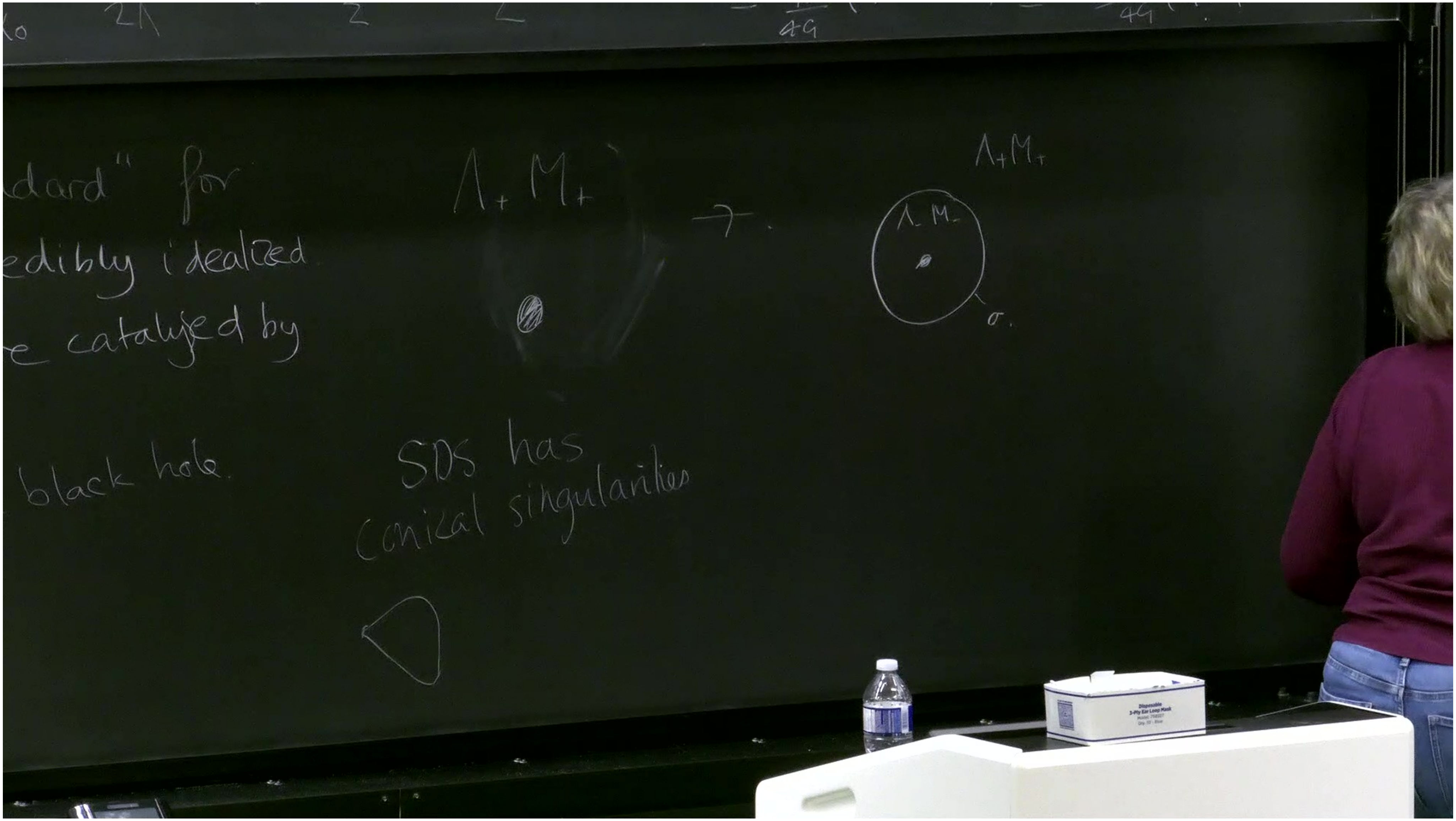
$$K_{\alpha\beta} = \Gamma^{\rho}_{\alpha\beta} = -\frac{1}{\rho_0} g_{\alpha\beta}$$

$$= \frac{1}{16\pi G}\int_{\chi<\chi_0} \sqrt{g}\,(\underbrace{R-2\Lambda}_{2\Lambda}) -$$

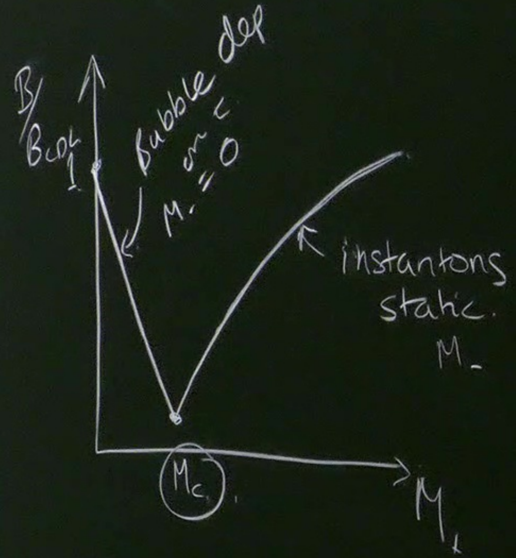
Then find $\sin\chi_0 = \dfrac{2(4\pi G\sigma\ell)^2}{1+(4\pi G\sigma\ell)^2}$

$$\propto B = \frac{\pi\ell^2}{G}\,\frac{(4\pi G\sigma\ell)^4}{\left[1+(4\pi G\sigma\ell)^2\right]^2}$$


CDL is the "gold standard" for
tunnelling, but is incredibly idealized
Most phase transitions are catalyzed by
impurities



dard" for
edibly idealized
e catalyzed by
black hole.
SDS has
conical singularities
Λ+ M+
Λ- M-
σ.



M_c Static

$M_- = 0$

Range of M_- for M_+

Lowest action dominant