**Title:** Lecture - Standard Model, PHYS 622

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## Strong CP Problem

$$\mathcal{L}_{SM} \supset -\frac{g_3^2\,\theta_3}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - \frac{g_2^2\,\theta_2}{32\pi^2} W^a_{\mu\nu}\tilde{W}^{a\mu\nu} - \frac{g_1^2\,\theta_1}{32\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu}$$

$$\tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\beta}G^{a\lambda\beta}$$

$$\frac{g_3^2}{32\pi} G^a_{\mu\nu} G^a_{\mu\alpha} = \partial_\mu K^\mu \qquad \text{where } K^\mu = \frac{g_3^2}{32\pi^2}\epsilon^{\mu\nu\delta\kappa}\left(G^a_\nu G^a_\delta \ldots\right.$$

$$\underbrace{-\frac{g_2^2\theta_2}{32\pi^2} W^a_{\mu\nu}\tilde{W}^{a\mu\nu} - \frac{g_1^2\theta_1}{32\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu}}_{\text{ignore these terms}}$$

where $$K^\mu = \frac{g_3^2}{32\pi^2}\epsilon^{\mu\nu\delta\kappa}\left(G^\alpha_\nu G^\alpha_{\delta\kappa} - \frac{g_3}{3} f_{\alpha\beta\gamma} G^\alpha_\nu G^\beta_\delta G^\gamma_\kappa\right)$$

$$\frac{g_3^2}{32\pi^2}\int d^3x \int_{t_i}^{t_f} dt\; G^{\alpha}_{\mu\nu}\tilde{G}^{\alpha\mu\nu} = \int d^3x\; K^0(x,t_f) - \int d^3x\; K^0(x,t_i) = N_{CS}(t_f) - N_{CS}(t_i)$$

$\int d^3x\; K^0 = N_{CS}$ Chern-Simons term (integers)

There exist gauge field configurations $(G^{\alpha}_{\mu})$ for which $G^{\alpha}_{\mu\nu}=0$ but $N_{CS}\neq 0$

$$\frac{g_3^2}{32\pi^2}\int d^3x \int_{t_i}^{t_f} dt\, G^{\alpha}_{\mu\nu}\tilde{G}^{\alpha\mu\nu} = \int d^3x\, K^0(x,t_f) - \int d^3x\, K^0(x,t_i) = N_{CS}($$

$$\int d^3x\, K^0 = N_{CS}$$ Chern-Simons

There exist gauge field configurations $(G^{\alpha}_{\mu})$ for which $G^{\alpha}_{\mu\nu} = 0$

A gauge field configuration $G^{\alpha}_{\mu}(x,t)$ w/ the property : $N_{CS}(t_f$

$$\int d^3x\, K^0(x,t_f) - \int d^3x\, K^0(x,t_i) = N_{CS}(t_f) - N_{CS}(t_i)$$

$\int d^3x\, K^0 = N_{CS}$ " Chern-Simons term (integers)

$(G^a_{\mu\nu})$ for which $G^a_{\mu\nu} = 0$ but $N_{CS} \neq 0$.

) w/ the property : $N_{CS}(t_f \gg 0) - N_{CS}(t_i \ll 0) = \underline{1}$ instantons.

A gauge field configuration $G^{a}_{\mu}(x,t)$ w/ the property

Vacuum is made up of degenerate states w/ different CS numbers.

$$N_{CS}|0,a\rangle = N_{CS}(a)|0,a\rangle$$

$$N_{CS}|0,b\rangle = N_{CS}(b)|0,b\rangle$$

Vacuum-to-vacuum transition.

$$\langle a|e^{-\beta H}|b\rangle = \langle a|e^{-\beta}$$

A gauge field configuration

⇒ Vacuum is made up of degenerate states w/ different CS numbers:

$$N_{CS}|0,a\rangle = N_{CS}(a)|0,a\rangle$$

$$N_{CS}|0,b\rangle = N_{CS}(b)|0,b\rangle$$

$$\boxed{|0\rangle_\theta = \sum_n e^{i\theta n}|n\rangle}$$

theta vacuum

vacuum-to-vacuum t

Vacuum

$$\langle a|e^{-\beta H}|b\rangle =$$

w/ the property : $N_{CS}(\ldots)$

w/ different CS numbers.

vacuum-to-vacuum transition.

$$\langle a| e^{-\beta H} |b\rangle = \langle a| e^{-\beta H} |b\rangle_{\theta_3 = 0} \, \underbrace{e^{-i\beta\theta_3 (N_{CS}(a) - N_{CS}(b))}}_{\text{phase shift due to}}$$

$\theta_3 G\tilde{G}$ : P and T violating
C and CPT conserving.

$$j^\mu_{q5} = i\bar{q}\,\gamma^\mu \gamma_5\, q$$

is not conserved because 1. quarks are massive

2. anomalies

assume massless quarks: $\partial_\mu j^\mu_{q5} = \frac{g_s^2}{16\pi^2} G^\alpha_{\mu\nu} \tilde{G}^{\alpha\mu\nu} \Rightarrow Q_{q5} = \int d^3x\, j^0_{q5} \Rightarrow$

$$Q_{q5} = \int d^3x \, j^0_{q5} \implies Q_{q5}(t_f) - Q_{q5}(t_i) = 2\Big[\underbrace{N_{cs}(t_f) - N_{cs}(t_i)}_{=1 \text{ for instanton processes}}\Big]$$

axial charge changes by 2

not vacuum anymore!

When all quarks are massive : $m_q \bar{q}_L q_R$ : allows you to annihilate the $q$-$\bar{q}$ pair

$$\bar{\Theta} = \Theta_3 + \sum_q \mathrm{Arg}(m_q) \Longrightarrow$$

this is the physical quantity

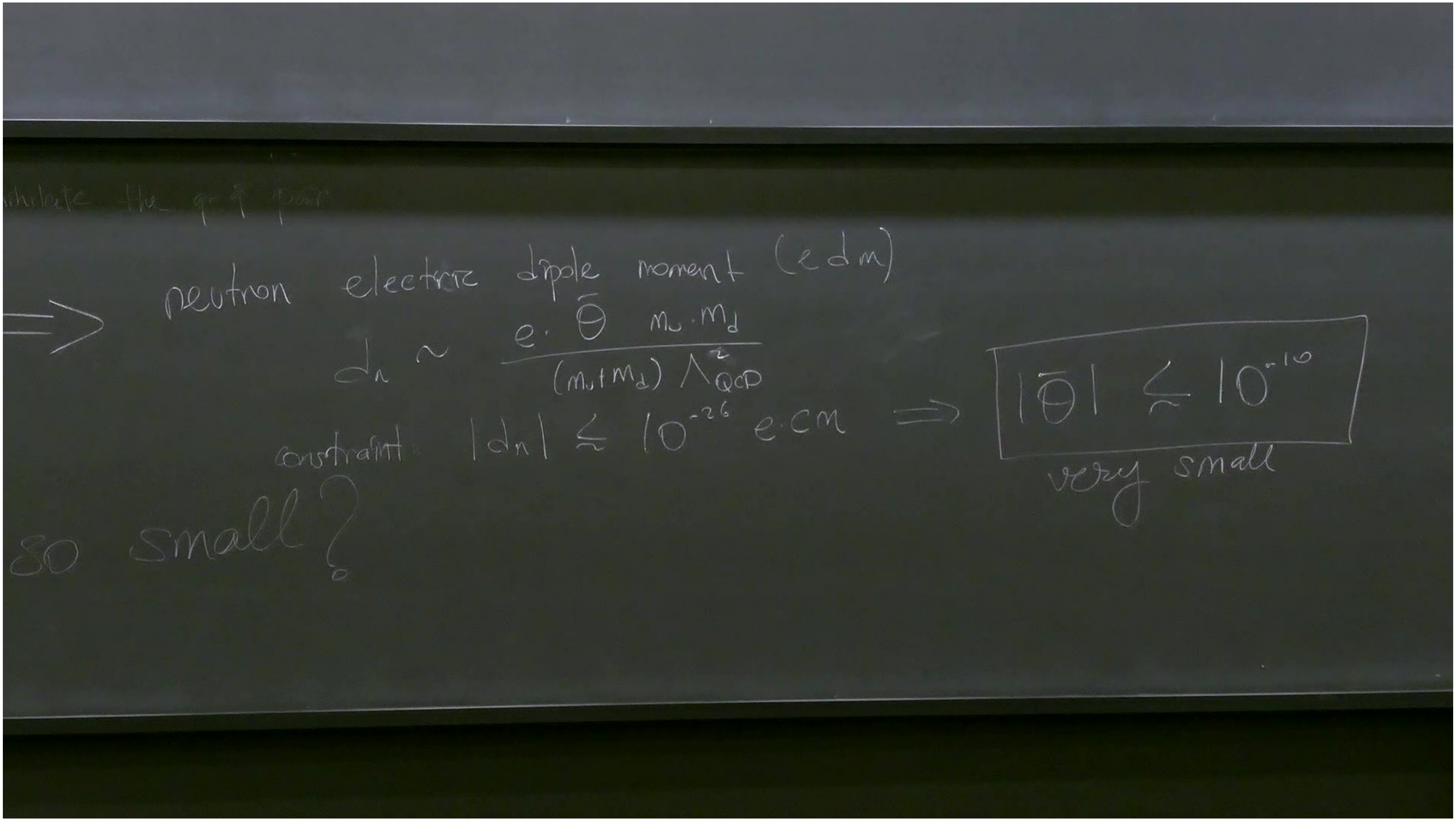
neutron electric

$d_n \sim$

constraint: $|d$

Why is $\bar{\Theta}$ so small?

The axion solution



...nihilate the $q$-$\bar{q}$ pair
$\Longrightarrow$ neutron electric dipole moment (edm)
$d_n \sim \frac{e \cdot \bar{\Theta} \; m_u \cdot m_d}{(m_u + m_d)\, \Lambda_{QCD}^2}$
constraint: $|d_n| \lesssim 10^{-26}\ e \cdot cm \Longrightarrow \boxed{|\bar{\Theta}| \lesssim 10^{-10}}$
very small
so small?

# The axion solution

(KSVZ model
DFSZ model)

Consider a complex field $\sigma$ w/ a global symmetry $U_{PQ}(1)$

small ?

symmetry $U_{PQ}(1)$ w/ a SSB potential.

$\frac{v_{PQ}^2}{2})^2$ $\quad$ $\sigma$: SM singlet.

$\Rightarrow$ after the SSB for $U(1)$

$\rho$ gets a mass.

$a(x)$ is massless. (NGB).

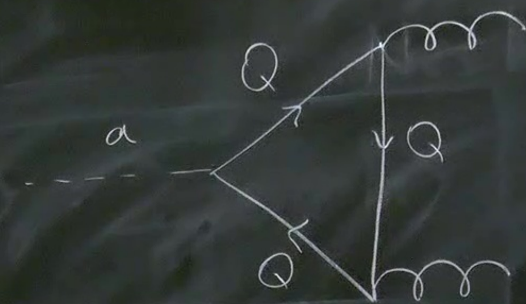
$v_{PQ}) \, e^{ia(x)/v_{PQ}}$ $\quad$ $v_{PQ} \gg v_{EW}$

A gauge field configuration

SVZ

Add quarks charged under $U_{PQ}(1)$

$$\mathcal{L}_{KSVZ} \supset y\,\sigma\,\bar{Q}_L Q_R + h.c.$$




axion

$$\Rightarrow \mathcal{L}_{eff} \supset \frac{1}{2}\partial_\mu a\,\partial^\mu a + \frac{\alpha_s}{8\pi}\frac{a}{f_a} G^{\alpha}_{\mu\nu}\tilde{G}^{\alpha\mu\nu}$$

$$f_a = \frac{v_{PQ}}{N_Q}$$ : axion scale.

solution

Consider a complex field $\sigma$ w/ a global symmetry $U_{PQ}(1)$ w/ a SSB potential

$$V_\sigma = -\lambda_\sigma \left( |\sigma|^2 - \frac{v_{PQ}^2}{2} \right)^2$$

$\sigma$: SM singlet.

$$\sigma = \frac{1}{\sqrt{2}} \left( \rho(x) + v_{PQ} \right) e^{i a(x)/v_{PQ}}$$

$v_{PQ} \gg v_{EW}$

$U(1)$ transf:

Axion-SM interactions generate a potential for the axion

(shift symm. will be broken $\Rightarrow$ axion is actually a pNGB.)

- Calculate the potential
- Calculate the energy shift between vacua: $|0, a_1\rangle \rightarrow |0, a_2\rangle$

$\rightarrow$ minimizing this potential picks $\langle a \rangle = -\bar{\theta} f_a$

al for the axion. $\Rightarrow V(a)$

a pNGB.)

$a_1\rangle \to |0, a_2\rangle$

$\overline{\theta} f_a$

$$m_a^2 = \left(\frac{\partial^2 V}{\partial a^2}\right) = \frac{\chi}{f_a^2}$$

where $\chi \simeq \frac{m_u m_d}{m_u + m_d} \Lambda_{QCD}^3$ topological susceptibility.

$$\boxed{m_a f_a \simeq m_\pi f_\pi}$$

QCD axion

A gauge field configuration

$$\mathcal{L}_{axion} \supset -\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu} \Longrightarrow g_{a\gamma\gamma}\, a \, \vec{E} \cdot \vec{B}$$

axion

$$+ \frac{\alpha_s}{8\pi} \frac{a}{f_a} G^{\alpha}_{\mu\nu} \tilde{G}^{\alpha\mu\nu}$$

$$g_{a\gamma\gamma} = \# \frac{\alpha}{2\pi f_a}$$

model dependent

$$f_a = \frac{v_{PQ}}{N_Q}$$ : axion scale.

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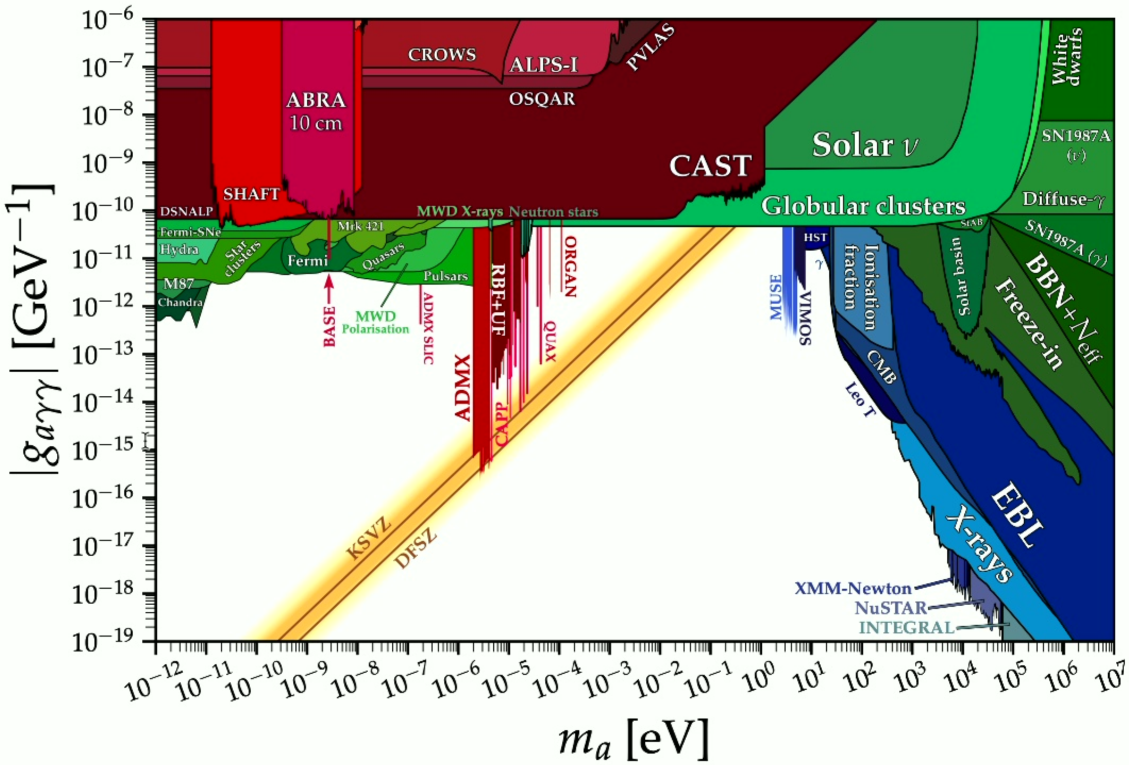


**Figure 90.1:** Exclusion plot for the ALP-photon coupling. Figure courtesy of Ciaran O'Hare [61], contains exclusions from refs. [62–168]. Constraints in red are from terrestrial experiments.