Title: The gravitational index of 5d black holes and black strings

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Collection/Series: Quantum Fields and Strings

Subject: Quantum Fields and Strings

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Abstract:

In this talk, I will discuss how one can use the gravitational path integral to compute the supersymmetric index of black holes as well as other black objects in string theory. The saddles that I shall describe admit a non-zero temperature, consequently lacking an infinitely long AdS throat that separates the horizon from the asymptotic region, and also admit periodic boundary conditions for fermionic fields around the thermal circle, consequently counting bosonic and fermionic states with an opposite sign. Since the microscopic calculation of these indices is oftentimes well understood yet the saddles that I shall describe now lack the conventional decoupling limit taken in AdS/CFT, our analysis represents a first step towards understanding holography for supersymmetric observables in flat space.

The gravitational index of black holes and black strings

Luca V. Iliesiu

A number of projects with Gustavo-Joaquin Turiaci, Sameer Murthy, Roberto Emparan, and Jan Boruch.

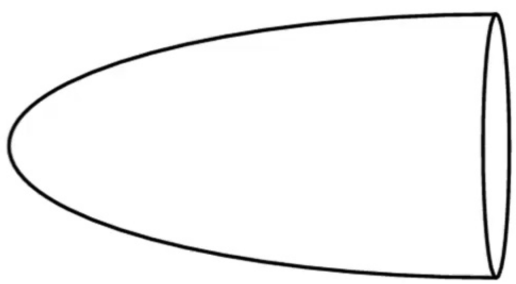
The big picture…

= Dual quantum system

Objects in gravity
e.g., a black hole

… even if the spacetime is asymptotically flat.

The big picture…



$\beta_{\text{flatspace}}$

$j(x)$

$$= \quad Z[\beta_{\text{flatspace}}, j(x)]$$

… even if the spacetime is asymptotically flat.

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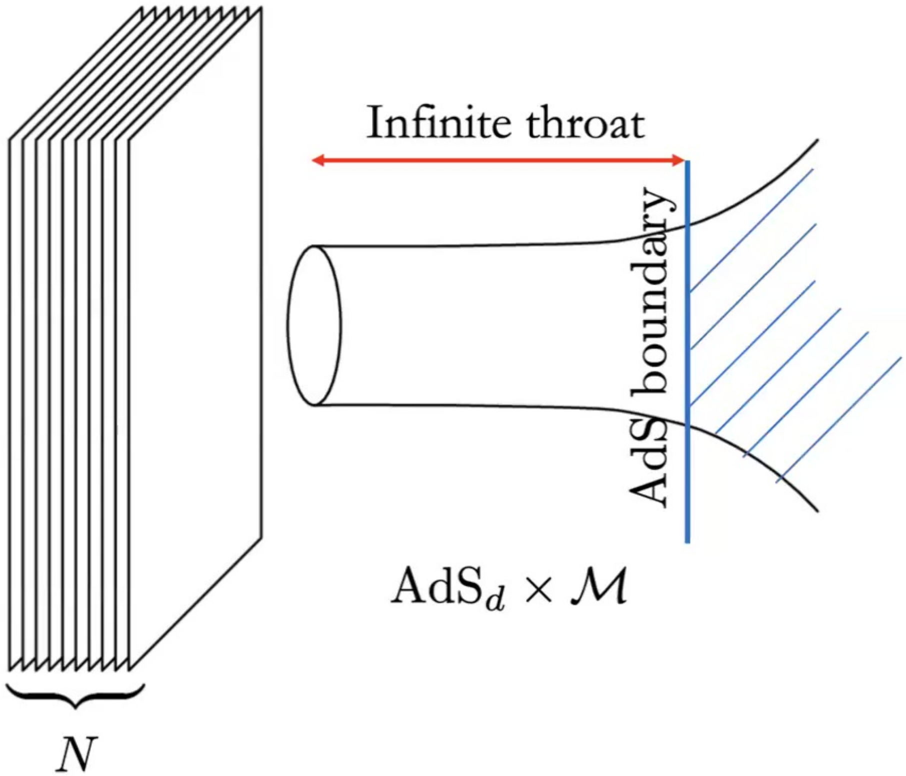
Holography realized in AdS/CFT



Asymptotically
$\mathbb{R}^{9,1}$ or $\mathbb{R}^{10,1}$
with
$\beta_{\text{flatspace}} \to \infty$
(the extremal limit)

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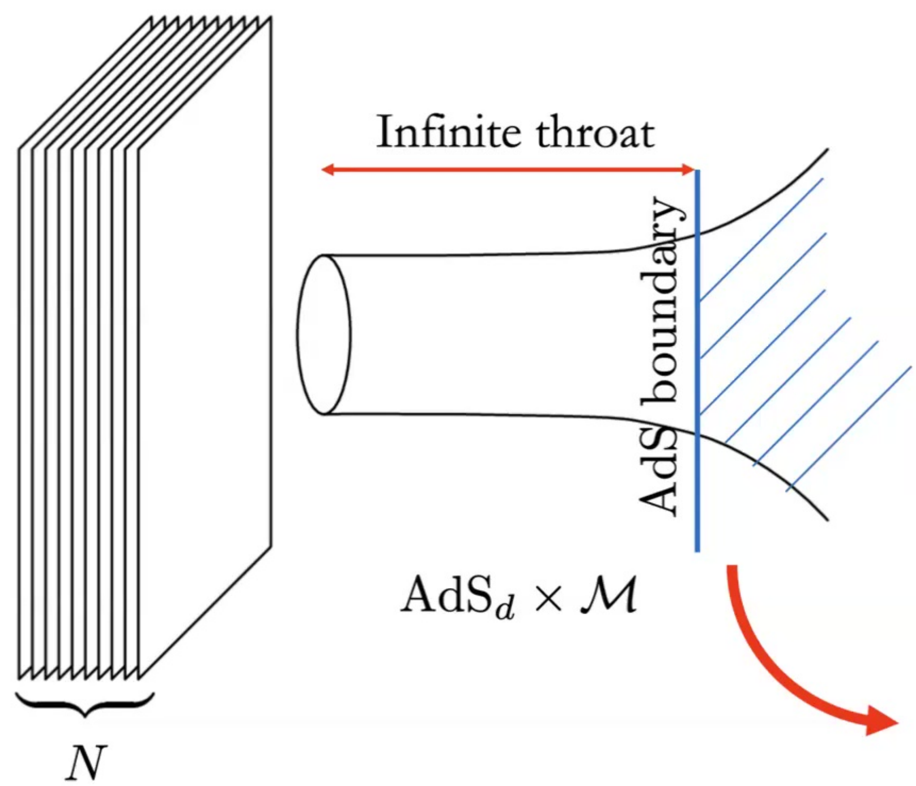
Holography realized in AdS/CFT



Asymptotically
$\mathbb{R}^{9,1}$ or $\mathbb{R}^{10,1}$
with
$\beta_{\text{flatspace}} \to \infty$
(the extreme limit)

$$Z_{\substack{\text{asymp.}\\ \text{AdS}}}(\beta, j(x)) = Z_{\text{CFT}}(\beta, j(x))$$

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Computations in AdS/CFT

=

properties of extremal black strings or branes in flatspace in the decoupling limit

Can we go away from the decoupling limit in a controlled fashion?

Yes, for at least some observables.

Primary goal of today: go beyond the decoupling limit for supersymmetric indices.

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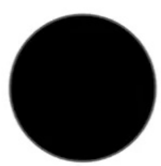
Why are indices helpful?

In a putative dual quantum system we would have,

$$\text{Index}_{Q_i}(\beta) = Tr_{\text{fix } Q_i}(-1)^F e^{\beta H} = \underbrace{(d_b - d_f)e^{-\beta E_{BPS}(Q_i)}}_{\text{A trivial } \beta\text{-dependence}}$$

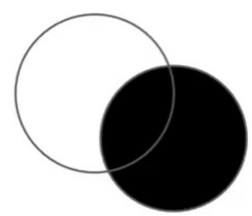
If we can really view gravitational objects as dual to quantum systems even in flatspace, we can interpolate between the decoupling (zero temperature limit) and finite temperature to find gravitational solutions that capture the index of the same quantum system.

Today:



S^2 or S^3

Index of black holes



$S^1 \times S^2$

Index of black strings

Indices have already long been associated to a decoupling limit … a misconception

A string theory index at weak string coupling	$\overset{?}{=}$	Area of an extremal black hole in a sector fixed charge

3 problems with this equality

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A string theory index at weak string coupling	=	Area of an extremal black hole in a sector fixed charge

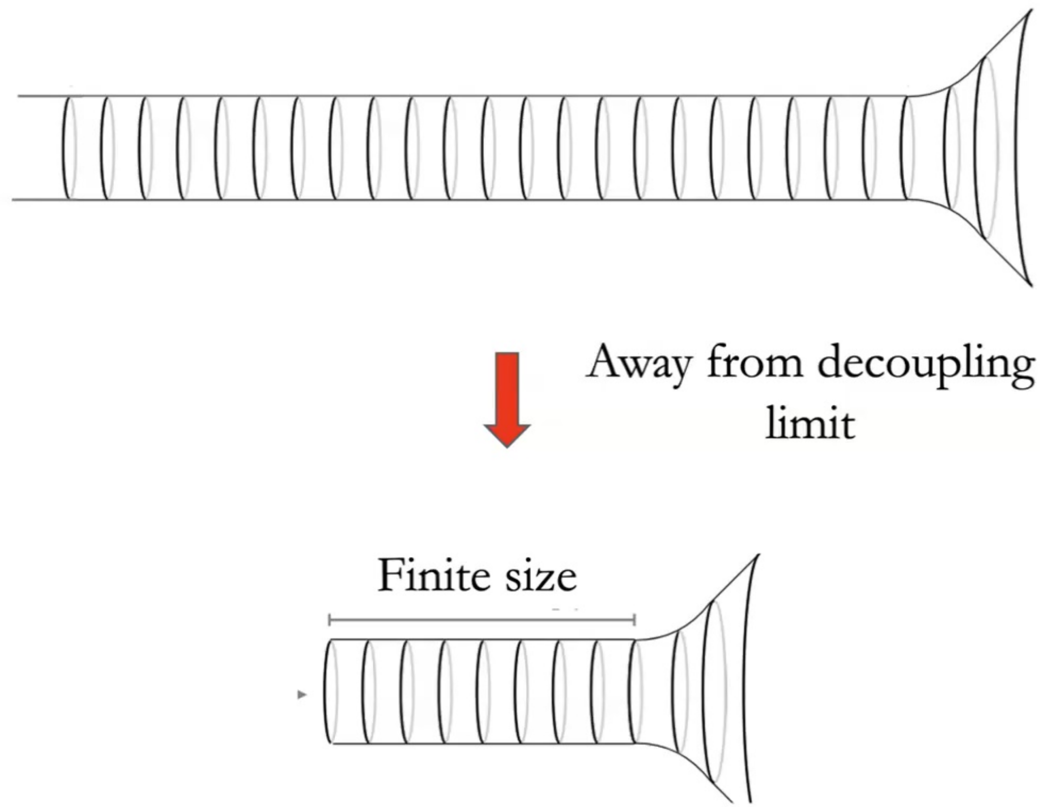
Problem I. The RHS is, at most, a degeneracy with $T \to 0$. The LHS is defined for any T

Problem II. The LHS counts bosonic and fermionic states with an opposite sign.

Problem III. How about all the other extremal black hole solutions that should contribute to the LHS? E.g., multicenter solutions

In fact, in AdS/CFT, the long held belief was that indices on the boundary side receive no contributions from black hole solutions [Witten `97]. We will review today why this belief is incorrect.

Starting point: Standard Reissner-Nordstrom black holes



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The index from gravity

$$\text{Index}_{Q_i}(\beta) = Tr_{\text{fix } Q_i}(-1)^F e^{\beta H} = (d_b - d_f)e^{-\beta E_{BPS}(Q_i)}$$

$$(-1)^F = e^{2\pi i J} \quad \Rightarrow \quad \Omega\beta = i\Omega_L\beta = 2\pi(1+2n)\,, \;\; n \in \mathbb{Z}$$

[Bizet, Cassani, Martelli, Murthy `18, Bobev, Charles, Min `20, LVI, Kologlu, Turiaci `20]

[for the purposes of this talk there is no need to distinguish the index and helicity super-trace]

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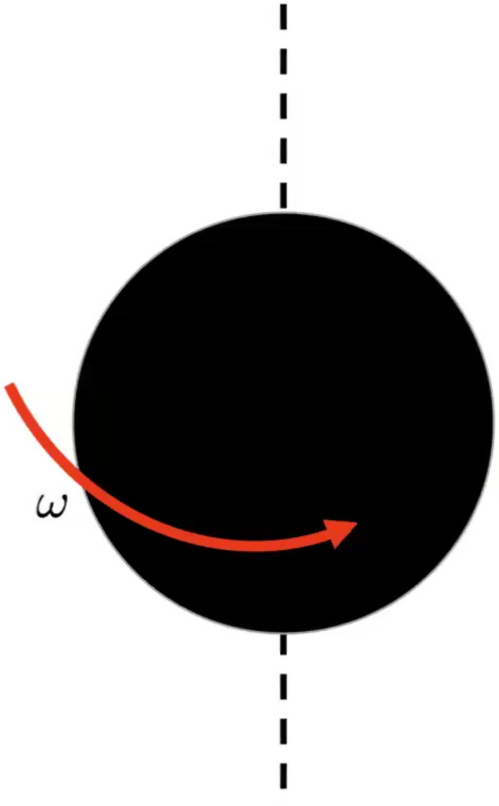
A black hole solution that we all hate (too complicated): the Kerr-Newman solution

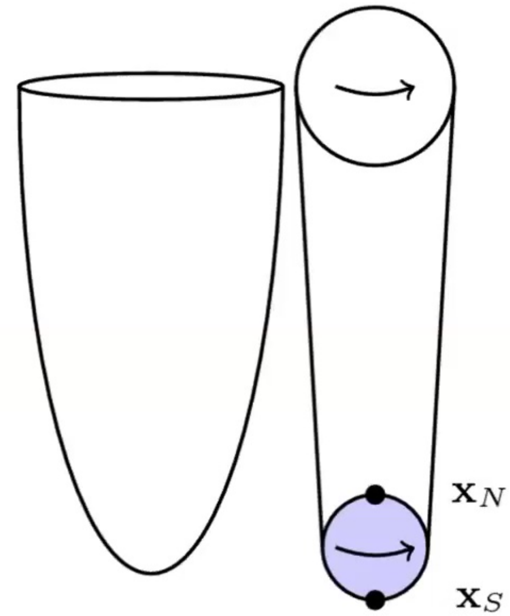


$$t \to it\,, \qquad \Omega = i\Omega_L\,,$$

$$(\tau, \phi) \sim (\tau + \beta, \phi + \beta\Omega)$$

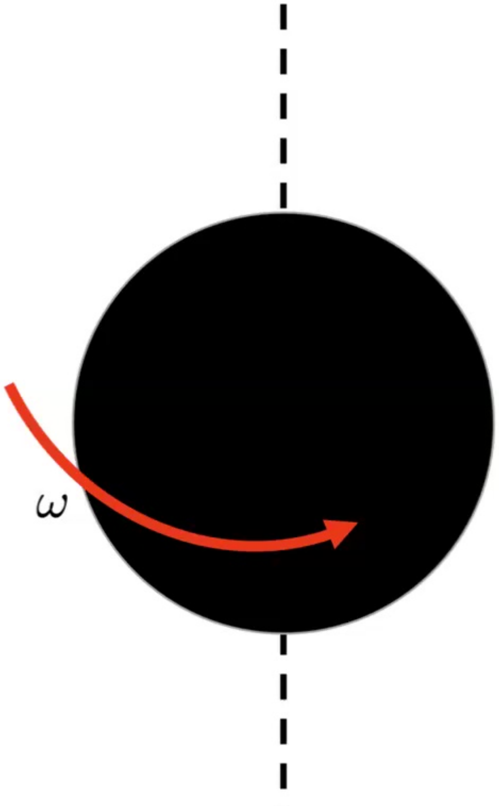
$$\psi(\tau, \phi) = -\psi(\tau + \beta, \phi + 2\pi)$$

$$\psi(\tau, \phi) = \psi(\tau + \beta, \phi)$$



[LVI, Kologlu, Turiaci `20]

A black hole solution that we all hate (too complicated):
the Kerr-Newman solution



$$t \to it\,, \qquad \Omega = i\Omega_L\,,$$

$$-I_E\Big(\beta, \Omega, Q\Big) = -\beta Q + \pi Q^2 + \frac{Q^3(4\pi^2 - (\beta\Omega)^2)}{2\beta} + \ldots$$

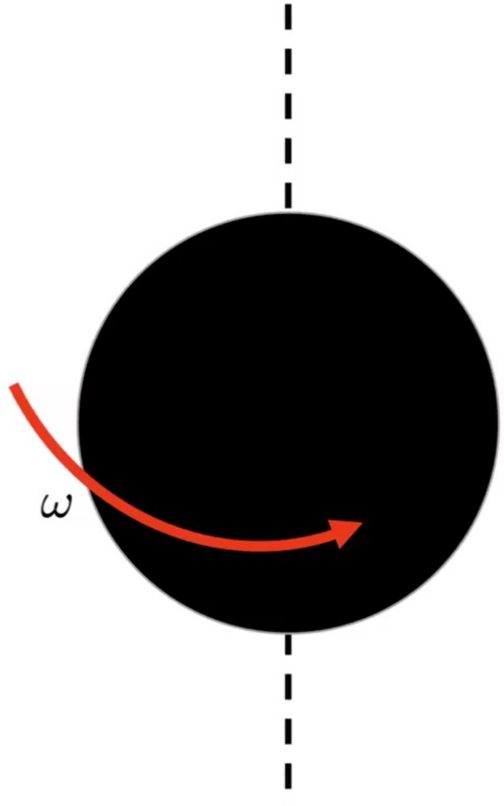
When $\beta\Omega = \pm 2\pi$

$$-I_E\Big(\beta, \Omega, Q\Big) = \underbrace{-\beta Q + \pi Q^2}_{\text{Form of an index}} \qquad \textit{(exact calculation)}$$

Geometry has Killing spinors (i.e. supersymmetric)

[LVI, Kologlu, Turiaci `20]

A black hole solution that we all hate (too complicated):
the Kerr-Newman solution



$$t \to it\,, \qquad \Omega = i\Omega_L\,,$$

$$-I_E\Big(\beta, \Omega, Q\Big) = -\beta Q + \pi Q^2 + \frac{Q^3(4\pi^2 - (\beta\Omega)^2)}{2\beta} + \dots$$

When $\beta\Omega = 2\pi(1+2n),\ n \neq 0,\ -1$

$-I_E$ does not take the form of an index

No Killing spinors

[LVI, Kologlu, Turiaci `20]

Subtlety:
the Kerr-Newman solution

$$= \text{Generated by the superpartner of } \partial_\tau$$

$$\int \frac{D\psi_{\mu\alpha}^{\text{Large}}}{\text{Super-isometries}} \cdots = 0$$

$$\neq 0$$

[LVI, Kologlu, Turiaci `20]

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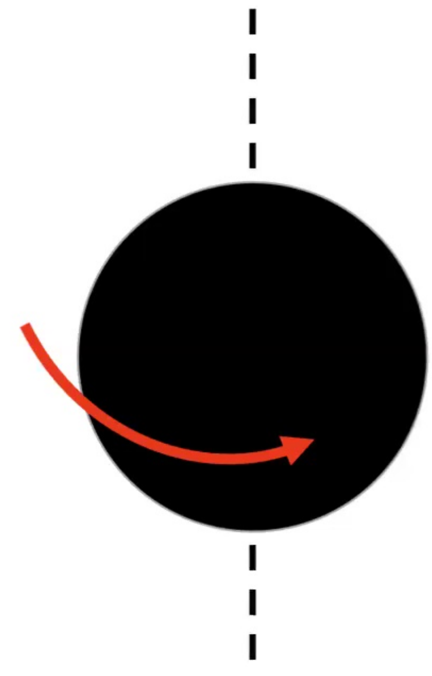
Alternative derivation of free energy: The thermodynamics relation

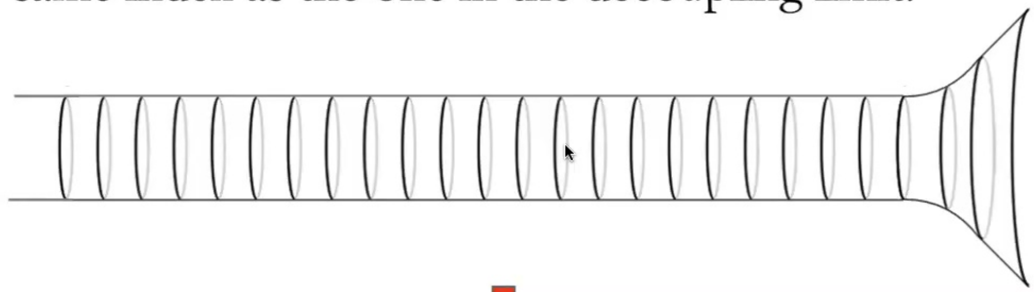
$$\underbrace{\text{Area}}_{\text{Depends on }\beta} + \underbrace{\beta\omega J}_{\text{Depends on }\beta} - \beta M = \pi Q^2 - \beta Q\,,$$



A baby version of the attractor mechanism.

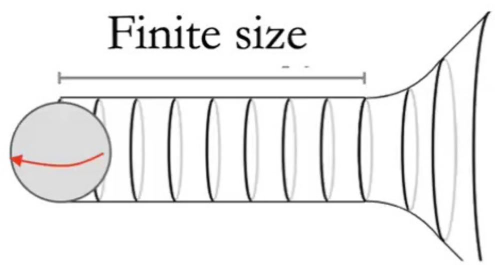
Take home message

Message I. There are flatspace geometries at finite β, that still captures the same index as the one in the decoupling limit.



Away from decoupling limit

Rotation of 2π when moving around thermal circle



[Bizet, Cassani, Martelli, Murthy `18, Bobev, Charles, Min `20, LVI, Kologlu, Turiaci `20]

Take home message

Message II. The free energy (on-shell action) of the black hole index has only trivial β-dependence even though their area has complicated β-dependence.

Message III.

$$\psi(\tau,\phi) = \psi(\tau+\beta,\phi)$$
$$\psi(\tau,\phi) = -\psi(\tau+\beta,\phi+2\pi)$$

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[Bizet, Cassani, Martelli, Murthy `18, Bobev, Charles, Min `20, LVI, Kologlu, Turiaci `20]

There is a much vaster set of allowed contributions to the index:

Multi-center black holes

General solution with Killing spinors:
The Israel-Wilson solution

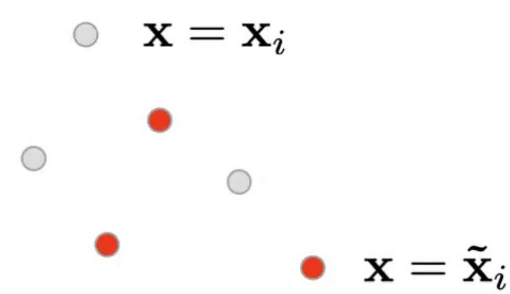
$$ds^2 = \frac{1}{U\tilde{U}}(d\tau + \omega_i dx^i)^2 + U\tilde{U}d\vec{x}^2, \qquad \nabla^2 U = \nabla^2 \tilde{U} = 0$$

$$U(\mathbf{x}) = 1 + \sum_i \frac{a_i}{|\mathbf{x} - \mathbf{x_i}|}, \qquad \tilde{U}(\mathbf{x}) = 1 + \sum_i \frac{\tilde{a}_i}{|\mathbf{x} - \tilde{\mathbf{x}}_\mathbf{i}|},$$

Asymptotically $\mathbb{R}^3 \times S^1$ or $\mathbb{R}^{3,1}$

=

\# ● = \# ○ = N

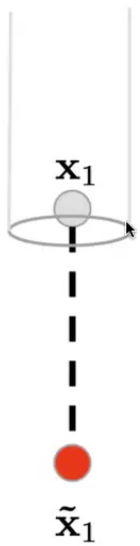


[Hartle, Hawking '72, Israel, Wilson '72, Whitt '84, Todd `85,Yuille '87, Maldacena, Michelson, Strominger `98, Dunajski, Hartnoll '06, …]

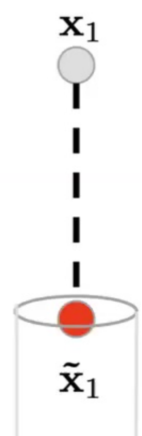
Imposing smoothness

If, $\mathbf{x}_i \neq \tilde{\mathbf{x}}_i$ $\qquad \epsilon_{ijk}\partial_j\omega_k = \tilde{U}\partial_i U - U\partial_i\tilde{U}$

$$\int_{\partial S_i} \omega = 4\pi a_i \tilde{U}(\mathbf{x}_i) \qquad\qquad \int_{\partial \tilde{S}_i} \omega = -4\pi a_i U(\tilde{\mathbf{x}}_i)$$



Singularity?



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Imposing smoothness

If, $\mathbf{x}_i \neq \tilde{\mathbf{x}}_i$ $\qquad$ $\epsilon_{ijk}\partial_j\omega_k = \tilde{U}\partial_i U - U\partial_i\tilde{U}$

$$\int_{\partial S_i} \omega = 4\pi a_i \tilde{U}(\mathbf{x}_i) \qquad\qquad \int_{\partial \tilde{S}_i} \omega = -4\pi a_i U(\tilde{\mathbf{x}}_i)$$

We can perform a coordinate transformation

$$t \to t + \lambda(x^i) \qquad \omega_i \to \omega_i + \partial_i\lambda(x) \qquad t \to \begin{cases} t - 2U(\tilde{\mathbf{x}}_j)\tilde{a}_j\,\phi\,, & \text{at } \mathbf{x}_j \\ t + 2\tilde{U}(\mathbf{x}_j)a_j\,\phi\,, & \text{at } \tilde{\mathbf{x}}_i \end{cases}$$

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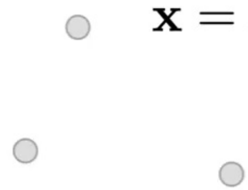
Imposing smoothness

Assuming poles are different, we can perform a coordinate transformation

$$t \to t + \lambda(x^i) \qquad \omega_i \to \omega_i + \partial_i \lambda(x) \qquad t \to \begin{cases} t - 2U(\tilde{\mathbf{x}}_j)\tilde{a}_j\,\phi\,, & \text{at } \mathbf{x}_j \\ t + 2\tilde{U}(\mathbf{x}_j)a_j\phi\,, & \text{at } \tilde{\mathbf{x}}_i \end{cases}$$

Lorentzian: $U(\mathbf{x})$ and $\tilde{U}(\mathbf{x})$ need to have the same poles

$$\mathbf{x} = \mathbf{x}_i = \tilde{\mathbf{x}}_i \quad \& \quad \omega = 0$$



$$U(\mathbf{x}) = \tilde{U}(\mathbf{x}) = 1 + \sum_i \frac{Q_i}{|\mathbf{x} - \mathbf{x}_i|}$$

Majumdar-Papapetrou extremal solution (AdS_2 fragmentation)

Imposing smoothness

Assuming poles are different, we can perform a coordinate transformation

$$t \to t + \lambda(x^i) \qquad \omega_i \to \omega_i + \partial_i \lambda(x) \qquad t \to \begin{cases} t - 2U(\tilde{\mathbf{x}}_j)\tilde{a}_j\,\phi\,, & \text{at } \mathbf{x}_j \\ t + 2\tilde{U}(\mathbf{x}_j)a_j\,\phi\,, & \text{at } \tilde{\mathbf{x}}_i \end{cases}$$

Euclidean: $t \sim t + \beta\,,$ $\quad \tilde{U}(\mathbf{x}_j)a_j = \frac{\beta}{4\pi}\,,$ $\quad U(\tilde{\mathbf{x}}_j)\tilde{a}_j = \frac{\beta}{4\pi}\,,$ $\quad \forall\, j$

$=$ $\qquad =$

At each horizon: $\frac{1}{2\Omega}$ $\qquad \frac{1}{2\Omega}$ → Once again, $\beta\Omega = 2\pi$

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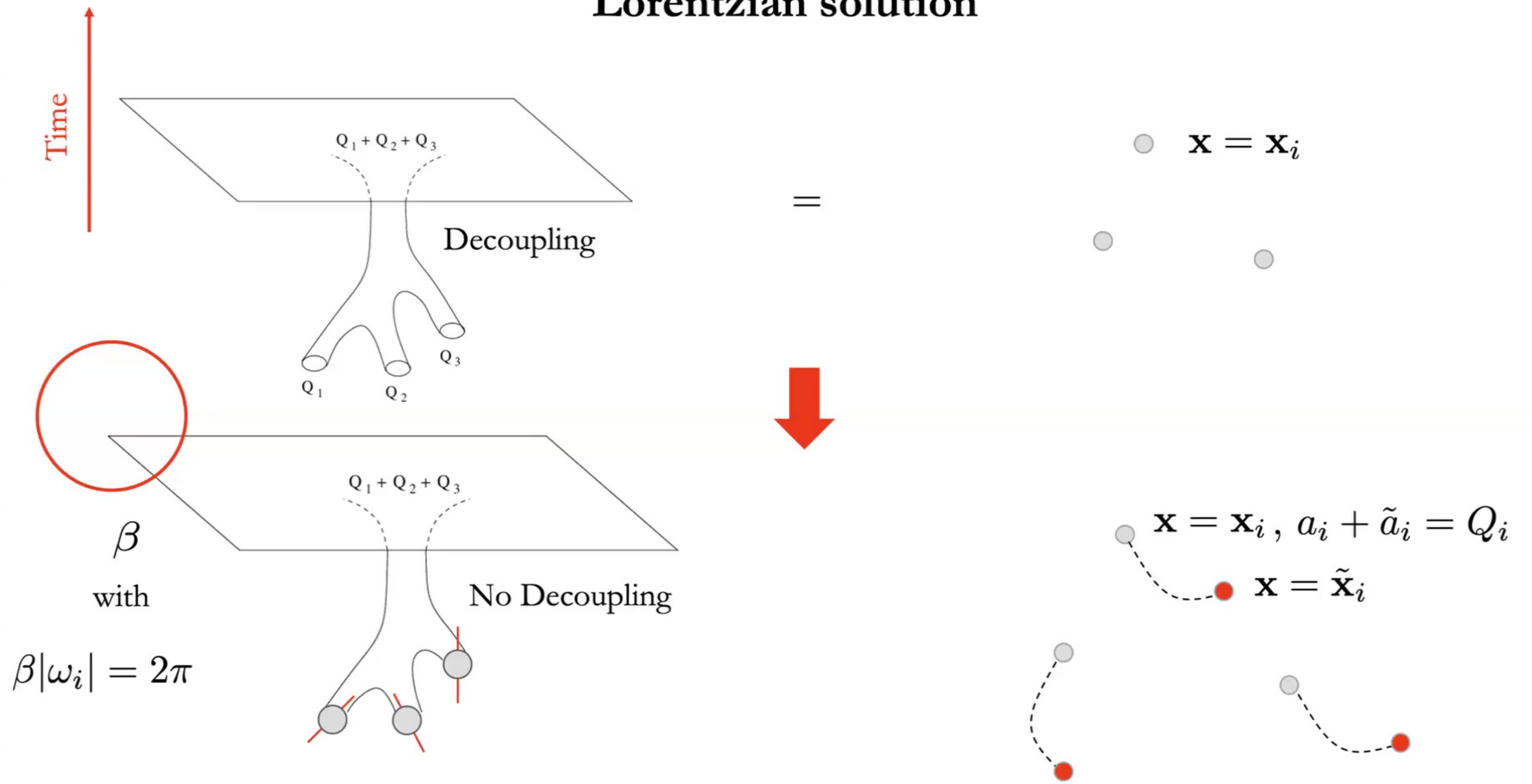
Lorentzian solution



Euclidean solution

with $2N - 1$ constraints

The most basic example:
One center and one anti-center

Example:

If there are only two poles,

we find that this is just the finite β Kerr-Newmann solution that has Killing spinors discussed earlier.

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The index for the black hole ensemble

$$-I_E = \underbrace{\frac{\pi}{2}\left(\sum_i a_i^2 + \sum_i \tilde{a}_i^2\right)}_{\log(d_b - d_f)} - \underbrace{\frac{\beta}{2}\left(\sum_i a_i + \sum_i \tilde{a}_i\right)}_{\beta E_{BPS}}$$

$$= \pi \sum_i Q_i^2 - \beta \sum_i Q_i$$

This takes precisely the form of the free energy associate to an index.

Once again, each individual area depends very non-trivially on β, but overall the Euclidean on-shell action has trivial β dependence.

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[upcoming work: LVI, Boruch, Murthy, Turiaci]

How about the black hole and black string solutions in string theory?

How about the black hole and black string solutions in string theory?

These are black holes in theories with vector multiplets (with **scalar moduli**):
I.e. *the attractor solutions* of [Ferrara, Kallosh, Strominger].

$$\mathbb{R}^{3,1} \times \mathrm{CY} \longrightarrow \mathbb{R}^{3} \times S^1_\beta \times \mathrm{CY}$$

Inverse temperature: β

Charges: Γ

New ingredient: X^{A}_{bdy}

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The attractor solutions

The metric:

$$ds^2 = \frac{1}{\Sigma(H)}(dt+\omega)^2 + \Sigma(H)dx^i dx^i,$$

For example for a compactification on a diagonal T^6 (effectively $n_v = 1$)

$$\Sigma(H) = \sqrt{\frac{1}{3}p^2q^2 - \frac{4\sqrt{3}}{9}q^3v - \frac{4}{3\sqrt{3}}p^3u + 2pquv - u^2v^2} \quad \text{where} \quad H(\mathbf{x}) = \underbrace{(v,p,q,u)}_{\text{Harmonic functions}}$$

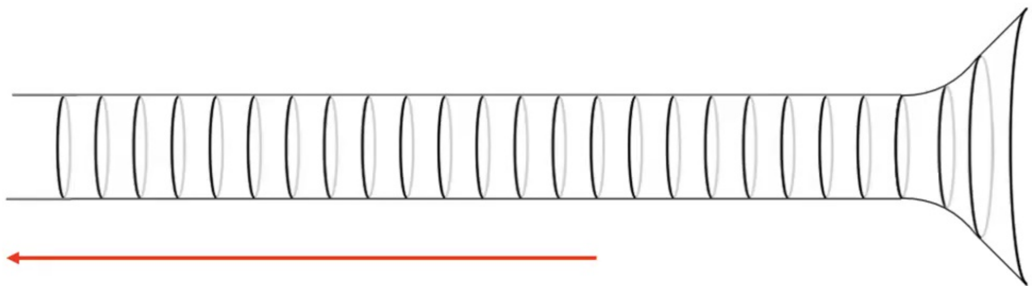
Depends on pre-potential

[Denef '00, Bates, Denef `03, Denef, Moore '07, …]

The Lorentzian solution

$$H = \mathcal{C} + \sum_{i=1}^{N} \frac{\Gamma_s}{|\mathbf{x} - \mathbf{x}_i|} \qquad (D6, D4, D2, D0)_{\text{at center } s}$$

Lorentzian: Infinite throat



Asymptotically $\mathbb{R}^{3,1}$

New ingredient: X^A_{bdy}

Properties of horizon are independent of X^A_{bdy}

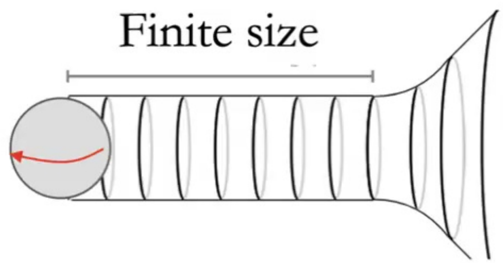
[Ferrara, Kallosh, Strominger `96, Denef '00, Bates, Denef `03, Denef, Moore '07, …]

The Euclidean solution

$$H = \mathcal{C} + \sum_{i=1}^{N} \frac{\Gamma_s}{|\mathbf{x} - \mathbf{x}_i|} \qquad (D6, D4, D2, D0)_{\text{at center } s}$$

Euclidean: finite throat

$\beta\Omega = 2\pi$



Asymptotically $\mathbb{R}^3 \times S^1_\beta$

New ingredient: X^A_{bdy}

Properties of horizon are dependent on X^A_{bdy} and β

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A new attractor mechanism

[LVI, Boruch, Turiaci, Murthy `23]

Area

J

Both depend on the moduli values and temperature

$$\frac{\text{Area}}{4} + \beta\Omega J \sim \sqrt{\hat{p}^3\hat{u}}$$

Spacetime is smooth

completely independent of the moduli values $(\mathcal{C} \text{ and } \beta)$

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Even though the index of such black holes has been computed in string theory we do not have a boundary CFT description for them.

Can we go away from the decoupling limit in a canonical example in AdS/CFT?

Need to go to higher dimensions.

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A new attractor mechanism

[LVI, Boruch, Turiaci, Murthy `23]

More generally, imposing smoothness yields an on-shell action with the same value:

$$-I_{\text{total}} = -\beta \overbrace{E_{\text{BPS}}}^{|Z(\Gamma;\Omega_\infty)|} + \underbrace{\pi\Sigma(\Gamma)}_{\substack{= \log\left(d_b - d_f\right) \\ = S_{\text{BH}}^{\text{extremal}}}}$$

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Solution that we will be looking for,

$$\begin{array}{ccc} \mathbb{R}^{3,1} \times \mathrm{CY} & \longrightarrow & \mathbb{R}^3 \times S^1_\beta \times \mathrm{CY} \\ \downarrow & \text{Uplift} & \downarrow \\ \mathbb{R}^{3,1} \times \mathrm{CY} \times S^1_M & \longrightarrow & \mathbb{R}^3 \times S^1_\beta \times \mathrm{CY} \times S^1_M \end{array}$$

[Gaiotto, Strominger, Yin `05 & `06]

Solution that we will be looking for,

$$\begin{array}{ccc} \mathbb{R}^{3,1} \times \mathrm{CY} & \longrightarrow & \mathbb{R}^3 \times S^1_\beta \times \mathrm{CY} \\ \downarrow & & \downarrow \\ \mathbb{R}^{3,1} \times \mathrm{CY} \times S^1_M & \longrightarrow & \mathbb{R}^3 \times S^1_\beta \times \mathrm{CY} \times S^1_M \\ \cup & & \\ \mathrm{AdS}_3 \times S^2 \times CY & & \\ = & & \\ \mathrm{MSW\ CFT}_2 & & \end{array}$$

[Maldacena, Strominger, Witten '97]

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Attractor saddles for 4d black hole index in $\mathbb{R}^3 \times S^1_{\beta_{4d}}$

$\Gamma^0 = 1$

$R_M / \ell_5 \to \infty$

↓

Attractor saddles for 5d black hole index in $\mathbb{R}^4 \times S^1_{\beta_{5d}}$

$S^2 \Rightarrow S^3$

Gravitational path integral confirmation that index of 5d black holes in M-theory

=

index of 4d black holes in IIA SUGRA

[see also Anupam,. Chowdury, Sen `24]

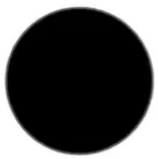
Attractor saddles for 4d black hole index in $\mathbb{R}^3 \times S^1_{\beta_{4d}}$

$\Gamma^0 = 1$
$R_M/\ell_5 \to \infty$

Attractor saddles for 5d black hole index in $\mathbb{R}^4 \times S^1_{\beta_{5d}}$

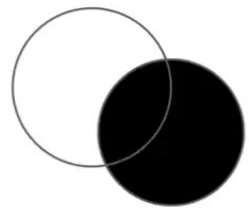
$\Gamma^0 = 0$

Attractor saddles for 5d black string index in $\mathbb{R}^3 \times S^1_M \times S^1_{\beta_{5d}}$



$S^2 \Rightarrow S^3$

Index of
5d black holes
=
Index of
4d black holes



$S^2 \Rightarrow S^1 \times S^2$

Index of
black strings

[see also Anupam,. Chowdury, Sen `24]

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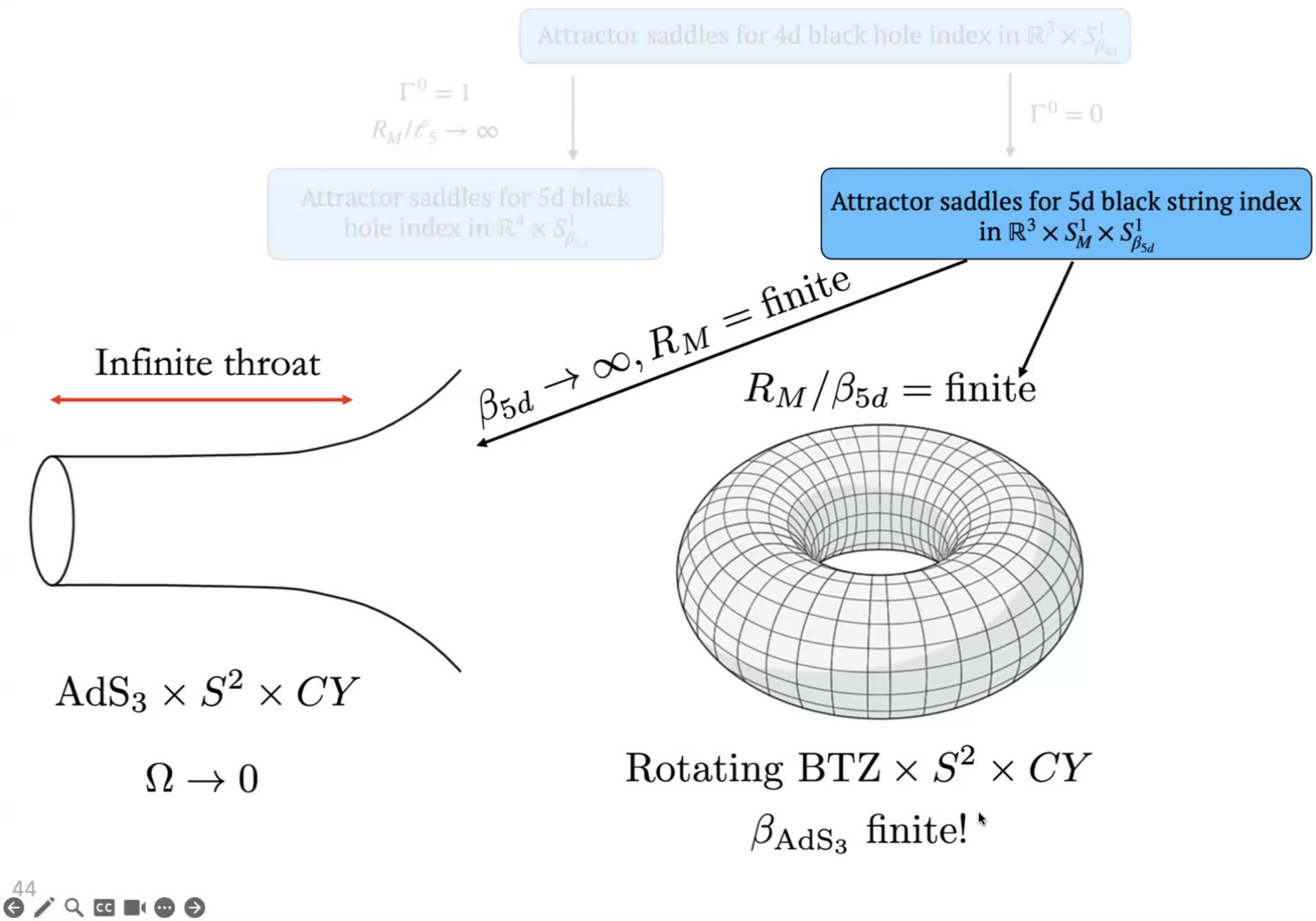
Attractor saddles for 4d black hole index in $\mathbb{R}^3 \times S^1_{\beta_{4d}}$
$\Gamma^0 = 1$
$R_M/\ell_5 \to \infty$
$\Gamma^0 = 0$
Attractor saddles for 5d black hole index in $\mathbb{R}^4 \times S^1_{\beta_{5d}}$
Attractor saddles for 5d black string index in $\mathbb{R}^3 \times S^1_M \times S^1_{\beta_{5d}}$
$\beta_{5d} \to \infty, R_M = \text{finite}$
$R_M/\beta_{5d} = \text{finite}$
Infinite throat
$\text{AdS}_3 \times S^2 \times CY$
$\Omega \to 0$
Rotating $\text{BTZ} \times S^2 \times CY$
β_{AdS_3} finite!
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What do these geometries have in common?

Their Euclidean actions compute the same index of the MSW CFT:

$$-I_E = 4\pi\sqrt{\frac{\Gamma_0 c_R}{24}}\ , \qquad c_R = D_{ABC}\Gamma^A\Gamma^B\Gamma^C$$

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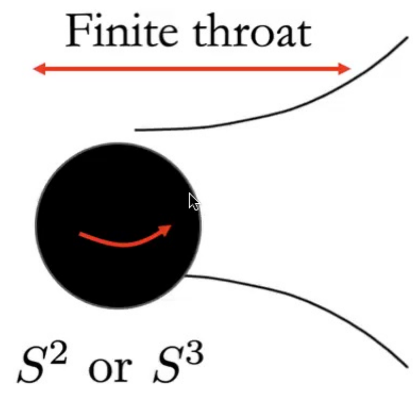
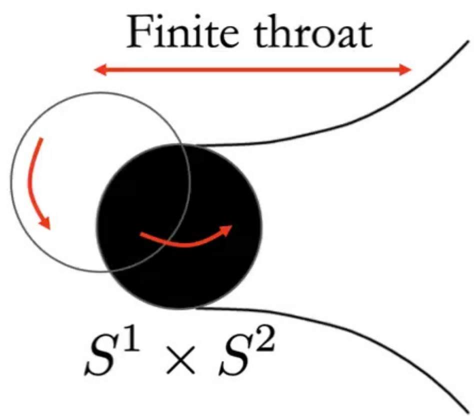
Take home message

We've found the gravitational solutions that contribute to the index,





These take black holes, black strings and black branes away from the decoupling limit but has the same microscopic description in the dual quantum system.

E.g., even though there is no AdS_3 portion of the spacetime we are still computing the index in a CFT!

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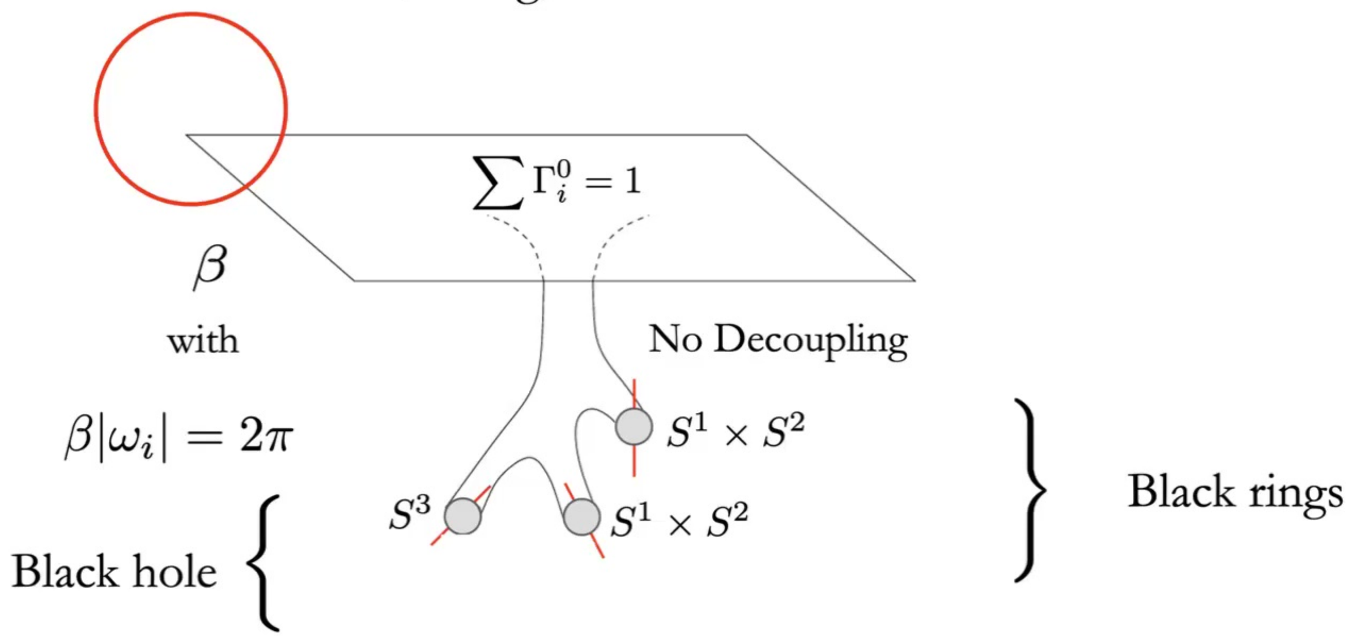
Outlook: **multi-center solutions**

[Boruch, Turiaci, Murthy (to appear)]

However there are more things contributing to this index than the single black hole/string/brane solution:



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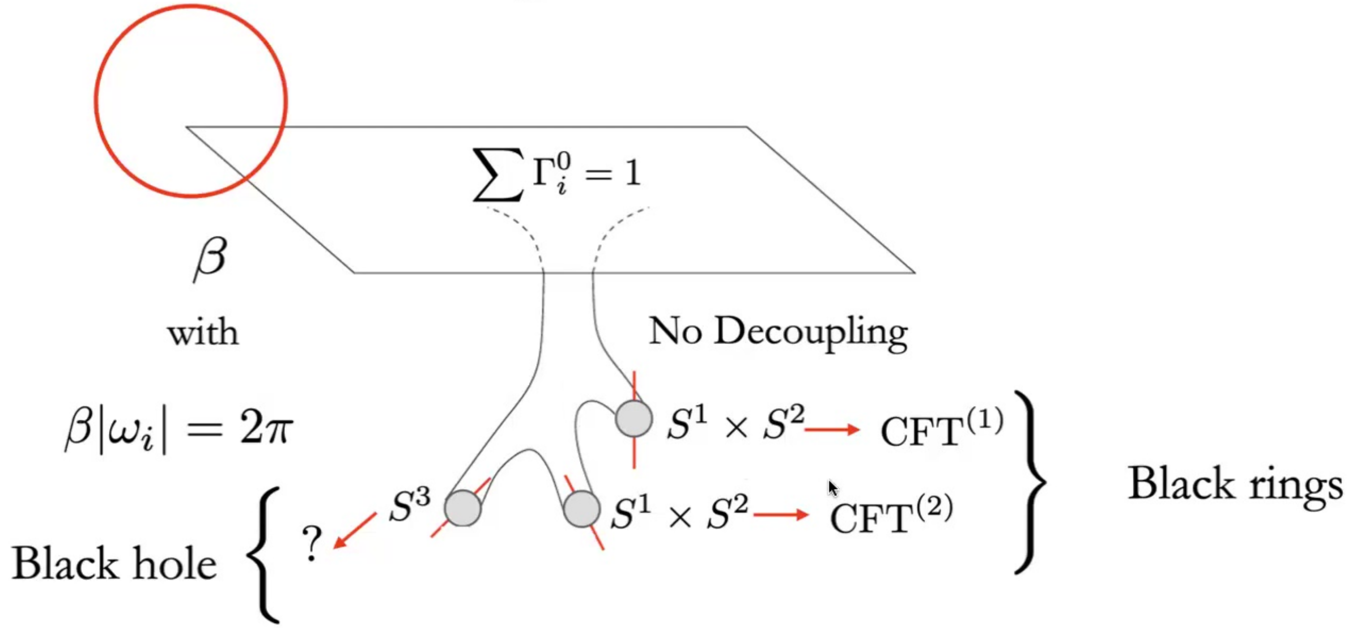

Multiple quantum systems interacting in the same spacetime.

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[Boruch, Turiaci, Murthy (to appear)]

Their Euclidean action factorizes into a sum of indices:

$$-I_E^{\text{total}}(\beta, \{\Gamma_i\}) = -\sum_i \log \text{Index}_{\Gamma_i}(\beta) \; + \; O(1) \, .$$

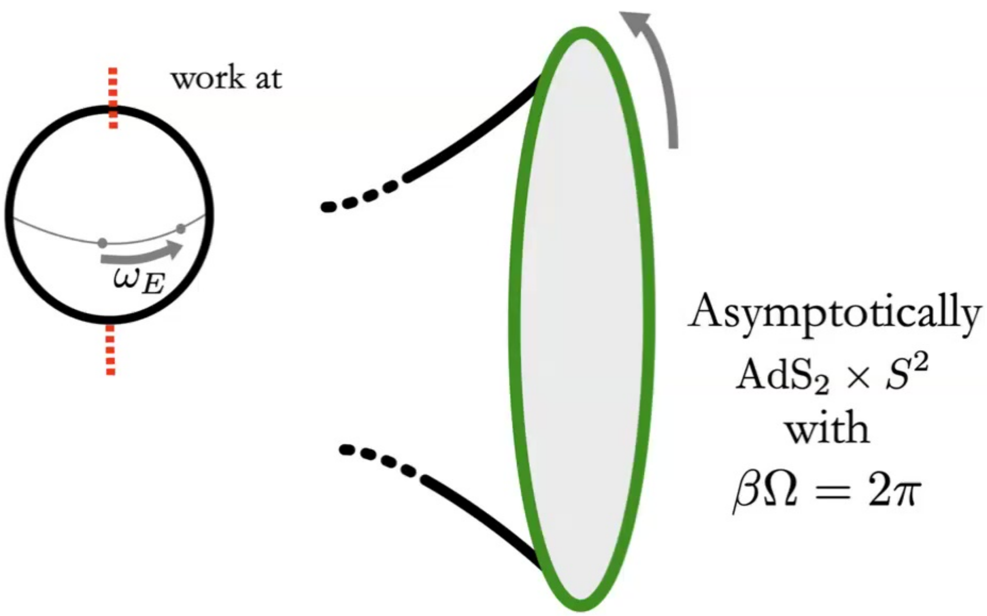
Richer moduli space of saddles compared to extremal solutions in both 4d and 5d supergravity.

Outlook: **supergravity localization**

Localization in supergravity as $\beta \to \infty$.

[LVI, Turiaci, Murthy `22]

Localization in supergravity on
$\mathrm{AdS}_2 \times S^2$ with index-b.c.



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Outlook: **supergravity localization**

Localization in supergravity as $\beta \to \infty$.

[LVI, Turiaci, Murthy `22]

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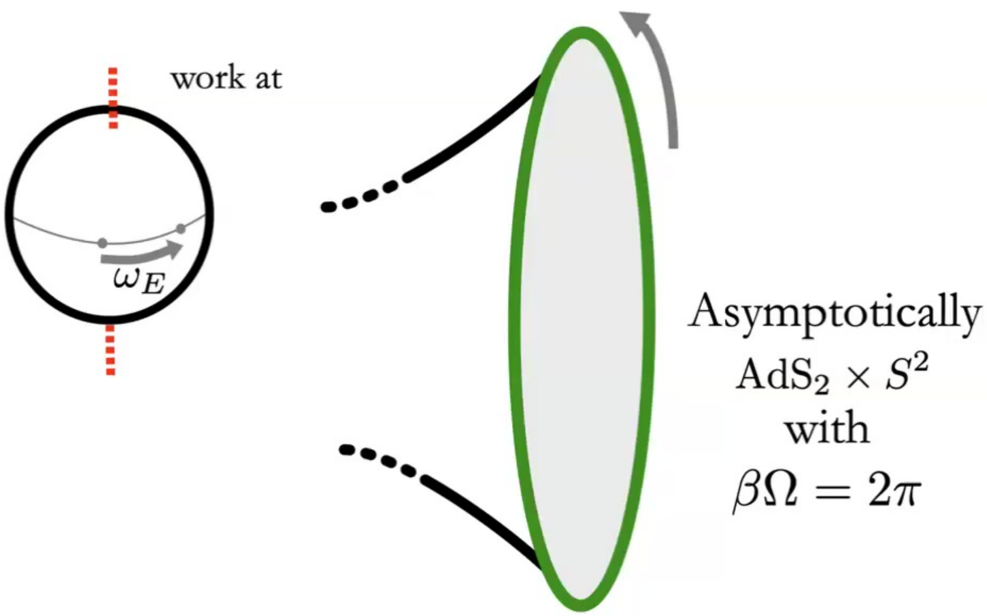

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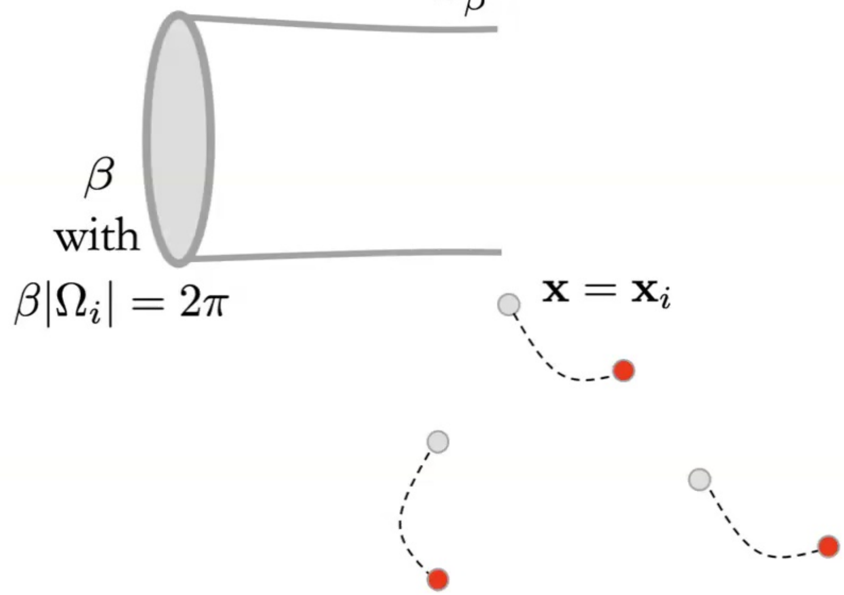
[WIP]

Today, we see that we can work at $\forall\, \beta$.

Opens up the possibility of localizing supergravity with $S^1_\beta \times \ldots$ index-b.c.



A dream… compute all CFT indices in one calculation.