Title: Celestial CFT from Dimensional Reduction of CFT

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Abstract:

The Celestial Holography conjecture posits the existence of a codimension two theory whose correlators compute the S-matrix in a conformal primary basis. Although resembling a CFT in several respects, the intrinsic definition of this proposed dual theory remains elusive. In this talk, I will discuss a conjecture suggesting that Celestial CFT (CCFT) is related to a dimensionally reduced CFT on the Lorentzian cylinder and present some concrete examples of celestial amplitudes constructed in this way.

Celestial CFT from Dimensional Reduction of CFT

Based on arXiv:2206.10547, arXiv:2303.10037 and arXiv:2405.07972 with Ana Raclariu

Motivation

The holographic principle proposes gravity theories are dual to non-gravitational theories in lower dimensions

AdS/CFT gives a concrete implementation [Maldacena, 1997; Gubser, Klebanov, Polyakov, Witten, 1998]

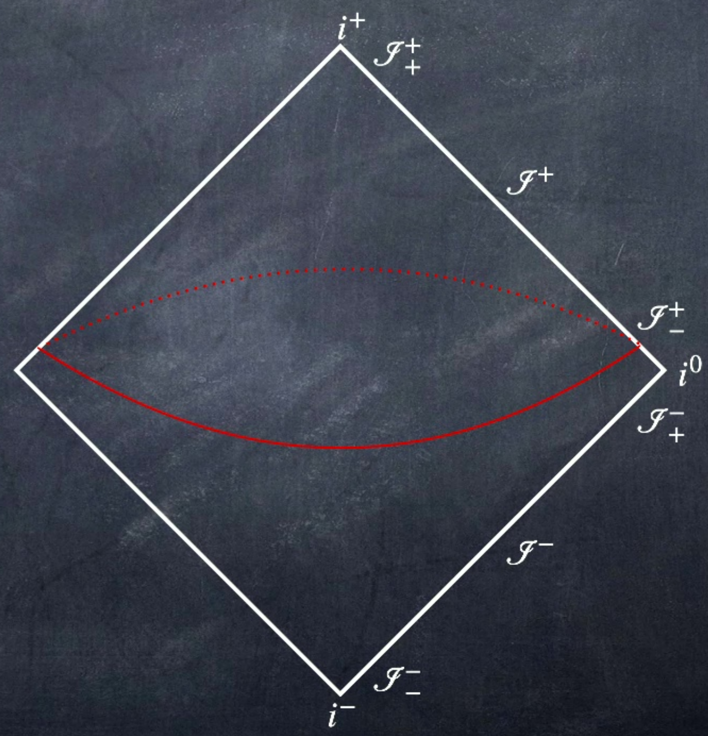
$$\text{Gravity in asymptotically AdS} = \text{CFT on } \partial\text{AdS}$$

Can holography be extended to asymptotically flat spacetime?

Flat Space Holography

Celestial holography conjectures a possibility

Gravity in Mink_{d+1} = Celestial CFT on $\mathcal{CS}^{d-1}$



Some Motivations

Writing scattering amplitudes in plane wave basis makes translation symmetry manifest

Introduce a conformal primary basis [Pasterski, Shao, 2017]

$$\varphi_\Delta(x;\hat{q}) = \int_0^\infty d\omega \omega^{\Delta-1} e^{\pm i\omega \hat{q}\cdot x} = \frac{(\pm i)^\Delta \Gamma(\Delta)}{(-\hat{q}\cdot x \pm i\epsilon)^\Delta}$$

Now $\mathrm{SO}(1,d) = \mathrm{Conf}(\mathscr{C}\mathscr{S}^{d-1})$ symmetry is manifest

Some Motivations

In this basis scattering amplitudes transform as correlators of quasi-primary operators

$$A_n(\Delta_1, \hat{q}_1; \ldots; \Delta_n, \hat{q}_n) = \langle \mathcal{O}_{\Delta_1}(\vec{z}_1) \cdots \mathcal{O}_{\Delta_n}(\vec{z}_n) \rangle$$

Bulk Lorentz symmetry maps to boundary conformal symmetry

Bulk soft theorems imply in further symmetries on the boundary

Some Motivations

Importantly, soft theorems imply in symmetries on the boundary

Subleading soft graviton symmetry in 4D implies 2D Virasoro symmetry [Kapec, Lysov, Pasterski, Strominger, 2014]

A 2D stress tensor can be defined from the subleading soft graviton [Kapec, Mitra, Raclariu, Strominger, 2016]

Soft gluon theorem in 4D implies in Kac-Moody symmetry in 2D [He, Mitra, Strominger, 2015]

Further extension to subleading soft orders is reorganized in terms of the $w_{1+\infty}$ algebra in gravity and s-algebra in gauge theory [Guevara, Himwich, Pate, Strominger 2021; Strominger 2021]

What is a CCFT?

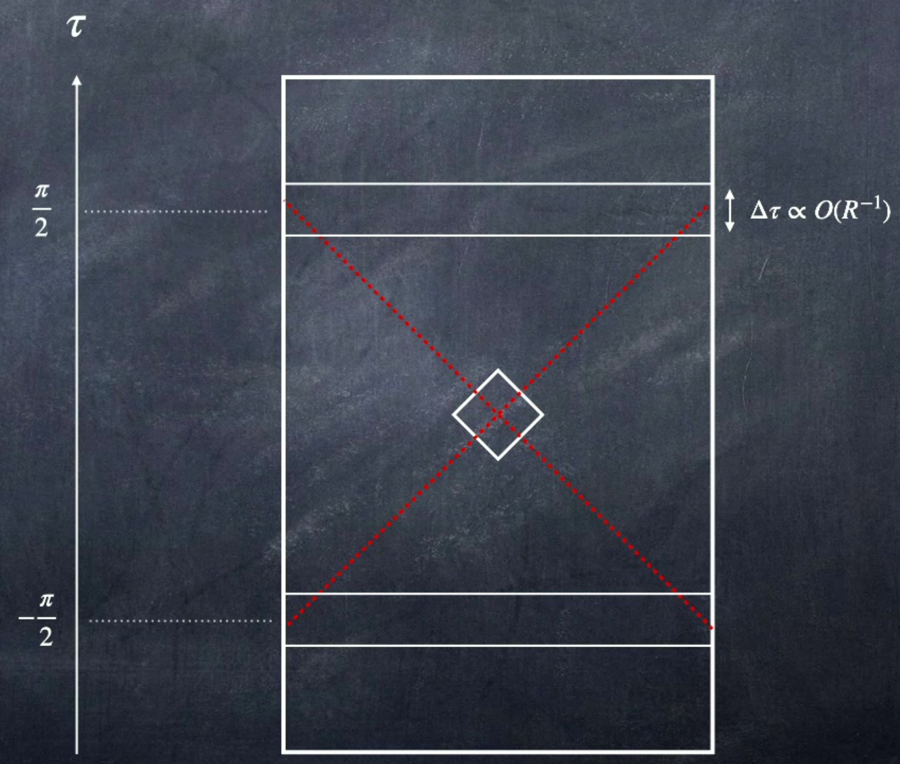
While many properties are understood, there is no complete understanding of what a CCFT is intrinsically

Constructions have been carried out on the self-dual sector [Costello, Paquette, Sharma, 2022; Costello, Paquette, Sharma, 2023]

Final goal of dimensional reduction proposal is provide a means to construct CCFT from standard CFT in (1,3) bulk signature and beyond the self-dual sector

Flat Space limit of AdS/CFT

For localized interactions in AdS we can take a flat space limit that zooms around the bulk point limit [Penedones, 2011; Hijano, Neuenfeld, 2020]



Flat Space limit of AdS/CFT

Holographic CFT correlators are computed by summing over Witten diagrams in AdS

These are made of bulk-to-boundary propagators $K_\Delta(P,X)$, bulk-to-bulk propagators $\Pi_\Delta(P,X)$ and interaction vertices

These can be expanded at large AdS radius

Flat Space limit of AdS/CFT

In the bulk we work in global AdS and change coordinates to

$$\tau = \frac{t}{R}, \quad \rho = \frac{r}{R}$$

In the boundary we expanded about the time slices in the bulk-point configuration

$$\tau = \pm\frac{\pi}{2} + \frac{u}{R}$$

Flat Space limit of AdS/CFT

The key observation then is that bulk-to-boundary propagators turn into conformal primary wavefunctions, while bulk-to-bulk propagators turn into Feynman propagators

Integration over AdS formally becomes integral over flat space

Therefore Witten diagrams turn into Feynman diagrams computing celestial amplitudes [de Gioia, Raclariu, 2022]

Flat Space limit of AdS/CFT

This is mainly motivation, but it is a non-rigorous analysis

It requires being able to exchange integrals over AdS for integrals over flat space in the $R \to \infty$ limit and moving the limit inside

We can then provide checks of the correctness of this proposal by working directly in the CFT side and bypassing the bulk

CFT Side

Restrict operators to infinitesimal strips around $\pm\frac{\pi}{2}$

$$\tau = \pm\frac{\pi}{2} + \frac{u}{R}$$

Taking $R \to \infty$ takes a Carrollian limit on the boundary

$$ds^2 = -\frac{du^2}{R^2} + \gamma_{AB}dz^A dz^B \to 0du^2 + \gamma_{AB}dz^A dz^B$$

Getting $\mathfrak{ebms}_4$ from reduction

Conformal Killing Equation

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \frac{2}{d}(\nabla \cdot \xi) g_{\mu\nu}$$

We can study that for the cylinder expanded about a time slice

$$ds^2 = -\frac{du^2}{R^2} + \gamma_{AB} dz^A dz^B$$

Getting $\mathfrak{ebms}_4$ from reduction

At finite R solutions generate $\mathfrak{so}(3,2)$ but as $R \to \infty$ we obtain an infinite-dimensional enhancement [de Gioia, Raclariu, 2023]

$$\epsilon_Y^{\pm} = \pm \frac{R}{2i} F_{\pm}(u) D \cdot Y \partial_u + F_{\pm}(u) Y^A \partial_A \qquad F_{\pm}(u) = \begin{cases} e^{\pm i\left(\tau_0 + \frac{u}{R}\right)}, & D \cdot Y \neq 0, \\ 1, & D \cdot Y = 0 \end{cases}$$

$$\epsilon_f = f(z, \bar{z}) \partial_u$$

Where $f(z, \bar{z})$ is an arbitrary function and $Y(z, \bar{z})$ is a (local) CKV on S^2

Getting $\mathfrak{ebms}_4$ from reduction

Define

$$T_f = \epsilon_f \qquad T_Y = i\frac{\epsilon_Y^+ + \epsilon_Y^-}{2R} \qquad L_Y = \frac{\epsilon_Y^+ - \epsilon_Y^-}{2}$$

Then these vectors reproduce $\mathfrak{ebms}_4$ as $R \to \infty$ [de Gioia, Raclariu, 2023]

$$[T_{f_1}, T_{f_2}] = O(R^{-2}) \quad [L_{Y_1}, L_{Y_2}] = L_{[Y_1,Y_2]} + O(R^{-2})$$

$$[T_f, L_Y] = T_{f'=\frac{1}{2}(D\cdot Y)-Y(f)} + O(R^{-2})$$

Action on CFT operators

Let $O_\Delta(\tau, \vec{z})$ be a primary operator. It transforms under a CKV as

$$\delta_\xi O_\Delta = -\left(\frac{\Delta}{d}\nabla\cdot\xi + \xi^\mu\nabla_\mu + \frac{i}{2}\nabla_{[\mu}\xi_{\nu]}S^{\mu\nu}\right)O_\Delta$$

We can study how primaries transform under the $\mathfrak{ebms}_4$ generators we found in a large R expansion

Getting $\mathfrak{ebms}_4$ from reduction

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Action on CFT operators

Under the 2D CKV the CFT_3 primary transforms as

$$\delta_{L_Y} O_\Delta = -\left[D_z Y^z \mathfrak{h} + Y^z(\partial_z - \Omega_z J_3) + D_{\bar{z}} Y^{\bar{z}} \bar{\mathfrak{h}} + Y^{\bar{z}}(\partial_{\bar{z}} - \Omega_{\bar{z}} J_3) + O(R^{-1})\right] O_\Delta$$

The weights $(\mathfrak{h}, \bar{\mathfrak{h}})$ are defined by

$$\mathfrak{h} = \frac{\hat{\Delta} + J_3}{2}, \quad \bar{\mathfrak{h}} = \frac{\hat{\Delta} - J_3}{2}, \quad \hat{\Delta} = \Delta + u\partial_u$$

Action on CFT operators

3D primaries transform as 2D primaries with same 3D spin and operator-valued dimension [de Gioia, Raclariu, 2023]

$$\hat{\Delta} = \Delta + u\partial_u$$

Dimensional reduction diagonalizes these dimensions

This analysis generalizes to CFT$_d$ straightforwardly

Proposal for $\mathrm{CFT}_d \to \mathrm{CCFT}_{d-1}$

Let a holographic CFT_d on the Lorentzian cylinder be given

For any operator $O_\Delta(\tau, \vec{z})$ define a continuum of operators

$$\mathcal{O}^{\pm}_{\delta}(\vec{z}) = N(\Delta, \delta) \int_{-\infty}^{\infty} du (u \pm i\epsilon)^{\Delta-\delta-1} O_\Delta \left(\pm\frac{\pi}{2} + \frac{u}{R}, \vec{z} \right)$$

This transform diagonalizes the $(d-1)$-dimensional slice dimensions

Soft Graviton Theorems

We can recover from the dimensional reduction the leading and subleading soft graviton theorems

These are implied by the stress tensor Ward identities

$$\langle T^{\mu}{}_{\mu}(x) X \rangle = - \sum_i \delta(x - x_i) \Delta_i \langle X \rangle$$

$$\langle T^{[\mu\nu]}(x) X \rangle = - i \sum_i \delta(x - x_i) S_i^{\mu\nu} \langle X \rangle$$

$$\nabla_\mu \langle T^{\mu\nu}(x) X \rangle = - \sum_i \delta(x - x_i) \frac{\partial}{\partial x_i^\nu} \langle X \rangle$$

Soft Graviton Theorems

The stress tensor Ward identities imply that the shadow stress tensor defined by

$$\widetilde{T}_{AB}(P) = \int D^d Y \frac{\partial_{P^A}\partial_{Y^C} \log(-P \cdot Y) \partial_{P^B}\partial_{Y^D} \log(-P \cdot Y)}{(-P \cdot Y)^{d-\Delta-2}} T^{CD}(Y)$$

Has insertions fixed to be [Kapec, Mitra, 2018; de Gioia, Raclariu, 2023]

$$\langle \widetilde{T}_{\mu\nu}(x) X \rangle = \frac{i}{4} \sum_i \frac{\varepsilon^{AB}_{\mu\nu}(x) P_A(x_i) P^D(x)}{P(x) \cdot P(x_i)} (\mathcal{J}_i)_{DB} \langle X \rangle$$

Soft Graviton Theorems

Taking components tangential to the S^{d-1} slice gives

$$\partial_u \langle \widetilde{T}_{ab} X \rangle = \sum_{i=1}^{n} \frac{\epsilon_{ab}^{AB} q_A(x_i) q^C(x_i)}{q(x) \cdot q(x_i)} \partial_{u_i} \langle X \rangle + O(R^{-1})$$

and

$$(1 - u\partial_u) \langle \widetilde{T}_{ab} X \rangle = i \sum_{i=1}^{n} \frac{\epsilon_{ab}^{AB}(x) q_A(x_i) q^C(x)}{q(x) \cdot q(x_i)} (\mathcal{J}_i)_{BC} \langle X \rangle + O(R^{-1})$$

Soft Graviton Theorems

Taking the time Mellin transform to map O_{Δ_i} to $\mathcal{O}^{\pm}_{\delta_i}$ now gives the leading and subleading soft theorems [de Gioia, Raclariu, 2023]

The ∂_{u_i} operators become the weight-shifting operators $e^{\partial_{\delta_i}}$ dual to flat space energies

The $(\mathcal{J}_i)_{BC}$ operators are the flat space Lorentz transformations acting on the fields

Soft Graviton Theorems

In this approach we identify $\partial_u \widetilde{T}_{ab}$ as the leading soft graviton and $(1 - u\partial_u)\widetilde{T}_{ab}$ as the subleading soft graviton

Importantly: only tangential components to S^{d-1} have been used

Result: conformal symmetry of CFT_3 implies in extended BMS_4 symmetry for the reduced theory

Two-Point Function Example

A CFT$_d$ two-point function is kinematically fixed

$$\langle O_\Delta(P_1) O_\Delta(P_2) \rangle = \frac{C_\Delta}{(-P_1 \cdot P_2)^\Delta}$$

Likewise in CCFT$_{d-1}$ it is also kinematically fixed

$$\langle \mathcal{O}^{\eta_1}_{\delta_1}(\vec{z}_1) \mathcal{O}^{\eta_2}_{\delta_2}(\vec{z}_2) \rangle = 2(2\pi)^{d+1} \delta_{\eta_1, -\eta_2} \delta(\delta_1 + \delta_2 - d + 1) \delta_{S^{d-1}}(\vec{z}_1, \vec{z}_2)$$

Two-Point Function Example

The delta function emerges from the large R expansion

$$\left(\frac{\epsilon}{\epsilon^2+z^2}\right)^\Delta = \left[\frac{\pi^{\frac{d-1}{2}}\Gamma(\Delta-\frac{d-1}{2})}{\Gamma(\Delta)\epsilon^{\Delta-d+1}}\delta^{(d-1)}(z)+O(\epsilon^{d+1-\Delta})\right]+\left[\frac{\epsilon^\Delta}{z^{2\Delta}}+O(\epsilon^{\Delta+2})\right]$$

To apply this one must appropriately continue to Lorentzian signature

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Two-Point Function Example

This identity implies in the expansion of the Wightman functions

$$\frac{C_\Delta}{(-P_1 \cdot P_2 + i\eta_1\zeta\epsilon)^\Delta} = \frac{\widetilde{C}_\Delta}{|u_1 - u_2|^{2\Delta-d+1}}\delta_{S^{d-1}}(z_1, z_2) + \cdots$$

Where $\eta_i = \pm 1$ labels incoming/outgoing and $\zeta = \pm 1$ picks the Wightman function $\langle\Omega|O_\Delta(P_1)O_\Delta(P_2)|\Omega\rangle$ or $\langle\Omega|O_\Delta(P_2)O_\Delta(P_1)|\Omega\rangle$

Two-Point Function Example

Taking the time Mellin transforms now reproduces the celestial amplitude when $\eta_1 = \zeta$ and zero otherwise. For the time-ordered we always get the right result [de Gioia, Raclariu, 2024]

$$\langle \mathcal{O}^{\eta_1}_{\delta_1}(\vec{z}_1)\mathcal{O}^{\eta_2}_{\delta_2}(\vec{z}_2)\rangle = 2(2\pi)^{d+1}\delta_{\eta_1,-\eta_2}\delta(\delta_1+\delta_2-d+1)\delta_{S^{d-1}}(\vec{z}_1,\vec{z}_2)$$

Importantly: to get the right normalization it is necessary to be in Lorentzian signature and keep track of the $i\epsilon$ prescription!

Three-Point Function Example

For the three-point function we directly start with time-ordered correlators

$$\langle O_1(P_1)O_2(P_2)O_3(P_3)\rangle = \frac{C_{123}}{(-P_1\cdot P_2+i\epsilon)^{\frac{\alpha_{12}}{2}}(-P_1\cdot P_3+i\epsilon)^{\frac{\alpha_{13}}{2}}(-P_2\cdot P_3+i\epsilon)^{\frac{\alpha_{23}}{2}}}$$

Where we defined

$$\alpha_{ij} = \alpha_i + \alpha_j - \alpha_k$$

Three-Point Function Example

We follow a different approach than the two-point. We write the Mellin representation of the three-point function and expand at large R

$$\prod_i \frac{1}{N_i}\langle O_1 O_2 O_3\rangle = \mathscr{C} R^{d+1-\sum_i \Delta_i} \prod_i \int_0^\infty d\omega_i^{\Delta_i - 1} \prod_{j<k} e^{i\eta_j\omega_j\eta_k\omega_k\left(|\Omega_{jk}|^2 - \frac{u_{jk}^2}{R^2}\right)}$$

Three-Point Function Example

The exponent can be recognized as

$$\sum_{i<j} \eta_i \omega_i \eta_j \omega_j |\Omega_{ij}|^2 = -\left(\sum_i \eta_i \omega_i \hat{q}_i \right)^2$$

We can then use a Gaussian delta function identity

$$\lim_{\epsilon \to 0} \frac{1}{(2\pi\epsilon)^{\frac{d+1}{2}}} e^{-\frac{x^2}{2\epsilon}} = \delta^{(d+1)}(x)$$

Three-Point Function Example

The large R limit therefore develops the distributional contribution as in the two-point function

$$\lim_{R\to\infty}\prod_i \frac{1}{N_i}\langle O_1 O_2 O_3\rangle = \mathscr{C}'\prod_i \int_0^\infty d\omega_i^{\Delta_i-1}\prod_{j<k} e^{i\eta_j\omega_j\eta_k\omega_k u_{jk}^2}\delta^{(d+1)}\left(\sum_i \eta_i\omega_i\hat{q}_i\right)$$

The Mellin integrals over u_i are now Gaussian an can be evaluated with an appropriate choice of contour.

Three-Point Function Example

The integrals give the right normalization and shift the CFT$_d$ dimensions into CCFT$_{d-1}$ dimensions [de Gioia, Raclariu, 2024]

$$\prod \int_{-\infty}^{\infty} du_i (u_i + \eta_i \epsilon)^{\Delta_i - \delta_i - 1} \lim_{R\to\infty} \prod_i \frac{1}{N_i} \langle O_1 O_2 O_3 \rangle = \prod_i \int_0^{\infty} d\omega_i^{\delta_i - 1} \delta^{(d+1)} \left(\sum_i \eta_i \omega_i \hat{q}_i \right)$$

The RHS is then manifestly the Mellin transform in energy of a flat space three-point function, reproducing the celestial amplitude

Three-Point Function Example

It was important to use the OPE coefficient for a contact interaction of three scalars derived from AdS/CFT

Therefore the contact interaction vertex in AdS maps to the contact interaction vertex in flat space as expected

Dynamical input was necessary at this level just in using the right OPE coefficient

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Future Directions

- 4-point functions in CCFT_2 from CFT_3
  - Conformal Block decompositions
- Celestial OPE from CFT OPE
  - Emergence of $w_{1+\infty}$ symmetry from CFT_3 physics (Work in Progress with Raclariu, Strominger and Wang)
- Application to $\text{AdS}_4 \times S^7$ reducing ABJM theory
  - Possible non self-dual celestial hologram?