Title: Higher dimensional Segal—Sugawara construction and fivebranes

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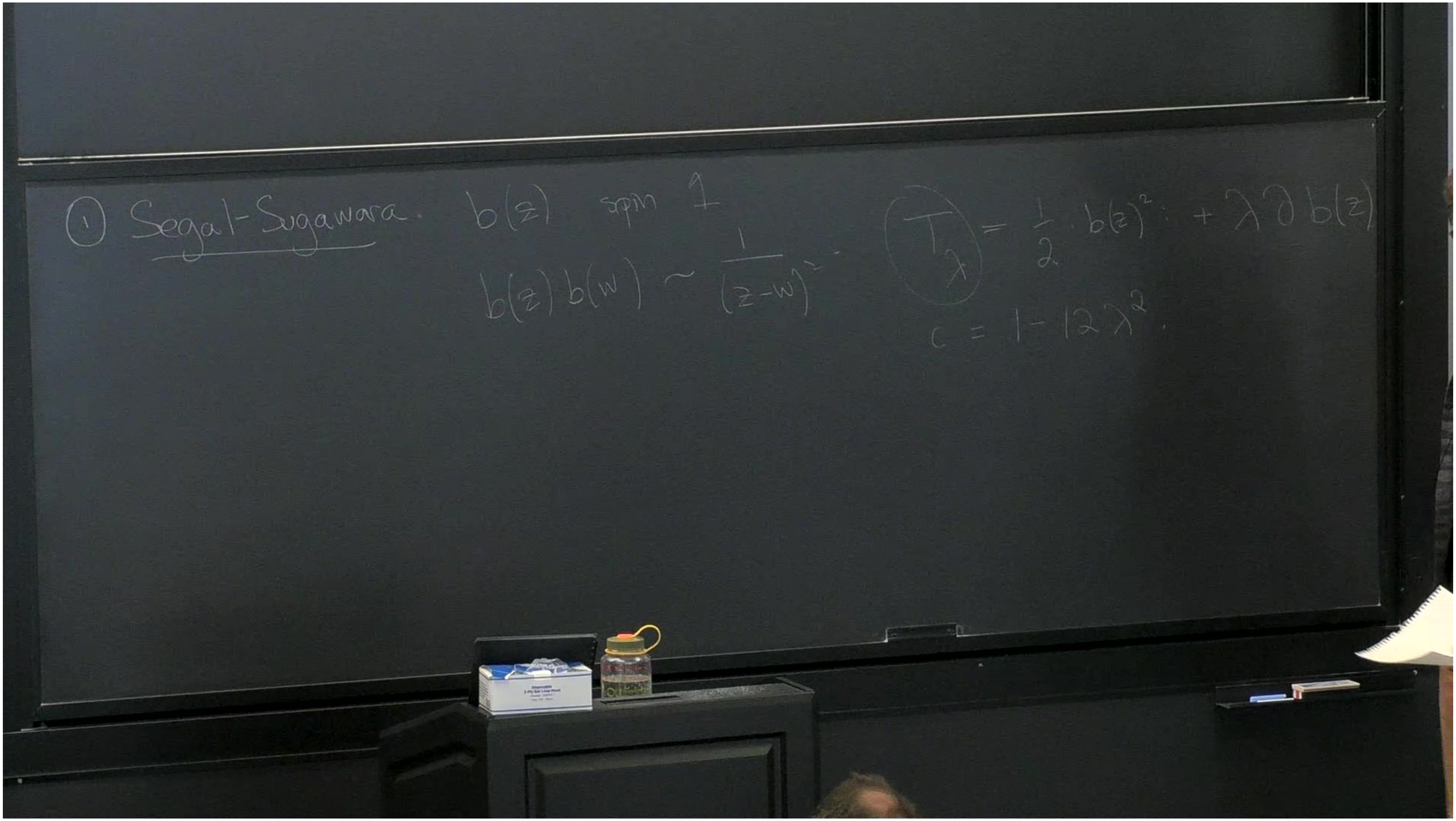
Abstract:

The correspondence of AGT sets up, in part, a connection between six-dimensional superconformal theories and 2d CFT. We will give a mathematical construction of 2d CFT from 6d SCFT which involves recent progress in our understanding of the holomorphic twist 6d superconformal symmetry. We then turn to the question of enhancement of familiar structures in 2d CFT to 6d including the well-known Segal—Sugawara construction. Finally, I will set up a Dolbeault enhancement of the AGT correspondence which is expected to probe coherent cohomology of the moduli space of instantons.

Higher Segal-Sugawara (Most parts w/ Raghavendran, Saberi),
+ fivebranes

$$\mathbb{C} \longrightarrow \mathrm{Vir} \longrightarrow \mathrm{Vect}(D^\times) \curvearrowleft \mathrm{Witt}$$

Q Universal symmetry of (twisted) 6d (2,0) superconformal theories?



① Segal-Sugawara. $b(z)$ spin 1
$b(z)\,b(w) \sim \frac{1}{(z-w)^2}$.
$T_\lambda = \frac{1}{2} :b(z)^2: + \lambda \partial b(z)$
$c = 1 - 12\lambda^2$.

① Segal-Sugawara: $b(z)$ spin 1

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$$c = 1 - 12\lambda^2.$$

$$J(f\partial_z) - \partial_z f$$

$\lambda = 0$

Invariantly: $b \in \Omega^{1,0}(\Sigma)$, $\bar\partial b = 0$.

$$\underset{0}{\Omega^{1,0}_\Sigma} \xrightarrow{\bar\partial} \underset{1}{\Omega^{1,1}_\Sigma} \quad \partial b$$

$\mu \in \mathrm{Vect}(\bar\Sigma)$

$$T(\mu) = \frac{1}{2}\int_\Sigma b\, \iota_\mu b + \lambda \int_{\Sigma_z} (J\mu)\, b.$$

$\hbar \to 0$ ↓ Poisson alg	↓ $\mathbb{P}_0$ alg $\{a,b\} = \Delta(ab) - \Delta(a)b \pm a\Delta(b)$.	↓ chiral boson VOA

$$\underset{0}{\Omega^{1,0}_\Sigma} \xrightarrow{\bar\partial} \underset{1}{\Omega^{1,1}_\Sigma} \quad \leftarrow \mathbb{P}_0\text{ - structure}$$

= Poisson bivector of degree +1

$$\pi = \partial \quad \Omega^{0,1}_{\Sigma} \longrightarrow \Omega^{1,1}_\Sigma \rightsquigarrow \pi \in \left(\Omega^{1,*}_\Sigma\right)^{\otimes 2}$$

QM	BV	
Algebra	BV alg $(A, \cdot, \Delta_{BV})$	Quantization of this object $\rightsquigarrow$ BV factorization algebra
$\hbar \to 0$ $\downarrow$	$\downarrow$	$\rightsquigarrow$
Poisson alg	$\mathbb{P}_0$-alg	chiral boson VOA

6d (2,0) superconformal theories,

$$T_{\partial} J_{\Sigma} \simeq H^0(\Sigma, \Omega^1)^*$$

$\rightsquigarrow$ chiral boson VOA defines a family over J_Σ.

X = cplx 3-fold.

$$IJ_X = H^3(X,\mathbb{R}) / H^3(X,\mathbb{Z}) \longleftrightarrow \left\{ \begin{array}{l} \text{cplx str s} \\ \text{on trivial holomorphic 2-gerbe} \end{array} \right\},$$

$$\mathbb{T}_{\bar\partial} IJ_X \simeq H^{\bullet}(X, \Omega^{3,cl}_X)^* [1].$$

The fields of the "chiral boson" on X is

$$\underset{-1}{\Omega^{2,\cdot}_X} \xrightarrow{\partial} \underset{0}{\Omega^{3,\cdot}_X}$$

$\bar\partial \circlearrowleft \qquad \bar\partial \circlearrowleft$

Has a natural BV str.

$$\pi = \partial$$

$$\left(\Omega^{3,1} \xrightarrow{\partial} \Omega^{2,2}\right)$$

"Action" $\frac{1}{2}\int_X \beta\, \bar\partial\, \partial^{-1} \beta$

The fields of the "chiral boson" on X is

$$\Omega^{2,\cdot}_X \xrightarrow{\partial} \Omega^{3,\cdot}_X$$

$-1 \qquad 0$

Has a natural BV str.

$\pi = \partial$

$(\Omega^{\leq 1} \xrightarrow{\partial} \Omega^{\geq 2})$

"Action" $\frac{1}{2}\int_X \beta\, \bar\partial\, \partial^{-1} \beta$

CL type I BS,

free limit

Prop (SW) The hol twist of 6d (1,0) tensor multiplet $\simeq$ "chiral boson"

chiral boson

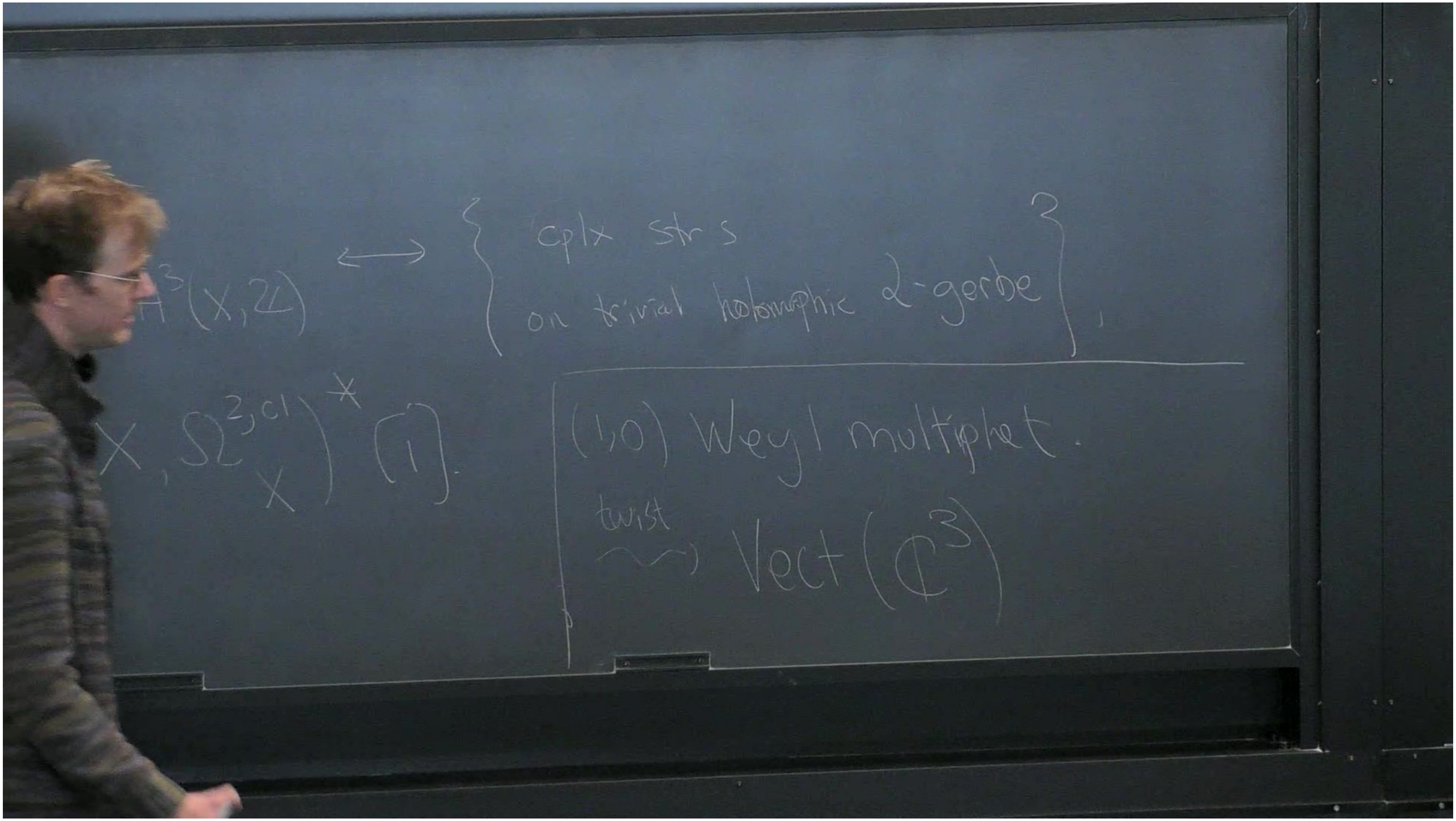
of 6d (1,0) tensor

≃ "chiral boson"

$\mu \in \mathrm{Vect}(X)$

$$T_\mu = \frac{1}{2}\int_X \beta\, \iota_\mu \beta + \underbrace{\# \int_X \beta\, \iota_\mu^2 \beta + \# \int_X \beta\, \iota_\mu^3 \beta}_{\text{"L_∞ terms"}}$$

$$\underline{\lambda \in H^\bullet(\mathrm{vect}(\mathbb{C}^3), \Omega^{2,cl})}$$


H^3(X,ℤ) ⟷ { cplx strs on trivial holomorphic 2-gerbe },
(X, Ω^{3,cl}_X)^× [1].
(1,0) Weyl multiplet.
twist
Vect(ℂ^3)

$, \mathbb{R}) / H^3(X, \mathbb{Z}) \longleftrightarrow \left\{ \begin{array}{l} \text{cplx str's} \\ \text{on trivial holomorphic 2-gerbe} \end{array} \right\}.$

$H^{\cdot}(X, \Omega^{3,cl}_X)^{\times}[1].$

(1,0) Weyl multiplet.

$\overset{\text{twist}}{\rightsquigarrow} \mathrm{Vect}(\mathbb{C}^3)$

$\xrightarrow{\text{twist}} \mathrm{Vect}(\mathbb{C}^3)$

$\longleftrightarrow \quad H_{loc}(\mathrm{Vect}(X)) \simeq \bigoplus_{k=0} H^k(X,\mathbb{C}) \otimes H^{2n+1-k}_{GF}(W_n)$

$H^{2n+1}_{GF}(W_n)$

$H^{2n+2}(BU(n))$

- $n=1$ $\quad H^3_{GF}(W_1) \simeq \mathbb{C}\, c_1^2$

$\mu \longmapsto \int J_\mu \partial J_\mu$

$n=2$: $H^6(BU(2)) = \langle c_1^3, c_1 c_2 \rangle \longleftrightarrow$ a, c anomalies

$N=1$ Weyl multiplet $\rightsquigarrow \mathrm{Vect}(\mathbb{C}^2)$

$($4d $N=2 \rightsquigarrow \mathrm{Vect}(\mathbb{C}^{2|1})$, anomalies $\simeq H^5(\mathrm{Vect}(2|1)) \simeq H^3(BU(2))$

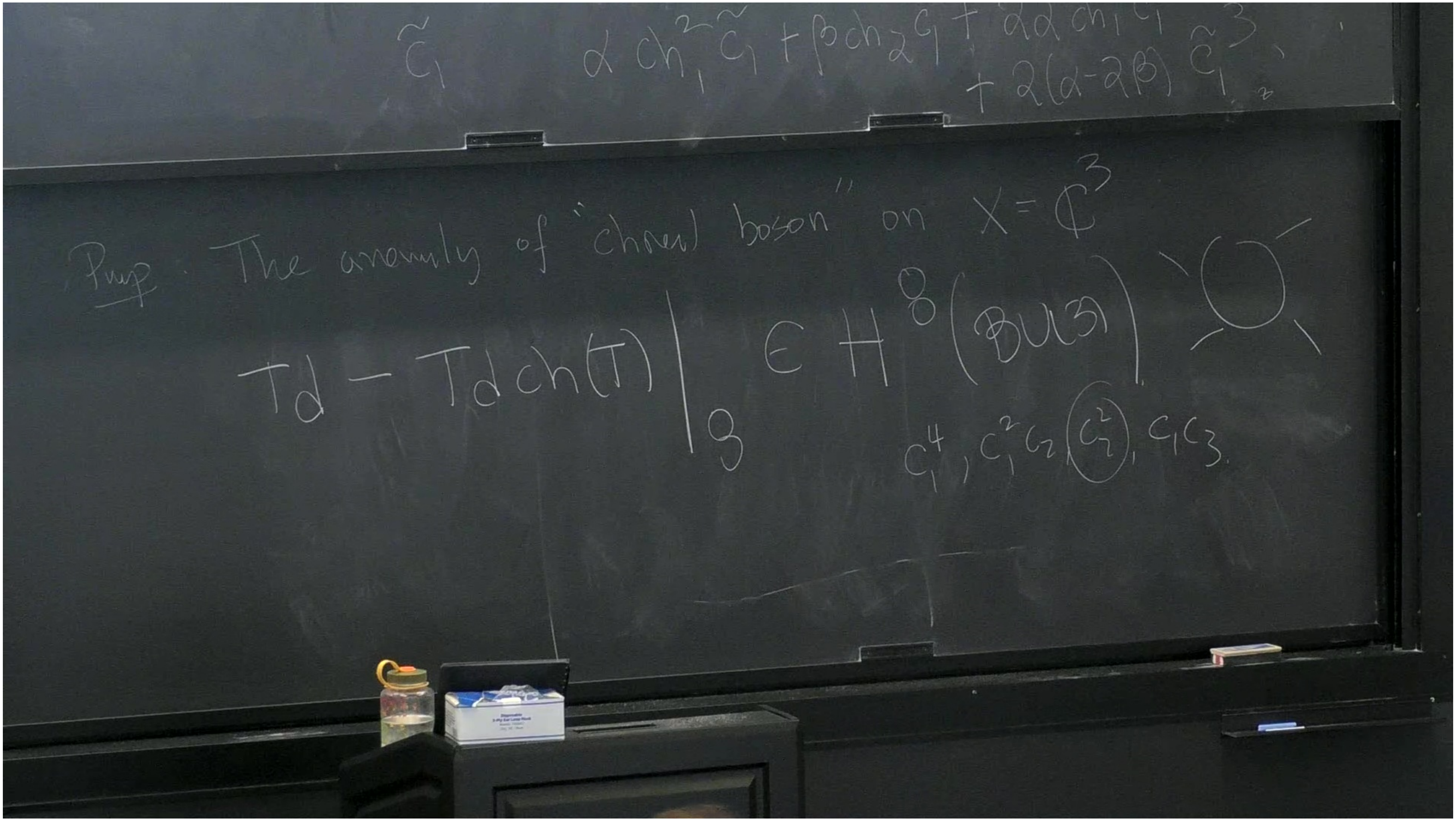
$\cong \mathbb{C}^2\langle \alpha, \beta \rangle$

$\mathrm{Vect}(\mathbb{C}^2) \oplus \mathcal{O} \otimes \mathfrak{gl}(1)$

$ch_1^3, \; ch_1 ch_2 \rightsquigarrow$

$\tilde{c}_1$

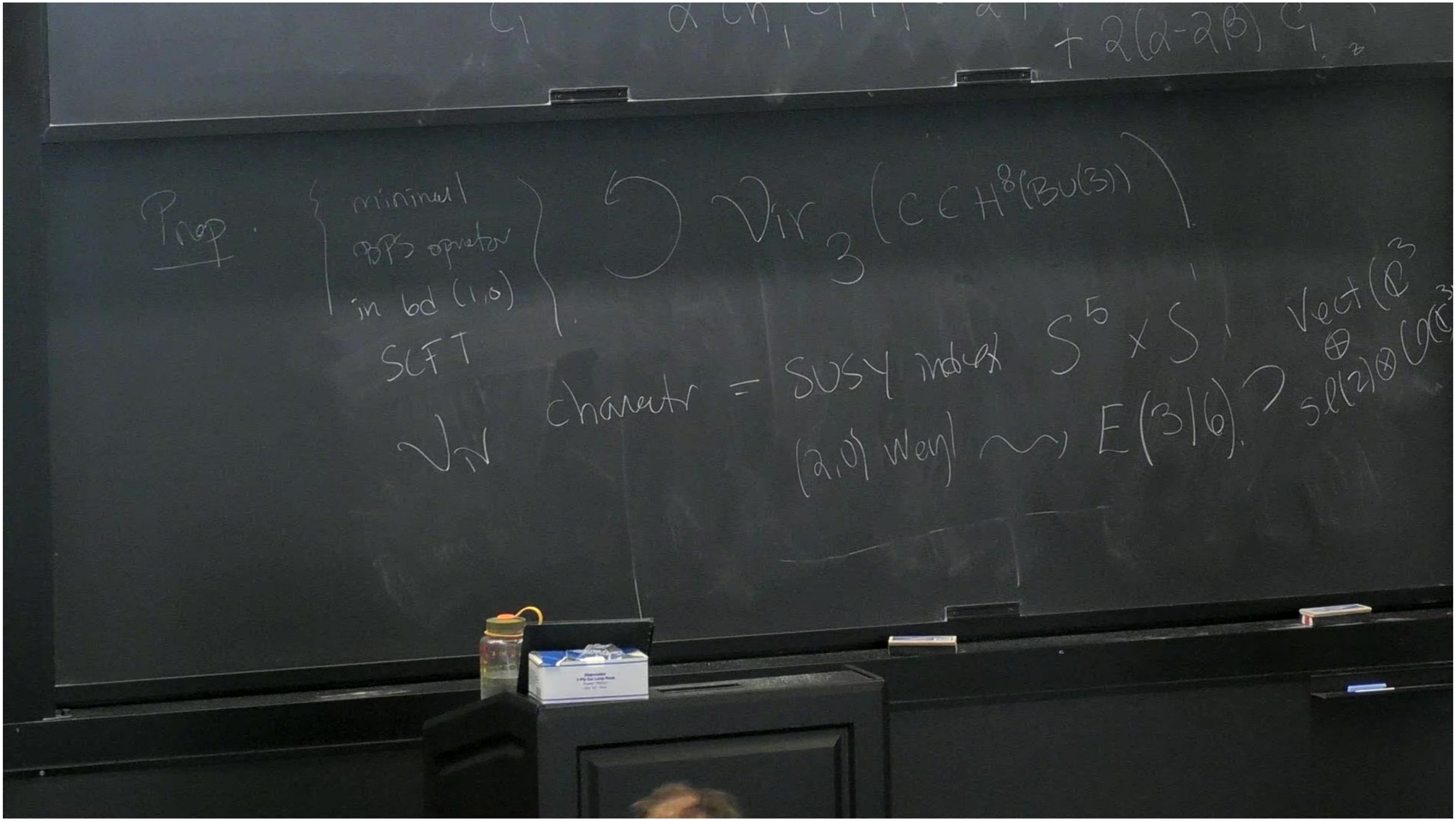
$\alpha\, ch_1^2 \tilde{c}_1 + \beta\, ch_2 \tilde{c}_1 + 2\alpha\, ch_1 \tilde{c}_1^2 + 2(\alpha - 2\beta)\tilde{c}_1^3$

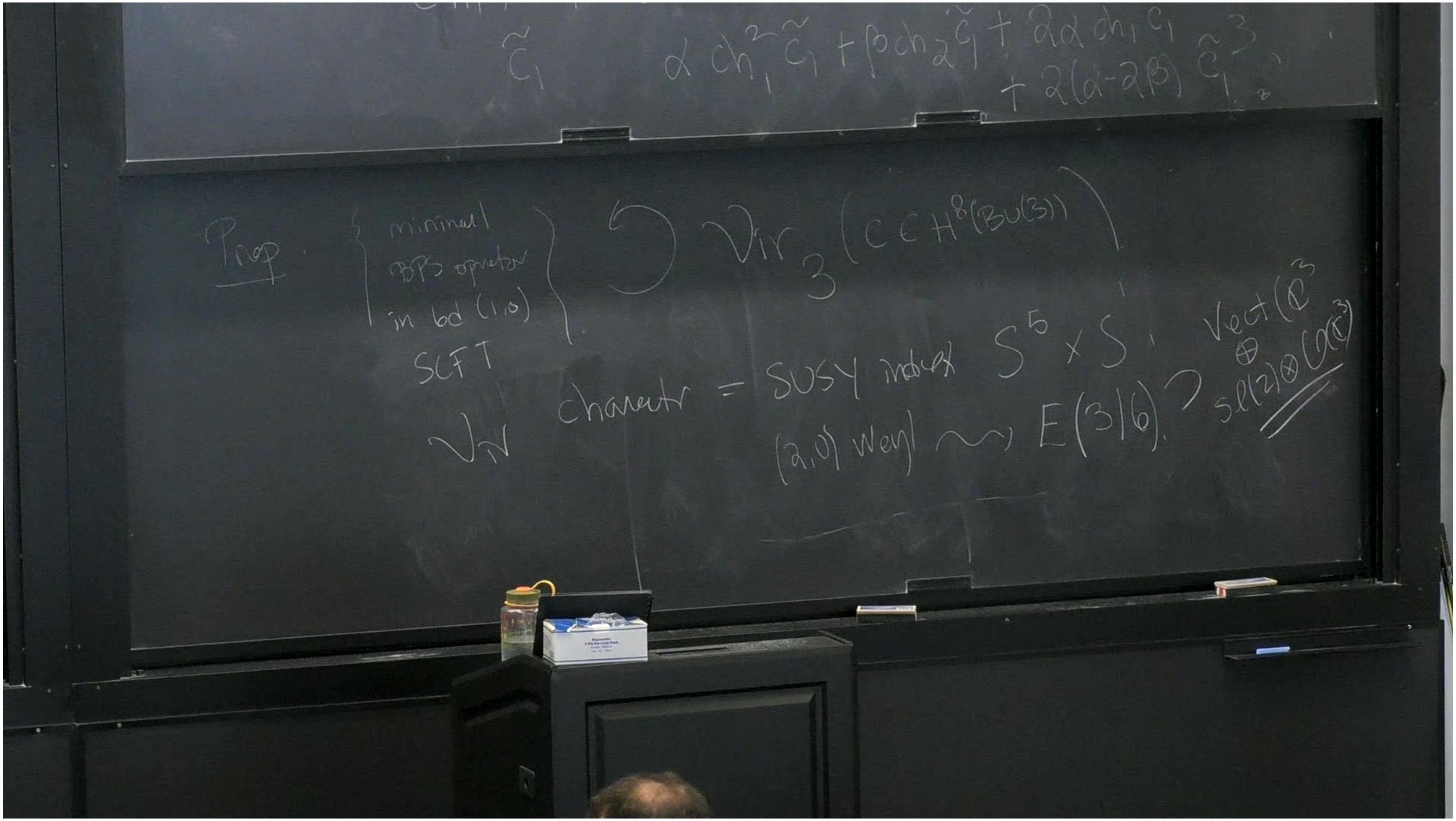

$\tilde{c}_1$ $\alpha\, ch_1^2 \tilde{c}_1 + \beta ch_2 c_1 + 2\alpha ch_1 c_1$ $\tilde{c}_1^3$
$+ 2(\alpha - 2\beta)\tilde{c}_1$
Prop: The anomaly of "chiral boson" on $X = \mathbb{C}^3$
$Td - Td\, ch(T)\big|_8 \in H^8(BU(3))$
$c_1^4,\ c_1^2 c_2,\ c_2^2,\ c_1 c_3.$

$\mathrm{Vect}(\ldots) \simeq \mathrm{Vect}(\mathbb{C}^{\ldots})$

$m_1, m_2 \rightsquigarrow (x,y,z) \mapsto \oint_{S^3} \mathrm{Tr}(Jx)\,\mathrm{Tr}(\partial J y\, \partial J z)$.

$+ \ldots$


+ 2(α-2β) c
Prop.
minimal BPS operator in 6d (1,0) SCFT
Vir_3 (C ⊂ H^8(BU(3)))
Vir character = SUSY index S^5 × S^1
(2,0) Weyl ~> E(3|6) ⊃ Vect(C^3) ⊕ sl(2) ⊗


c̃₁   α ch₁² c̃₁ + β ch₂ c̃₁ + 2α ch₁ c₁ c̃³
+ 2(α−2β) c̃₁
Prop.
minimal
BPS operator
in 6d (1,0)
SCFT
Vir₃ (c ∈ H⁸(BU(3)))
Vir character = SUSY index S⁵ × S¹
(2,0) Weyl ⇝ E(3|6) ⊃ Vect(ℂ³) ⊕ sl(2)⊗O(ℂ³)

$$C_{m_1} C_{m_2} \to (X, Y, Z) \mapsto g \, \mathrm{Tr}(JX) \, \mathrm{Tr}(\partial J Y \, \partial J Z).$$

$$H^7(E(3|6)) \longrightarrow H^7(E(3|6)_+)^{S_3} \quad + \cdots$$

Thm: Conformal SUGRA + 26 free (2,0) this is anomaly free