**Title:** Area metrics: Modified gravity from quantum gravity

**Speakers:** Johanna Borissova

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**Abstract:**

Area metrics define generalized geometric backgrounds to describe spacetime. They are suggested at various places in classical field theory and arise within different approaches to quantum gravity. On these grounds, after introducing the notion of an area metric, I will consider covariant area-metric actions to second order in fluctuations and derivatives. I will then show how these give rise to effective length-metric actions with a distinct nonlocal Weyl-curvature squared term beyond general relativity. Finally, I will point out possible implications and routes for phenomenology.

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# Area metrics: Modified gravity from quantum gravity

Johanna Borissova

collaborations with Bianca Dittrich, Astrid Eichhorn, Pei-Ming Ho, Kirill Krasnov, Marc Schiffer

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# Area metric

[Schuller & Wohlfahrt 2005, Punzi et al 2006]

$G_{\mu\nu\rho\sigma} = G_{\rho\sigma\mu\nu} = - G_{\nu\mu\rho\sigma}$

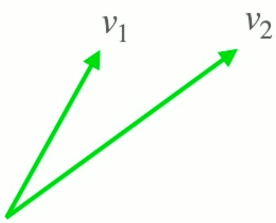
$G_{\mu[\nu\rho\sigma]} = 0$ **cyclicity**

**Metric**

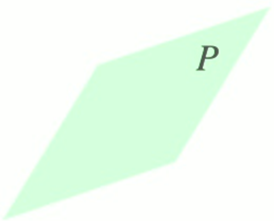
$g_{\mu\nu} = g_{\nu\mu}$

$g$**-induced area metric:** $G^{g}_{\mu\nu\rho\sigma} = g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\rho\nu}$

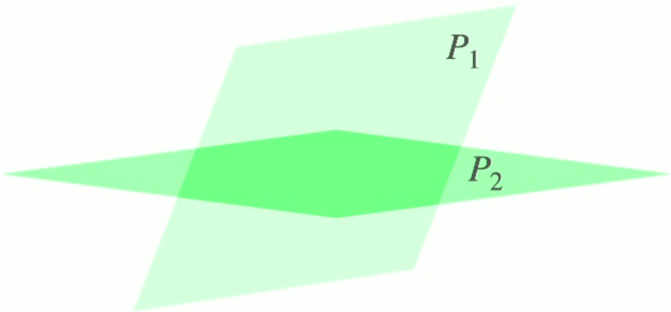

$g(v, v) : l_v^2$




$g(v_1, v_2) : \cos(\angle_{2d} v_1, v_2)$




$G(P; P) : A_P^2$




$G(P_1; P_2) : \cos(\angle_{3d} P_1, P_2)$

$v$ : vector

$P$ : (simple) bi-vector

# Area (metric) variables in QG

**LQG**
[Rovelli & Smolin 1994, Ashtekar & Lewandowski 1996]
$A_j = \gamma l_{pl}^2 \sqrt{j(j+1)}$ area discreteness

**(black-hole) entropy**
[Bekenstein 1972-3, Hawking 1974]
$S_{\text{BH}} = \frac{A}{4 l_{pl}^2}$
[Ryu & Takayanagi 2008]
"area law" for holographic entanglement entropy

**fundamental action(s)**
(Q)ED, YM: $S \supset G^g_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$
Nambu-Goto: $S = \text{Area}(\Sigma)$

$G^g \mapsto G$ generalized background:
e.g. string worldsheet interactions
[JB & Ho 2024]

**modified non-chiral Plebanski theories***
[Plebanski 1997, Reisenberger 1995, Pietri & Freidel 1999, Krasnov 2006-9, Alexandrov & Krasnov 2008, Freidel 2008, Speziale 2010]
GR (classical) = BF (topological) + 20 constraints

[JB & Dittrich 2022]
constraints: impose 10 sharp + 10 "weak" (potential) $\rightarrow S[G]$
$$\mathcal{L}_{eff}(h) = \mathcal{L}_{EH}(h) - \frac{1}{2} C_{\mu\nu\rho\sigma} \left( \frac{1}{\Box + m_+^2} + \frac{1}{\Box + m_-^2} \right) C^{\mu\nu\rho\sigma}$$
**low-energy spin-foam effective action**

**semiclassical regime of (effective) spin foams***
[Dittrich 2021, Dittrich & Kogios 2022]
Area-Regge continuum limit: $S[A_t] \rightarrow S[G]$
$G \leftrightarrow g$ + massive d.o.f.
$\mathcal{L}_{eff}(h) = \mathcal{L}_{EH}(h) + \text{Weyl}^2 + \mathcal{O}(\lambda^4)$

**twisted simplex geometries***
[Dittrich, Padua-Argüelles 2023]
coherent simplex $\sigma$ $\leftrightarrow$ microscopic $G_\sigma$

*strong indications for area metrics from spin-foam QG!

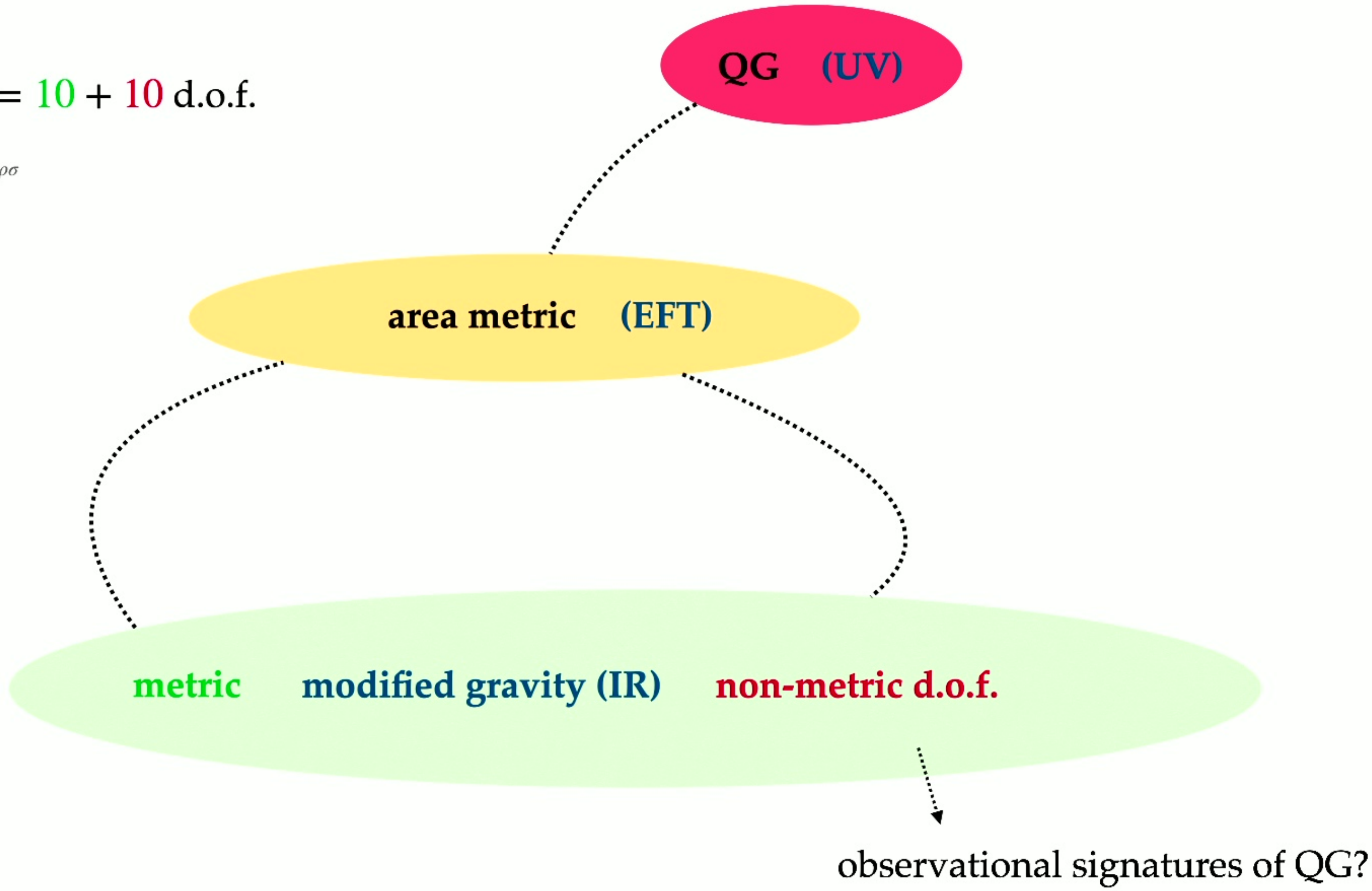
Area metrics = **unified framework for QG**?
- possible route towards establishing **connections between distinct QG theories** and understanding **universal features of QG**

# Aspiration

$d = 4 : 21 - 1 = 10 + 10$ d.o.f.

$G_{\mu\nu\rho\sigma} = G_{\rho\sigma\mu\nu} = - G_{\nu\mu\rho\sigma}$

$G_{\mu[\nu\rho\sigma]} = 0$



# Covariant (perturbative) area metric gravity

## Generally covariant 2nd-order quadratic Lagrangian for area metric perturbations [JB, Dittrich & Krasnov 2023]

1. **expansion** $G = G^\delta$ + fluctuations $a$ $\leftrightarrow$ $(0,0) \oplus (1,1) \oplus (2,0) \oplus (0,2)$ $\leftrightarrow$ $(h_{\mu\nu}, \chi^+_{\mu\nu}, \chi^-_{\mu\nu})$, $\partial^\mu \chi^\pm_{\mu\nu} = 0$, $\delta^{\mu\nu}\chi^\pm_{\mu\nu} = 0$ transverse-traceless
2. **most general ansatz** $\mathscr{L}^{(2)}(a) = \mathscr{L}^{(2)}(h_{\mu\nu}, \chi^+_{\mu\nu}, \chi^-_{\mu\nu})$
3. **linearized diffeomorphism invariance** $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$, $\chi^\pm_{\mu\nu} \to \chi^\pm_{\mu\nu}$

$$\mathscr{L}^{(2)}(a) = \mathscr{L}^{(2)}_{EH}(h) + \frac{1}{2}\sum_\pm \alpha_\pm h_{\mu\nu}\chi^{\pm\mu\nu}p^2 + \frac{1}{2}(\beta_\pm p^2 + m^2_\pm)\chi^\pm_{\mu\nu}\chi^{\pm\mu\nu}$$ $\to$ 4 new free parameters compared to GR!

($\mathscr{L}$ **parity-invariant** $\Leftrightarrow$ couplings "+" $\stackrel{!}{=}$ "−")

## Effective metric action and propagator $\rho_\pm \equiv \frac{\alpha^2_\pm}{\beta_\pm}$, $M^2_\pm \equiv \frac{m^2_\pm}{\beta_\pm}$

$$\mathscr{L}^{(2)}_{eff}(h) = \mathscr{L}^{(2)}_{EH}(h) - \frac{1}{2}{}^{(1)}C_{\mu\nu\rho\sigma}(h)\left(\frac{\rho_+}{\Box + M^2_+} + \frac{\rho_-}{\Box + M^2_-}\right){}^{(1)}C^{\mu\nu\rho\sigma}(h)$$

**Modified gravity (from area metrics) = quasi-local Einstein-Weyl gravity**

**Special subclass:**

$M_+^2 \equiv M_-^2 \equiv M^2, \rho_+ + \rho_- = 2$ (additional shift symmetry):

$\left(\text{Prop}\right)^{\text{spin-2}} \propto \frac{1}{p^2} + \frac{1}{M^2}$ **ghostfree**

$\alpha_\pm \leftrightarrow \gamma_\pm$
$\beta_\pm \leftrightarrow \gamma_\pm$

**Area metric actions for the continuum limit of spin foams**

[JB & Dittrich 2022]

$$\mathscr{L}(a) = \mathscr{L}_{EH}(h) + \frac{1}{2}\sum_\pm \gamma_\pm h_{\mu\nu}\chi^{\pm\mu\nu}p^2$$
$$+\frac{1}{2}(\gamma_\pm p^2 + M_\pm^2)\chi^\pm_{\mu\nu}\chi^{\pm\mu\nu}, \quad \gamma_\pm = 1 \pm \frac{1}{\gamma}$$

**Hamiltonian analysis:** (after Wick-rotation to Lorentzian signature)

2 **massless spin-2 modes (graviton)** + 5 **massive modes with negative energy** (decoupled — stable to this order)

$\gamma$-**dependent mixing of + and × polarizations for spin-2 mode**: $\lambda(\gamma, M^2) \propto \frac{\mathfrak{R}[\alpha_+^L]\,\mathfrak{I}[\alpha_+^L]}{M^2}$

towards GW signatures from area discreteness?

## Spherically symmetric solutions in higher-derivative quasi-local Einstein-Weyl gravity

[JB & Dittrich w.i.p.]

$$S[g] = \frac{m_{\text{Pl}}^2}{2}\int d^4x\sqrt{-g}\left(R - \mu C_{\mu\nu\rho\sigma}\frac{1}{\eta\Box + m^2}C^{\mu\nu\rho\sigma}\right) \rightarrow$$ **localization** via $\psi_{\mu\nu\rho\sigma} \equiv -\left(\eta\Box + m^2\right)^{-1}C_{\mu\nu\rho\sigma} \rightarrow$ **EOM** for $g_{\mu\nu}, \psi_{\mu\nu\rho\sigma}$

**Static spherically symmetric ansatz:** $ds^2 = -A(r)dt^2 + B(r)dt^2 + r^2 d\Omega^2$ + function $\psi(r)$

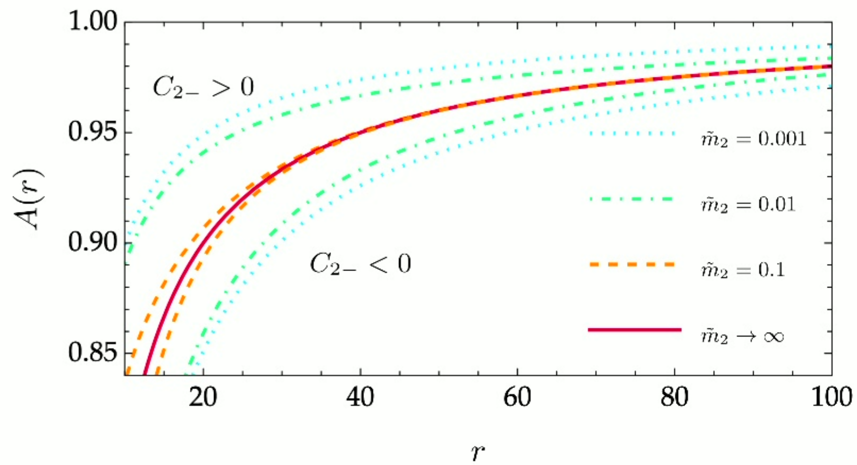
$\mu \to 0$ : GR
$\eta \to 0$ : **Einstein-Weyl gravity —**
$\exists$ spin-2 ghost with squared mass $m_2 \equiv \sqrt{\frac{m^2}{2\mu}}$
$m^2 \to 0$ : non-local Einstein-Weyl gravity

### Weak-field limit

$A(r) = 1 + \delta a(r), B(r) = 1 + \delta b(r), \ \psi(r) = \delta c(r) \ \rightarrow$ solve EOM up to $\mathcal{O}\left(\delta^2\right) \rightarrow$ 2 free parameters for asymptotically flat solutions

$$A(r) = 1 - \frac{2M}{r} + C_{2-}\frac{e^{-\tilde{m}_2 r}}{r}, \ \tilde{m}_2 = \sqrt{\frac{m^2}{2\mu - \eta}}$$

**Yukawa corrections suppressed for** $\mu \to \frac{1}{2}\eta$
**(≙ not GR but ghostfree higher-derivative theory)**



# Prospects

**Take-home message: Area-metric gravity** = (one of the few) **modified-gravity theories supported by QG**

**Future plan:** Take this fact seriously and explore…

- **classical stability** at higher orders

- **phenomenology** associated with the presence of **non-metric d.o.f. + $\gamma$ as parity-breaking parameter**
  - black holes and mimickers: Frobenius-series expansion around $r = 0$ + numerical matching of solutions in different regimes [JB & Dittrich w.i.p.]
  - cosmology: implications of (torsion-induced?) parity breaking for FRW universes

- viability as **quantum EFT**: unitarity,…

- properties in the context of **non-perturbative QFT + asymptotic safety**
  - **RG flows** for masses $M_{\pm}^2$ and $\gamma$ [JB, Dittrich, Eichhorn & Schiffer w.i.p.]