Title: A noncommuting charge puzzle

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Collection/Series: Quantum Information

Subject: Quantum Information

Date: January 08, 2025 - 11:00 AM

URL: https://pirsa.org/25010069

Abstract:

The assumption that conserved quantities, also known as charges, commute underpins many basic physics derivations, such as that of the thermal state's form and Onsager coefficients. Yet, the failure of operators to commute plays a key role in quantum theory, e.g., underlying uncertainty relations. Recently, the study of systems with noncommuting charges has emerged as a growing subfield of quantum many-body physics and revealed a conceptual puzzle: noncommuting charges can hinder thermalization in some ways, yet promote it in others.

In this talk, we address this puzzle in two distinct settings. First, we introduce noncommuting charges into monitored quantum circuits—a toolbox for studying entanglement dynamics. Numerical results reveal a critical phase with long-range entanglement, replacing the area-law phase typically observed in such circuits. This enhanced entanglement indicates noncommuting charges promote entanglement generation, which accompanies thermalization. Second, we consider systems with dynamical symmetries, which are known to violate the Eigenstate Thermalization Hypothesis (ETH), leading to non-stationary dynamics and preventing equilibration, let alone thermalization. We demonstrate that each pair of dynamical symmetries corresponds to a specific charge. Importantly, introducing new charges that do not commute with the existing charges disrupts the associated non-stationary dynamics, thereby facilitating thermalization.

Together, these results shed light on the complex interplay between noncommuting charges, entanglement dynamics, and thermalization in quantum many-body systems.

A noncommuting charge puzzle:
To thermalize or not to thermalize

Majidy, Nature Communications (2024)
Majidy, Braasch, Lasek, Updahyaya, Kalev, Yunger Halpern, Nature Reviews Physics (2023)
Majidy, Agrawal, Gopalakrishnan, Potter, Vasseur, Yunger Halpern PRB (2023)
Majidy, Lasek, Huse, Yunger Halpern PRB (2023)
Yunger Halpern and **Majidy** npj QI (2022)

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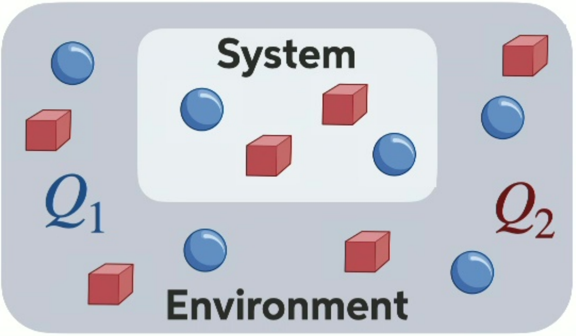
Noncommuting charges

What are they

- Systems exchanging quantities (energy, particles, etc.)
- "Charges" Q_α if conserved globally
- Prevalent and implicit assumption: $[Q_1, Q_2] = 0$
- What if $[Q_1, Q_2] \neq 0$?



Physical differences[1]:

- Reduce entropy-production rates[2] ❄️
- Anomalous deviation from thermal state[3] ❄️
- Restrict dynamics more harshly[4] ❄️

Physical example

[1] Majidy, et al. *Nat Rev Phys* (2023), [2] Manzano, Parrondo, Landi. *PRXQ* (2022), [3] Murthy, Babakhani et al. *PRL* (2023),
[4] Marvian *Nat Pays* (2022)

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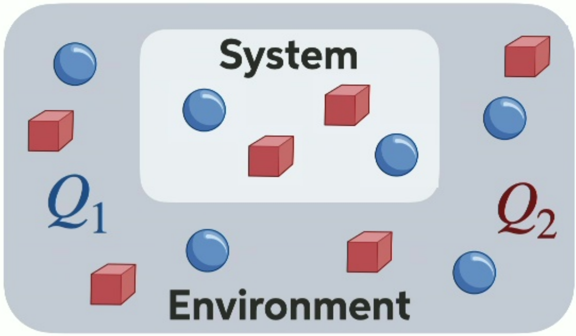

Physical differences[1]:

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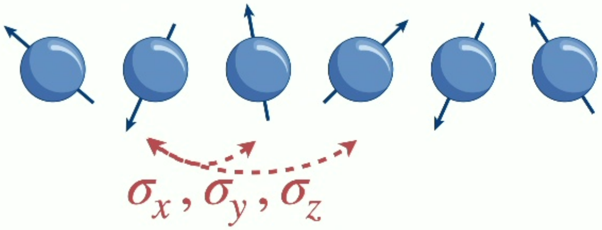
Physical example

- Simple example: Heisenberg spin-chain[5]
- Experimental test with trapped-ions[6]



[1] Majidy, et al. *Nat Rev Phys* (2023), [2] Manzano, Parrondo, Landi. *PRXQ* (2022), [3] Murthy, Babakhani et al. *PRL* (2023),
[4] Marvian *Nat Pays* (2022), [5] NYH, Majidy *npj QI* (2022), [6] Kranzl, Lasek et al. *PRX Quantum* (2022)

To thermalize, or not to thermalize?

Effect on entanglement?[6]

- Isolate charges' noncommutation? with ?
- Construct analogous models
- Noncommuting model has more entanglement on average (Page curves).

Entanglement dynamics[7]

- Introduce noncommuting charges into monitored quantum circuits
- Critical phase

Thermalization of local observables[8]

- Most systems thermalize, some don't
- Hamiltonian's with dynamical symmetries
- Noncommuting charges eliminate dynamical symmetries

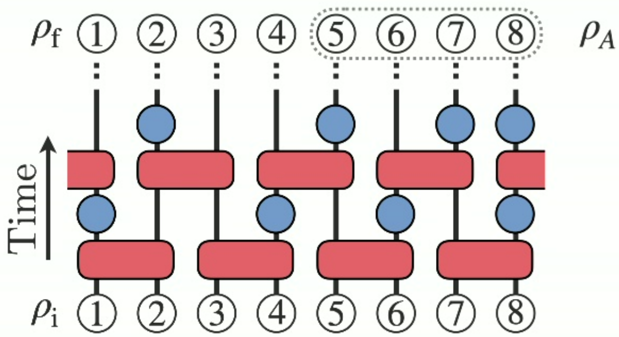
Dynamical Symmetry

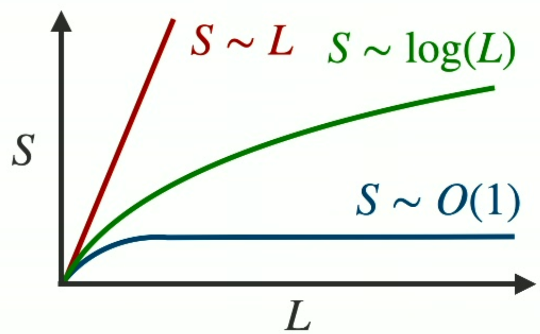
Operator

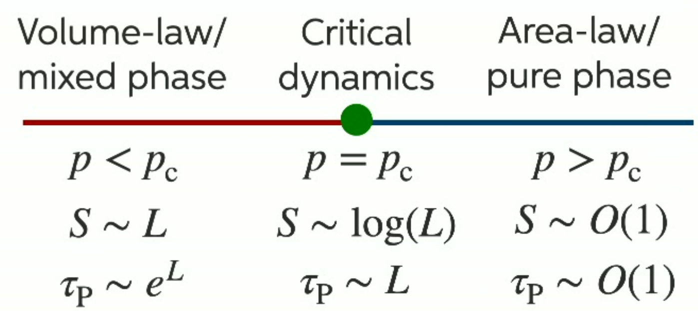
[6] Majidy, Lasek, Huse, NYH *PRB* (2023), [7] Majidy, Agrawal, Gopalakrishnan, Potter, Vassuer, NYH *PRB* (2023), [8] Majidy *Nat Comm* (2024)

Monitored quantum circuits[9-11]

- L qubits in an state ρ_i
- Random unitaries & projective measurements with probability p
- Entanglement phase transition: $S := S(\rho_A)$ at $p = p_c$[12-13]
- Purification phase transition: Initial mixed state purifies at time scale τ_P[14]







[9] Potter, Vasseur *Springer* (2021), [10] Fisher, Khemani, Nahum, Vijay *Annual Reviews* (2022), [11] Skinner *arXiv* (2023)
[12] Skinner, Ruhman, Nahum *PRX* (2019), [13] Li, Chen, Fisher *PRB* (2018), [14] Gullans, Huse *PRX Quantum* (2020),

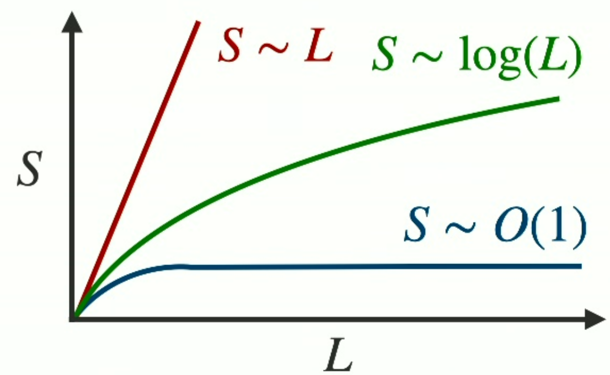
Monitored quantum circuits

Second phase transition when you add a charge[15]

- Charge: $S_z^{\text{tot}} = \sum_{j=1}^{L} \sigma_z^{(j)}$, m is the eigenvalue of S_z^{tot}
- Gates and measurement conserve m.
- Charge-sharpening transition:
  - $p > p_{\text{cs}}$: One can learn the global charge's value from local measurements efficiently
  - $p < p_{\text{cs}}$: One cannot

What if the charges are noncommuting?

- Can you learn the charge?
- Do the entanglement dynamics change?

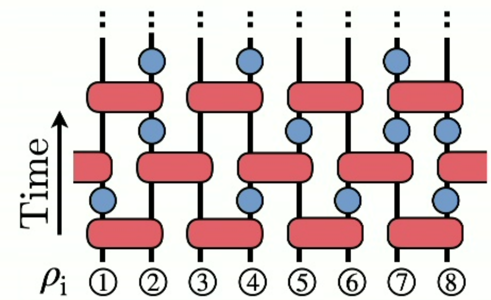


[15] Agrawal, Zabalo, et al. *PRX* (2022)

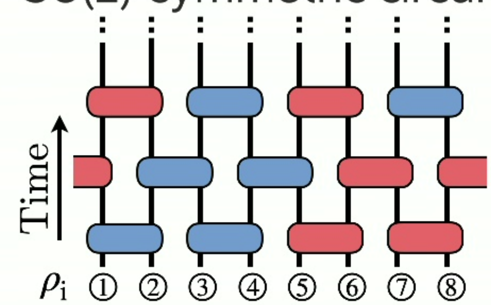
Set-up: SU(2)-symmetric monitored circuit

- Charges are $S_\alpha^{\text{tot}} = \sum_{j=1}^{L} \sigma_\alpha^{(j)}$
- Unitaries, U, and projections operators P:
  $[U, S_\alpha^{\text{tot}}] = 0 = [P, S_\alpha^{\text{tot}}]$ for all α
- $U = \cos(\phi)\mathbb{1} - i\sin(\phi)\text{SWAP}$
- $|\text{s}_0\rangle$ = two-qubit singlet state
  $|\text{t}_m\rangle$ = two-qubit eigenvalue-m triplet state.
- $P_{s_0} = |\text{s}_0\rangle\langle\text{s}_0|$ and $P_{t_m} = \sum_m |\text{t}_m\rangle\langle\text{t}_m|$

Symmetry-free circuit



SU(2)-symmetric circuit



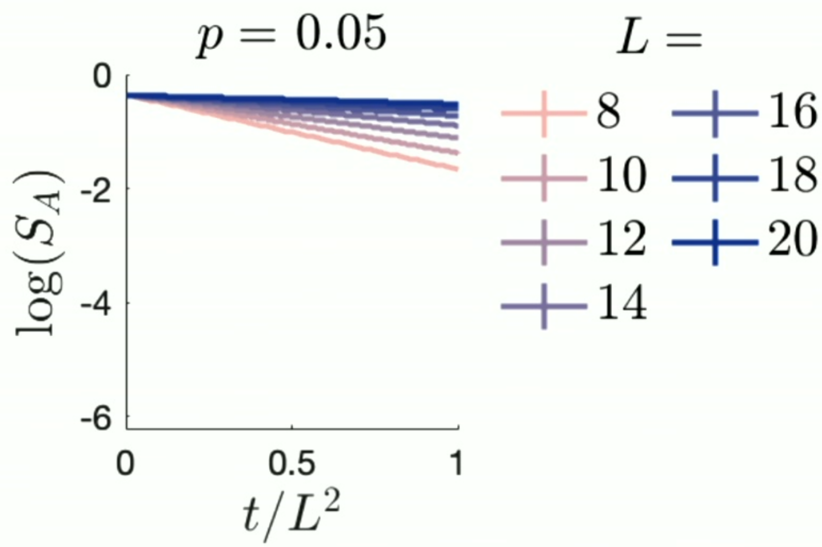
Critical-phase

(Evidence 1) Scale invariance

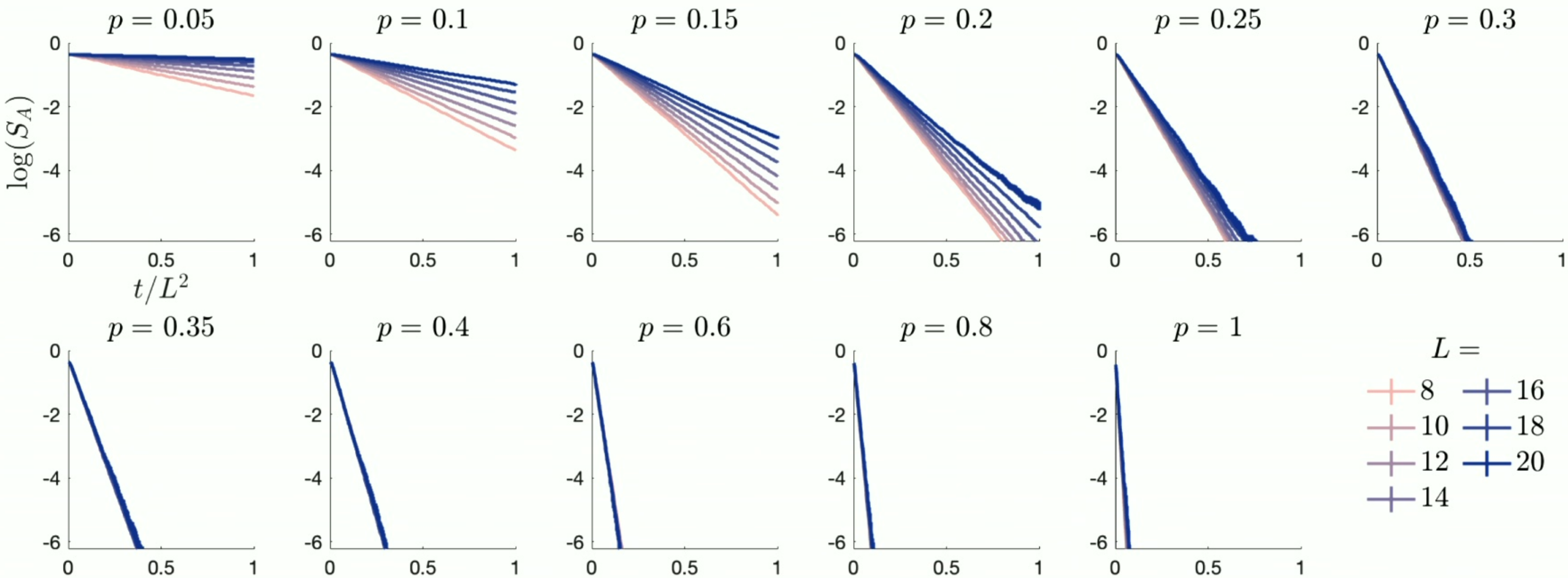
- Scale invariance: $f(L) \to f(\lambda L) = \lambda^{\nu} f(L)$. E.g., power laws: $\tau_{\mathrm{p}} = aL^{b}$
- Measuring τ_{p}[14]: $|\chi_0\rangle$ and $|\chi_1\rangle$ are orthogonal states from the same charge sector.
- Entangle an ancilla: $|\tilde{\psi}_i\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|\chi_0\rangle + |1\rangle_A|\chi_1\rangle\right)$
- Measure the entanglement entropy of the ancilla, S_A. If it purifies, so has the circuit.

[14] Gullans, Huse *PRX Quantum* (2020)

Critical-phase



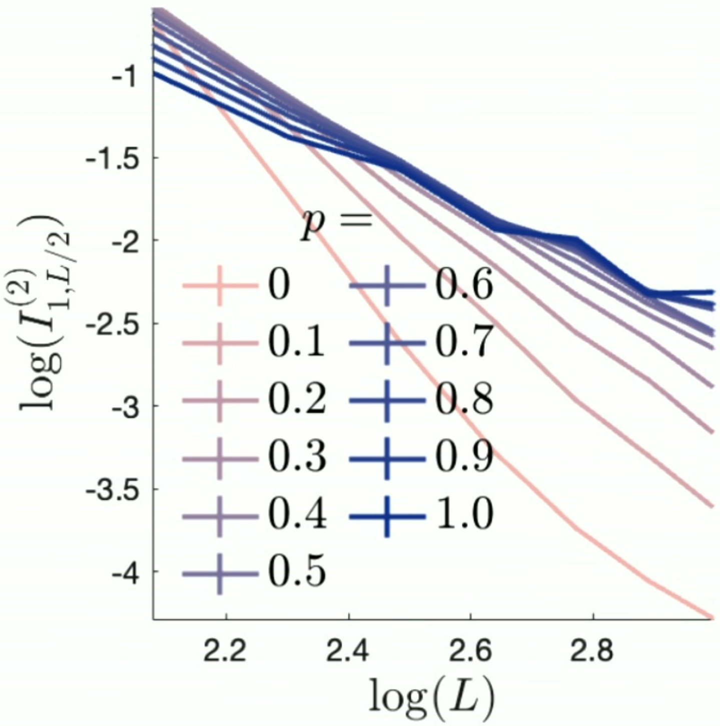
Critical-phase



Critical-phase

(Evidence 2) Large mutual information

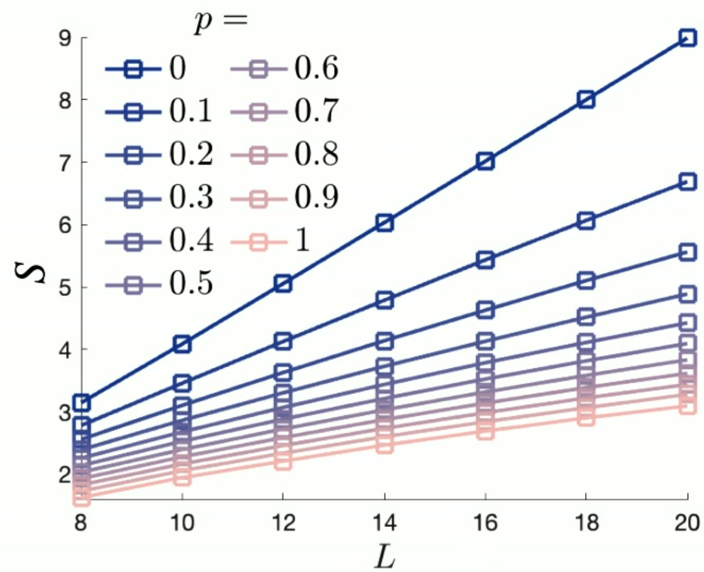
- Mutual information between antipodal pairs of sites, $I^{(2)}_{1,L/2}$
- Critical point: $I^{(2)}_{1,L/2}$ grows quickly as $p \to p_c$, peak, and then quickly decrease.
- Critical phase: $I^{(2)}_{1,L/2}$ grows quickly as $p \to p_c$, and stays large

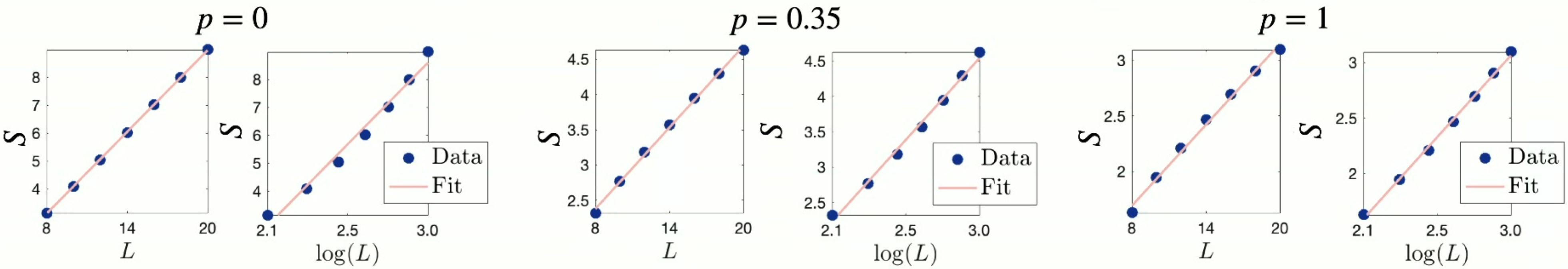


Critical-phase

(Evidence 3) Log entanglement growth

- Clearly, no area-law phase.
- $p < p_c$: Expected volume-law, with $S \sim L$.
- $p > p_c$: Hard to distinguish[16]
- Two-site measurements are not cause for concern





[16] A Moharramipour, LA Lessa, C Wang, TH Hsieh, S Sahu *PRX Q* (2024)

Summary of comparison

Symmetry-free and U(1) cases

Volume-law/ mixed phase	Critical dynamics	Area-law/ pure phase
$p < p_c$	$p = p_c$	$p > p_c$
$S \sim L$	$S \sim \log(L)$	$S \sim O(1)$
$\tau_P \sim e^L$	$\tau_P \sim L$	$\tau_P \sim O(1)$

SU(2) case

Volume-law/ mixed phase		Critical phase
$p < p_c$	$p = p_c$	$p > p_c$
$S \sim L$		$S \sim \log(L)$ *
$\tau_P \sim e^L$		$\tau_P \sim L^2$

- The volume-law phase is expected ($S \sim L, \tau_P \sim e^L$)
- Purification numerics demonstrate $\tau_P \sim L^2$
- Entanglement numerics clearly show no area-law and are consistent with $S \sim \log(L)$
- Mutual information numerics also indicate a critical phase

Again, suggestive of more entanglement

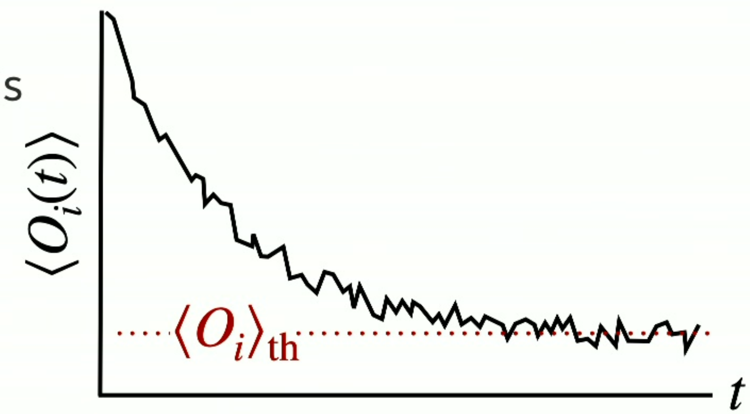
Noncommuting charges remove a type of non-thermalizing behaviour

S Majidy *Nature Communications* (2024)

Thermalization of quantum systems

How can unitary dynamics lead to thermalization?

- Eigenstate thermalization hypothesis (ETH)[28-29]
- Local observables O_i, state $|\psi\rangle$, exp val $\langle O_i(t)\rangle$, & thermal exp val $\langle O_i\rangle_{\text{th}} := \text{tr}[e^{-\beta H}O_i]$.
- Dictates physical conditions for which this expectation holds



Violations of the ETH

- Stationary, non-thermal: Many-body localization (MBL)[30]
- Non-stationary dynamics: Quantum scars[31,32] and dynamical symmetries[33]

Effects of noncommuting charges

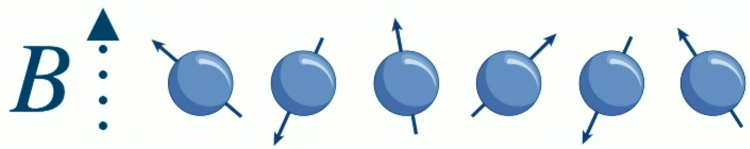
- Force noncommuting charges onto ETH violations
- Destabilize MBL[34]. But, what about non-stationary dynamics?

[28] Srednicki *PRE* (1994), [29] Deutsch *PRA* (1991), [30] Nandkishore, Huse *Annual Reviews* (2014) , [31] H. Bernien et al. *Nature* (2017)
[32] E. Heller *PRL* 1984 , [33] Buča, Tindall, Jaksch *Nat Comm* (2019), [34] A Potter, R Vasseur *PRB* (2016)

Dynamical symmetries

What are they?

- Extensive operators $A_{\pm\gamma}$ s.t. $\boxed{[H, A_{\pm\gamma}] = \pm\lambda A_{\pm\gamma}}$ $(\lambda \in \mathbb{R}, \lambda \neq 0)$.
- All O_i with $\mathrm{tr}[O_i A_{\pm\gamma}] \neq 0$ won't thermalize.
- Sufficient conditions. Exist in open/closed systems. Responsible for quantum time crystals, quantum attractors, etc…
- Not conventional "symmetries"
- Example: $H = B\sum_{j=1}^{N}\sigma_z^{(j)}$. $A_{\pm\gamma} = \sum_{j=1}^{N}(\sigma_x^{(j)} \pm i\sigma_y^{(j)})$. $O_i = \sum_i \sigma_x^{(i)}$ has persistent and potentially large—up to $\mathcal{O}(N)$ in this example—oscillations



The connection

Dynamical Symmetries —[Lemma 1: Always]→ Charges

Theorem 1: For every pair of dynamical symmetries that a Hamiltonian has, there exists a charge $Q_\beta = [A_{+\beta}, A_{-\beta}]$.

- *Proof:* Conserved, $[H, Q_\beta] = 0$. Hermitian, $Q_\beta^\dagger = Q_\beta$. Extensive, $A_{\pm\beta}$ are extensive.

Theorem 2: For a wide class of charges, there exists a Hamiltonian that conserves those charges and has dynamical symmetries which can be found from the charges algebraic structure.

Noncommuting charges block dynamical symmetries

Example ($\tau_\alpha^{\text{tot}} = \sum_{j=1}^{N} \tau_\alpha^{(j)}$, τ_i are Gell—Mann matrices):

$$H = \frac{J}{2}\Big(\sum_{\alpha}\sum_{\langle j,k\rangle}\sum_{\langle\langle j,k\rangle\rangle} \tau_\alpha^{(j)}\tau_\alpha^{(k)}\Big) + \frac{B_1}{2}\Big(\sum_j \tau_3^{(j)}\Big) + \frac{B_2}{2}\Big(\sum_j \tau_8^{(j)}\Big)$$

$$\tau_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

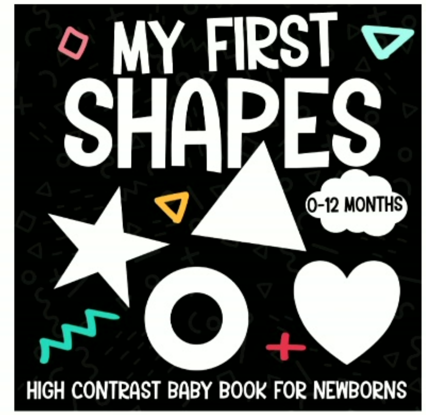
$$\tau_8 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

- Abelian subalgebra of $\mathfrak{su}(3)$: $B_1 \neq 0 \neq B_2$.
  - 2 Charges: $\tau_3^{\text{tot}}, \tau_8^{\text{tot}}$
  - 6 Dynamical symmetries: Sums over $A_{\pm k}$
- Full $\mathfrak{su}(3)$: $B_1 = 0 = B_2$

$$A_{\pm 1} = \tau_1 \pm i\tau_2$$
$$A_{\pm 2} = \tau_4 \pm i\tau_5$$
$$A_{\pm 3} = \tau_6 \pm i\tau_7$$

Schematic depiction of main result

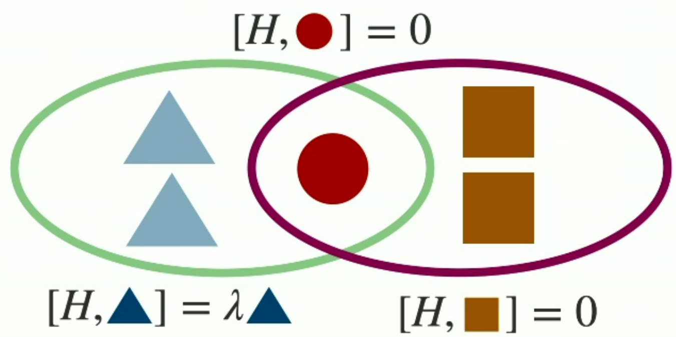
- Start with a Hamiltonian H with **dynamical symmetries**



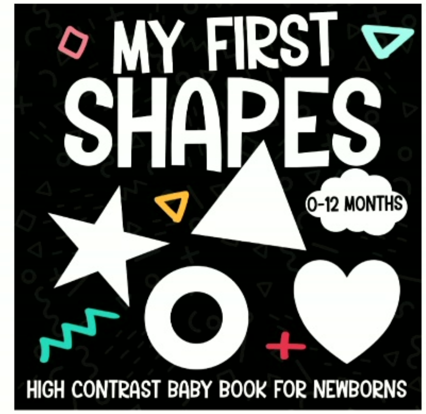
$$[H, \blacktriangle] = \lambda \blacktriangle$$

Schematic depiction of main result

- Start with a Hamiltonian H with **dynamical symmetries**
- Theorem 1: From the dynamical symmetries identify the **charges**
- Dynamical symmetries and charges necessarily form a basis for a non-Abelian algebra (Cartan-Weyl basis)



- Consider a set of charges that do not commute with the existing ones
- The set of noncommuting charges form a basis for the algebra



Summary of examples

	Subalgebra of su(2): **Abelian**	Two subalgebras of su(2): **Abelian**	Subalgebra of su(3): **Abelian**	Full su(2): **Non-abelian**	Full su(3): **Non-abelian**
Example	XXX model + field	Hubbard model	Eq. 26 of main text	XXX model	Eq. 26 of main text
Charge	S_z^{tot}	$S_z^{\text{tot}}, \eta_z^{\text{tot}}$	$\tau_3^{\text{tot}}, \tau_8^{\text{tot}}$	$S_\alpha^{\text{tot}}, \alpha = x, y, z$	$\tau_i^{\text{tot}}, i = 1,\ldots,8$
Dynamical Symmetry	$S_{\pm z}^{\text{tot}}$	$S_{\pm z}^{\text{tot}}, \eta_{\pm z}^{\text{tot}}$	$A_{\pm 1}^{\text{tot}}, A_{\pm 2}^{\text{tot}}, A_{\pm 3}^{\text{tot}}$		

Summary

- Address how noncommuting charges affect thermalization in many-body systems
- Each pair of dynamical symmetries corresponds to a specific charge
- Introduce commuting charges, dynamics are in tact.
- Introduce non-commuting charges, eliminate non-stationary dynamics

The bigger ~~picture~~ puzzle

Hinder thermalization:

- Reduce entropy-production rates[2]
- Anomalous deviation from thermal state[3]
- Constraint global unitaries implementable locally more than commuting charges[18]
- Invalidate two derivations of the thermal state's form[19-20]

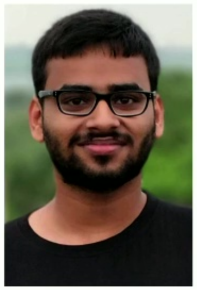
Promote thermalization:

- Increased entanglement[21-22]
- Blocks various forms of non-thermalizing dynamics
  - Many-body localization[23]
  - Dynamical symmetries[24]
  - Quantum scars[25]

[2] Manzano, Parrondo, Landi. *PRX Quantum* (2022), [3] Murthy, Babakhani et. al. *PRL* (2023), [18] Marvian *Nat Phys* (2022),
[19] NYH et al. *Nat Comm* (2016), [20] NYH *J Phys A* (2018), [21] Majidy, Lasek, Huse, NYH *PRB* (2023), [22] Majidy, Agrawal, et al. *PRB* (2023)
[23] Potter, Vasseur *PRB* (2016), [24] Majidy, *Nat Comm* (2024), [25] O'Dea, Burnell, Chandran, Khemani *PRR* (2022)

Thanks for listening

Collaborators 🙏



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