Title: Black hole spectroscopy

Speakers: Emanuele Berti

Collection/Series: Strong Gravity

Subject: Strong Gravity

Date: January 16, 2025 - 1:00 PM

URL: https://pirsa.org/25010068

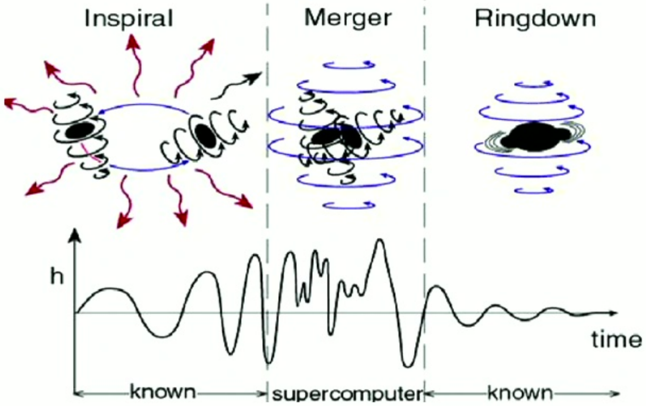
Abstract:

According to general relativity, the remnant of a binary black hole merger is a perturbed Kerr black hole. Perturbed Kerr black holes emit "ringdown" radiation which is well described by a superposition of damped exponentials ("quasinormal modes”), with frequencies and damping times that depend only on the mass and spin of the remnant. The observation of gravitational radiation emitted by black hole mergers might finally provide direct evidence of black holes, just like the 21 cm line identifies interstellar hydrogen. I will review the current status of this "black hole spectroscopy" program. I will focus on: (1) the role of nonlinearities in ringdown modeling, (2) the current observational status of black hole spectroscopy, and (3) future prospects for the observability of modified gravity effects and nonlinear modes.

Black hole spectroscopy

Emanuele Berti, Johns Hopkins University

Gravity Seminar, Perimeter Institute
Waterloo, Canada
January 16 2025



Talk based largely on:

Cotesta+ 2201.00822, Cheung+ 2208.07374, Baibhav+ 2302.03050, Redondo-Yuste+ 2308.14796, Cheung+ 2310.04489, Yi+ 2403.09767, Berti-Cardoso-Carullo+, Berti-Cheung-Yi (to appear)

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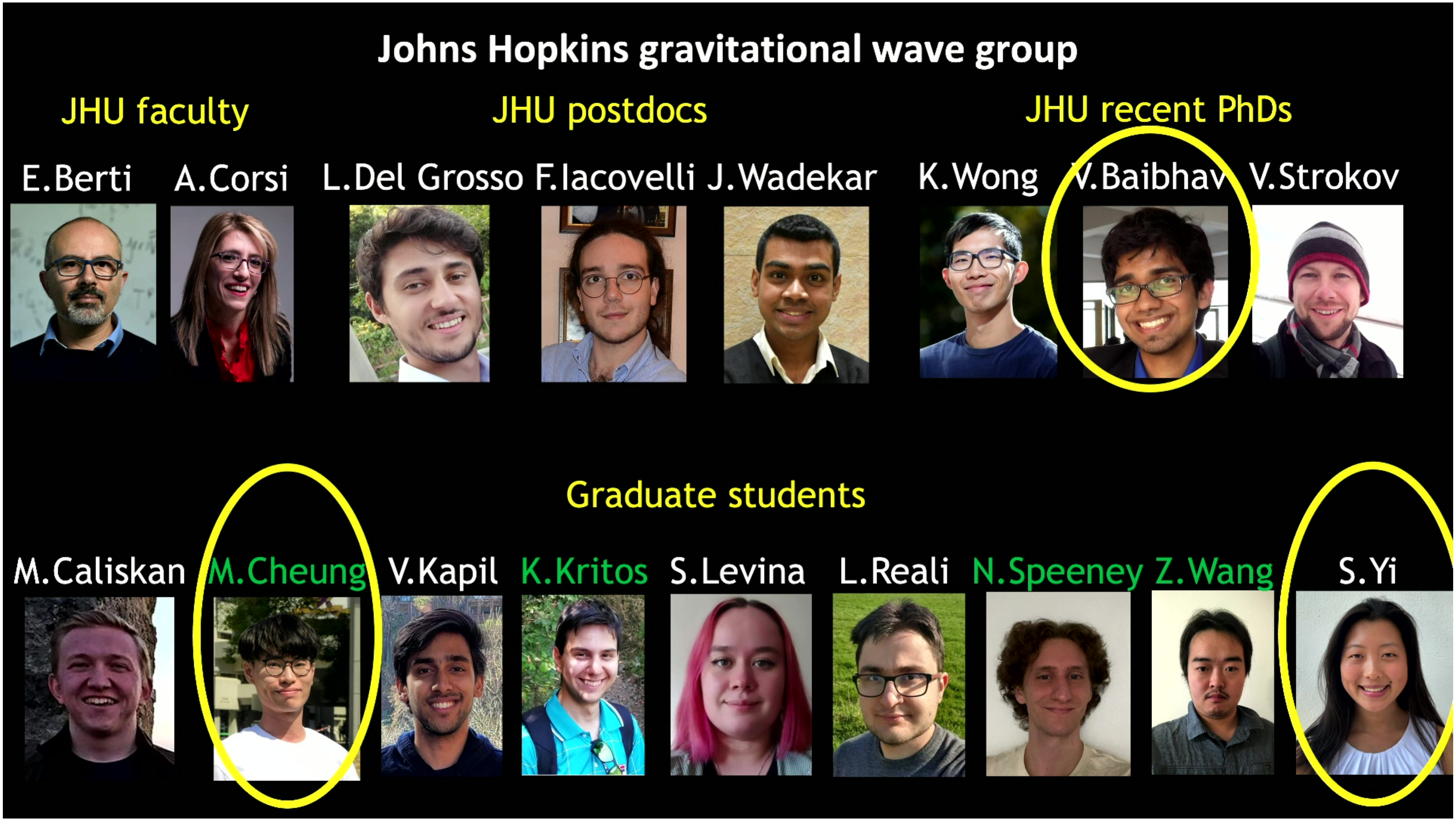


Johns Hopkins gravitational wave group

JHU faculty

E.Berti A.Corsi

JHU postdocs

L.Del Grosso F.Iacovelli J.Wadekar

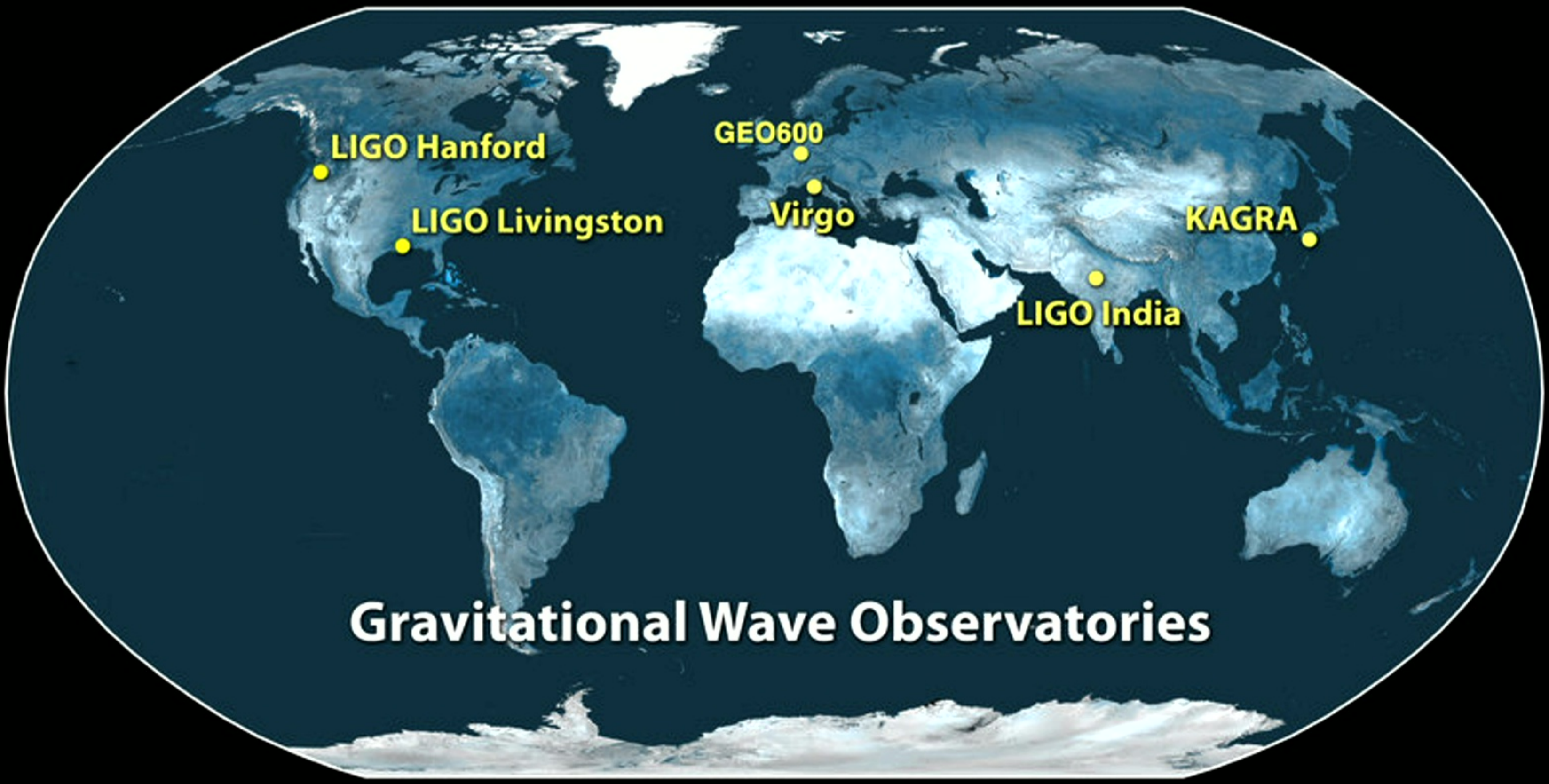
JHU recent PhDs

K.Wong V.Baibhav V.Strokov

Graduate students

M.Caliskan M.Cheung V.Kapil K.Kritos S.Levina L.Reali N.Speeney Z.Wang S.Yi

Present and future detectors

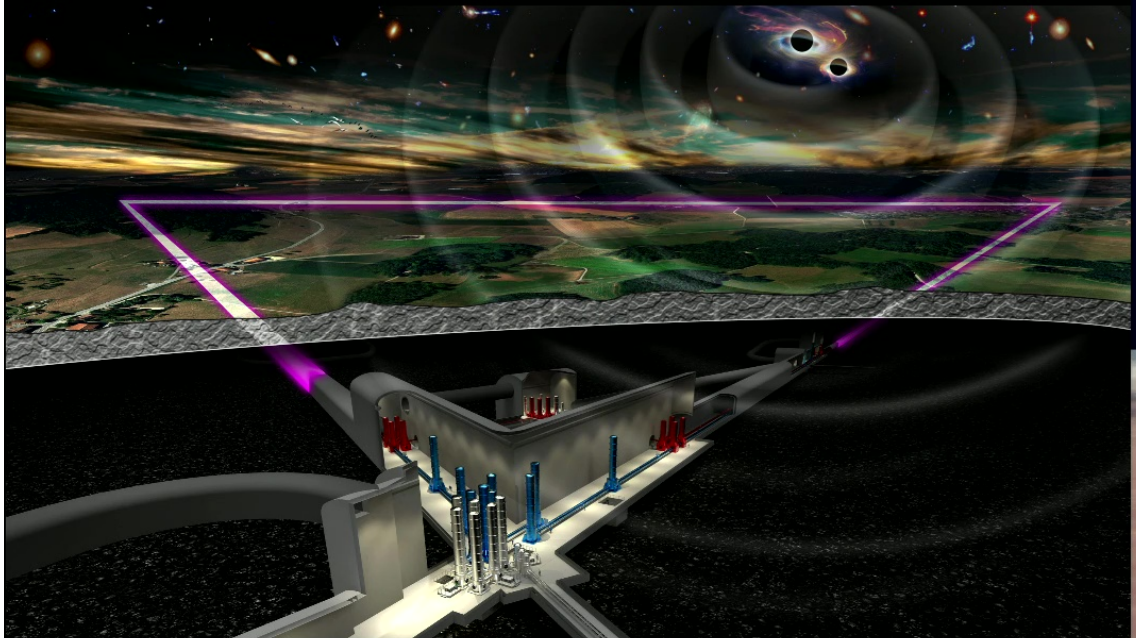


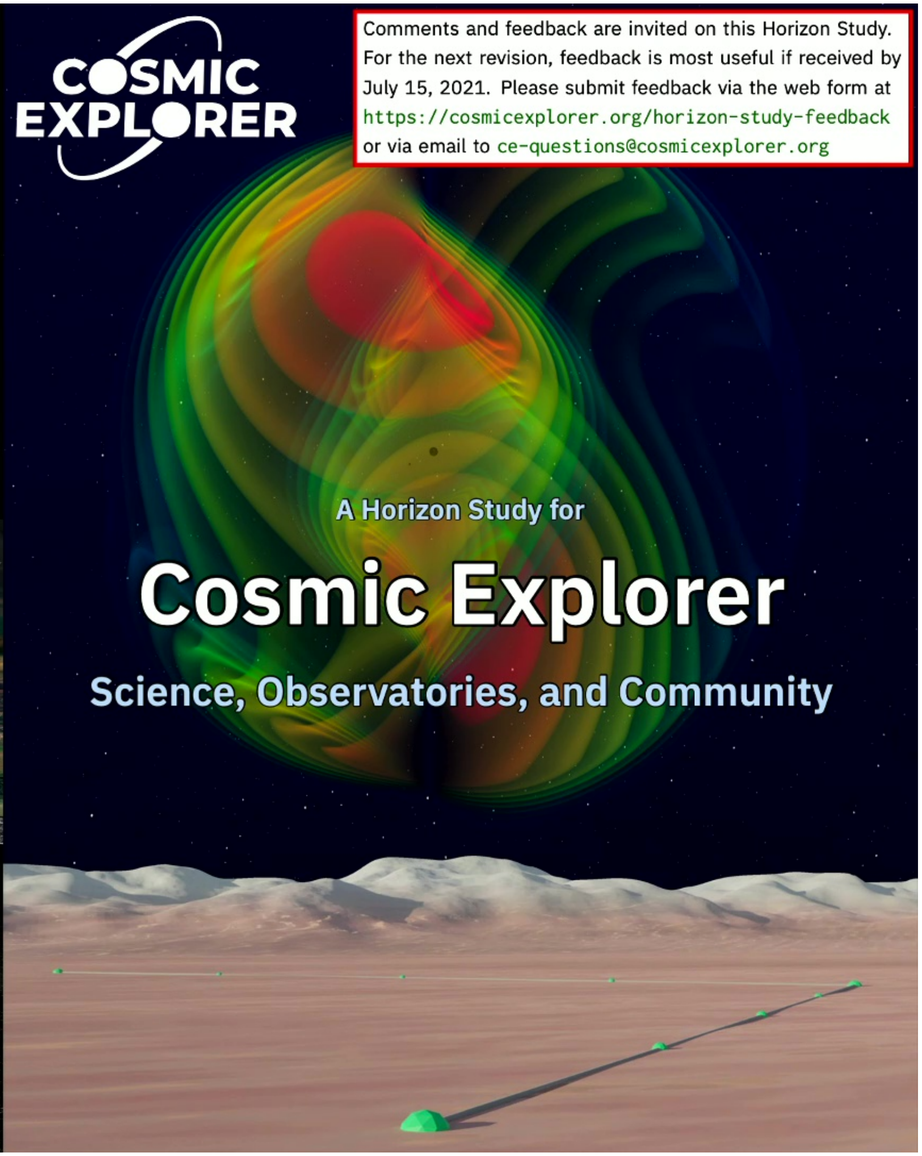
Also “current generation”: PTAs, A+, A#

Next generation (XG)

Einstein Telescope 2021 ESFRI, site(s)?
Cosmic Explorer Horizon Study, MPSAC

Moon? (LGWA, LILA, LSGA…)
Atom interferometry, NEMO, MHz





COSMIC EXPLORER

Comments and feedback are invited on this Horizon Study. For the next revision, feedback is most useful if received by July 15, 2021. Please submit feedback via the web form at `https://cosmicexplorer.org/horizon-study-feedback` or via email to `ce-questions@cosmicexplorer.org`

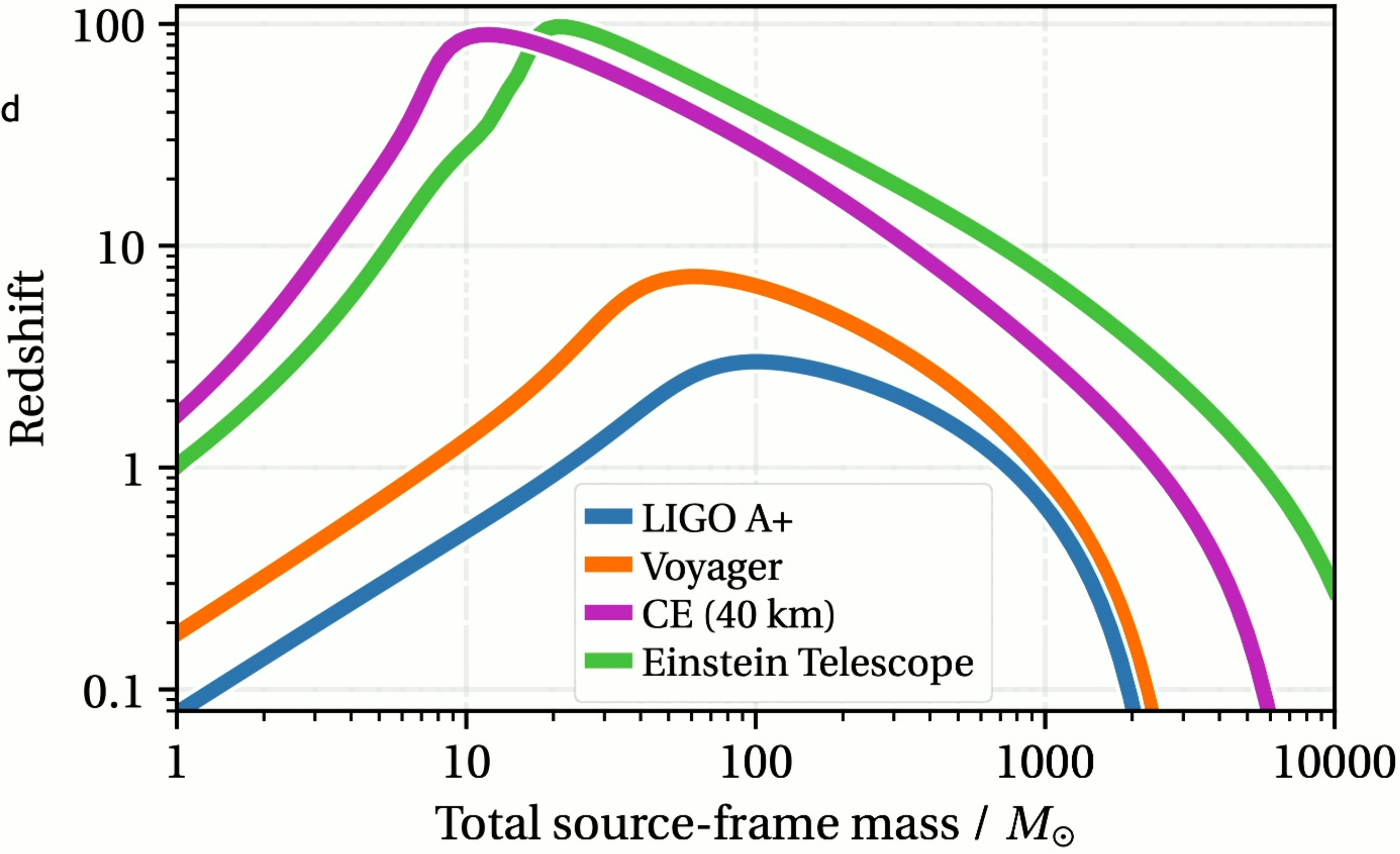
A Horizon Study for

Cosmic Explorer

Science, Observatories, and Community

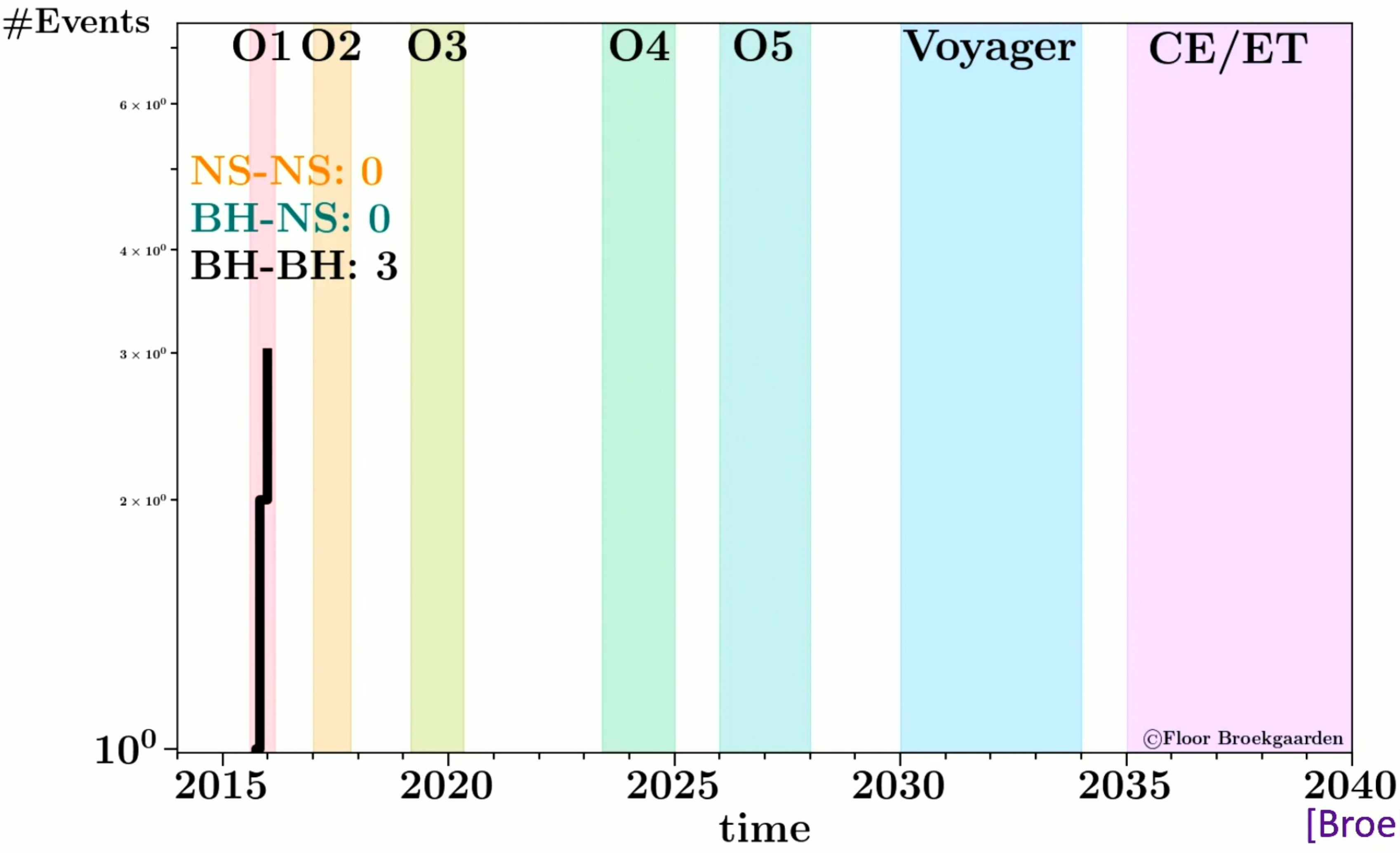
From 2G to XG: opportunities and challenges

- Factor of 10 in sensitivity and lower frequency bound
- ET: triangle, or two L-shaped
- CE: CE40 and CE20
- Hundreds of signals every day!
- Hundreds of BNSs/ thousands of BBHs every year with SNR>100
- **Opportunities:**
  New synergies, null stream?
  New algorithms (MDCs)
- **Challenges:**
  Long waveforms
  Overlapping signals
  Waveform systematics
  Strong foreground



[Credits: Sathya]

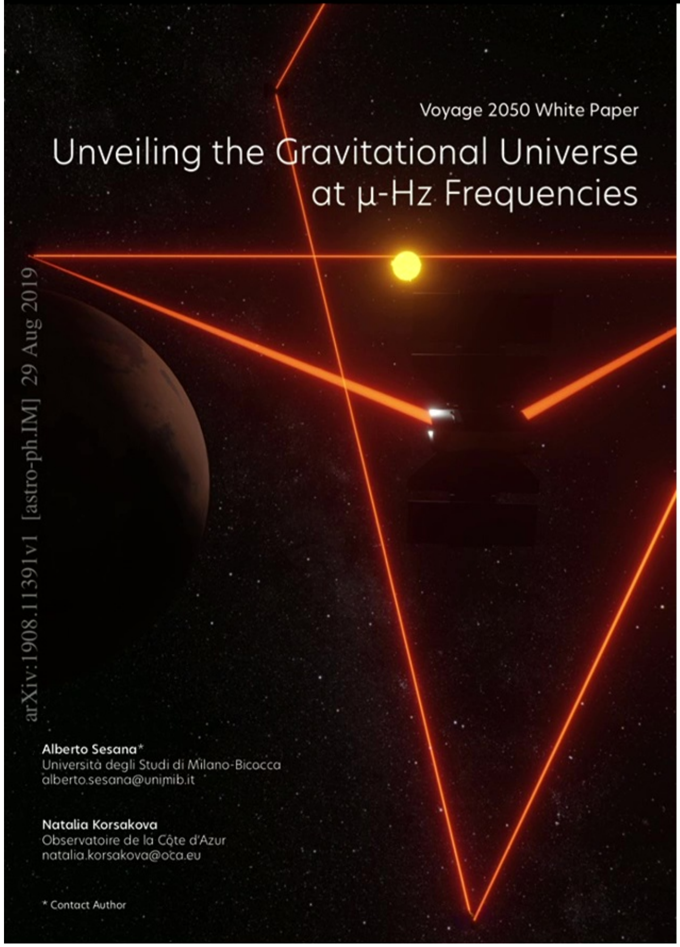
Gravitational wave astronomy in the 2030s: Big Data



[Broekgaarden]

Space: LISA, TianQin, Taiji, and far future detectors beyond LISA

Lower frequencies? Better sensitivity? Decihertz?



arXiv:1908.11391v1 [astro-ph.IM] 29 Aug 2019

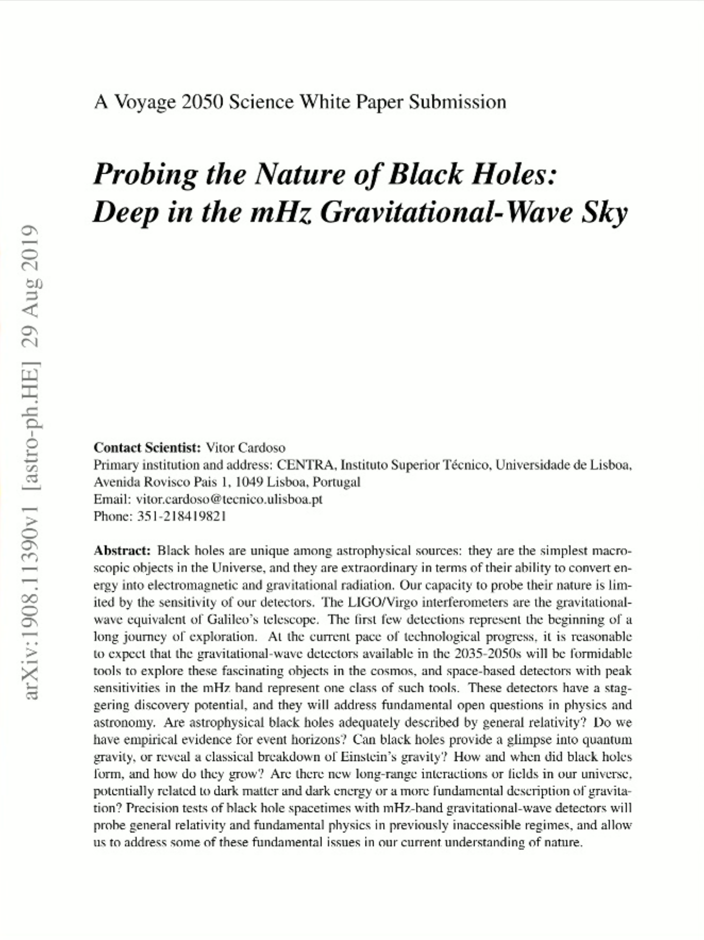
Voyage 2050 White Paper

Unveiling the Gravitational Universe at μ-Hz Frequencies

Alberto Sesana*
Università degli Studi di Milano-Bicocca
alberto.sesana@unimib.it

Natalia Korsakova
Observatoire de la Côte d'Azur
natalia.korsakova@oca.eu

* Contact Author



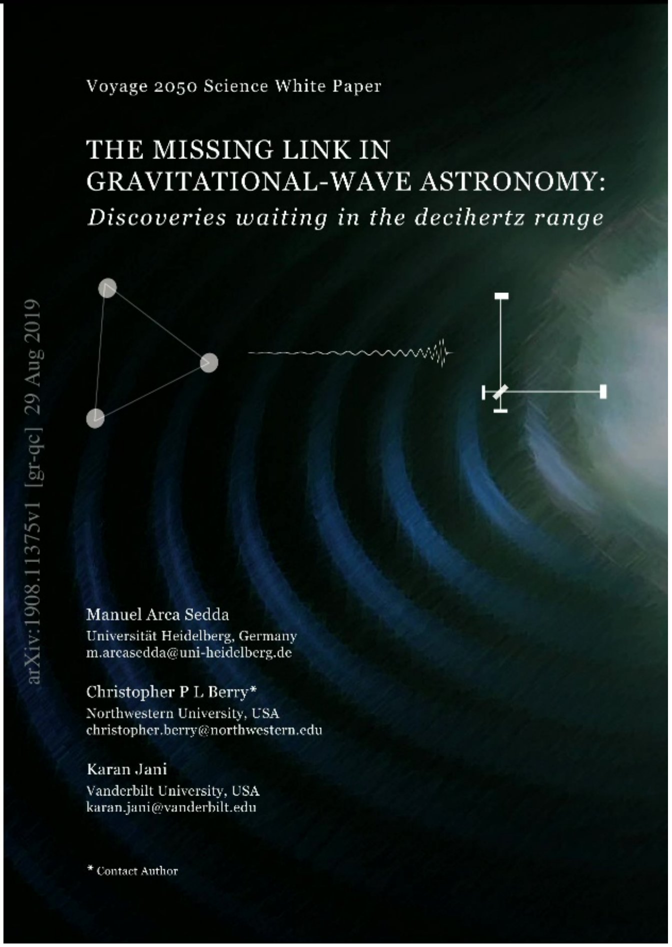
arXiv:1908.11390v1 [astro-ph.HE] 29 Aug 2019

A Voyage 2050 Science White Paper Submission

Probing the Nature of Black Holes: Deep in the mHz Gravitational-Wave Sky

Contact Scientist: Vitor Cardoso
Primary institution and address: CENTRA, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal
Email: vitor.cardoso@tecnico.ulisboa.pt
Phone: 351-218419821

Abstract: Black holes are unique among astrophysical sources: they are the simplest macroscopic objects in the Universe, and they are extraordinary in terms of their ability to convert energy into electromagnetic and gravitational radiation. Our capacity to probe their nature is limited by the sensitivity of our detectors. The LIGO/Virgo interferometers are the gravitational-wave equivalent of Galileo's telescope. The first few detections represent the beginning of a long journey of exploration. At the current pace of technological progress, it is reasonable to expect that the gravitational-wave detectors available in the 2035-2050s will be formidable tools to explore these fascinating objects in the cosmos, and space-based detectors with peak sensitivities in the mHz band represent one class of such tools. These detectors have a staggering discovery potential, and they will address fundamental open questions in physics and astronomy. Are astrophysical black holes adequately described by general relativity? Do we have empirical evidence for event horizons? Can black holes provide a glimpse into quantum gravity, or reveal a classical breakdown of Einstein's gravity? How and when did black holes form, and how do they grow? Are there new long-range interactions or fields in our universe, potentially related to dark matter and dark energy or a more fundamental description of gravitation? Precision tests of black hole spacetimes with mHz-band gravitational-wave detectors will probe general relativity and fundamental physics in previously inaccessible regimes, and allow us to address some of these fundamental issues in our current understanding of nature.



arXiv:1908.11375v1 [gr-qc] 29 Aug 2019

Voyage 2050 Science White Paper

THE MISSING LINK IN GRAVITATIONAL-WAVE ASTRONOMY:
Discoveries waiting in the decihertz range

Manuel Arca Sedda
Universität Heidelberg, Germany
m.arcasedda@uni-heidelberg.de

Christopher P L Berry*
Northwestern University, USA
christopher.berry@northwestern.edu

Karan Jani
Vanderbilt University, USA
karan.jani@vanderbilt.edu

* Contact Author

The Schwarzschild metric

November 18, 1915:
Schwarzschild metric

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$r = 0$: physical curvature singularity

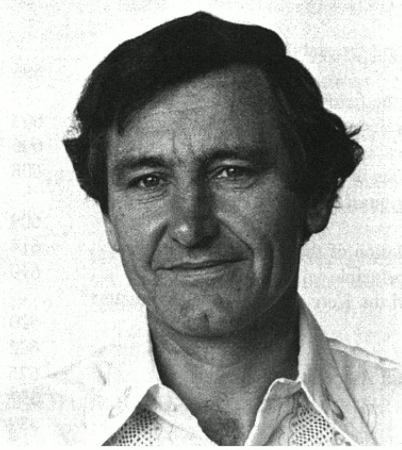
$r = \dfrac{2GM}{c^2}$: “Schwarzschild radius”

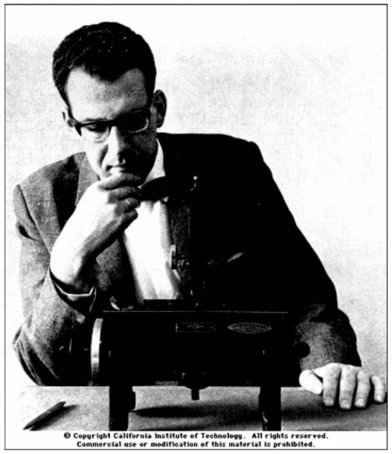
Key questions:

1) **Is the Schwarzschild “singularity” the end point of gravitational collapse?**
   1939: Oppenheimer-Snyder, yes (for dust, in spherical symmetry)
   1963: Lifshitz-Khalatnikov, not generically
   Wheeler (following Schmidt’s discovery of a quasar): do nuclear forces halt collapse?
   Answer: Penrose-Hawking singularity theorems

2) **If so, is the Schwarzschild solution stable?**
   Answer: black hole perturbation theory, quasinormal modes (QNMs) and black hole spectroscopy

Are black holes stable? The Golden Age (1963-1970s)





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1963:

✓ Roy Kerr: **rotating black holes**

✓ Maarten Schmidt at Caltech discovers the first quasar, 3C273 at z=0.15 – extragalactic!

✓ Must be compact and outshines the brightest galaxies: first supermassive black hole

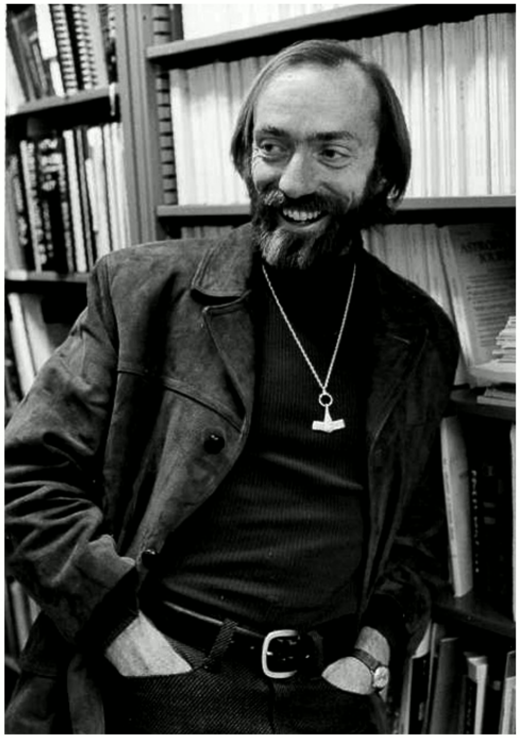
✓ Giacconi-Gursky propose orbital satellite to study X-ray sources
1964: Cygnus X-1, first stellar-mass black hole

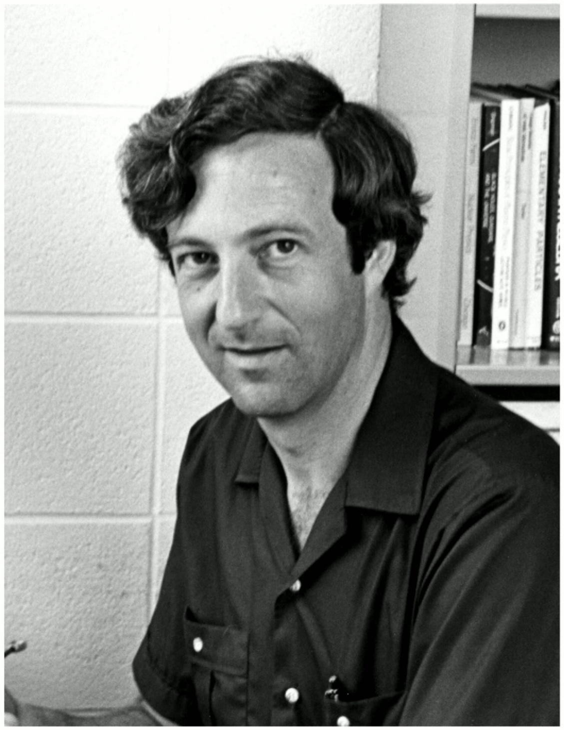
Late 1960s and 1970s:

✓ "Golden Age" of black hole physics

✓ Misner-Thorne-Wheeler, "Gravitation"

✓ Kip Thorne and students (including Saul Teukolsky) lay the foundations to understand black hole stability and dynamics





QNMs and overtones: some milestones. Phase 1 – theory development

1957 – Regge-Wheeler axial (odd-parity) perturbations as a scattering problem, boundary conditions not understood
1970 – Zerilli polar (even-parity) perturbations, much harder!

Scalar, electromagnetic and gravitational perturbations of a Schwarzschild BH: Regge-Wheeler/Zerilli equations

$$f\frac{d}{dr}\left(f\frac{d\Phi}{dr}\right)+\left[\omega^2-fV_{\pm}\right]\Phi=0$$

$$V_s=\frac{\ell(\ell+1)}{r^2}+\left(1-s^2\right)\frac{r_H}{r^3}$$

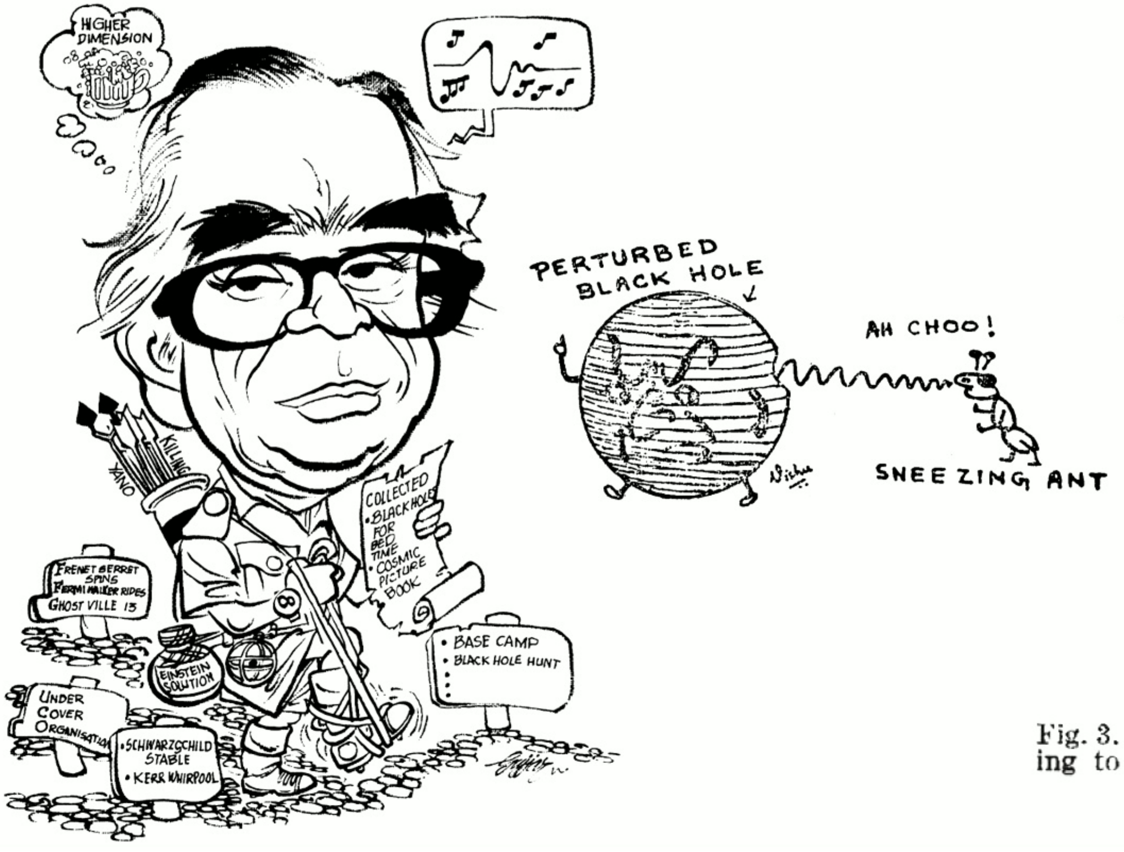
$$V_-=\frac{\ell(\ell+1)}{r^2}-\frac{3r_H}{r^3}$$

$$V_+=\frac{9\lambda r_H^2 r+3\lambda^2 r_H r^2+\lambda^2(\lambda+2)r^3+9r_H^3}{r^3\left(\lambda r+3r_H\right)^2}$$

[for reviews: Berti-Cardoso-Starinets, gr-qc/0905.2975; EB-Cardoso-Carullo+, to appear]

QNMs and overtones: some milestones. Phase 1 – theory development

1957 – Regge-Wheeler axial (odd-parity) perturbations as a scattering problem, boundary conditions not understood
1970 – Zerilli polar (even-parity) perturbations, much harder!
1970 – Vishveshwara now boundary conditions are clear: scattering experiment, “ringdown waves”



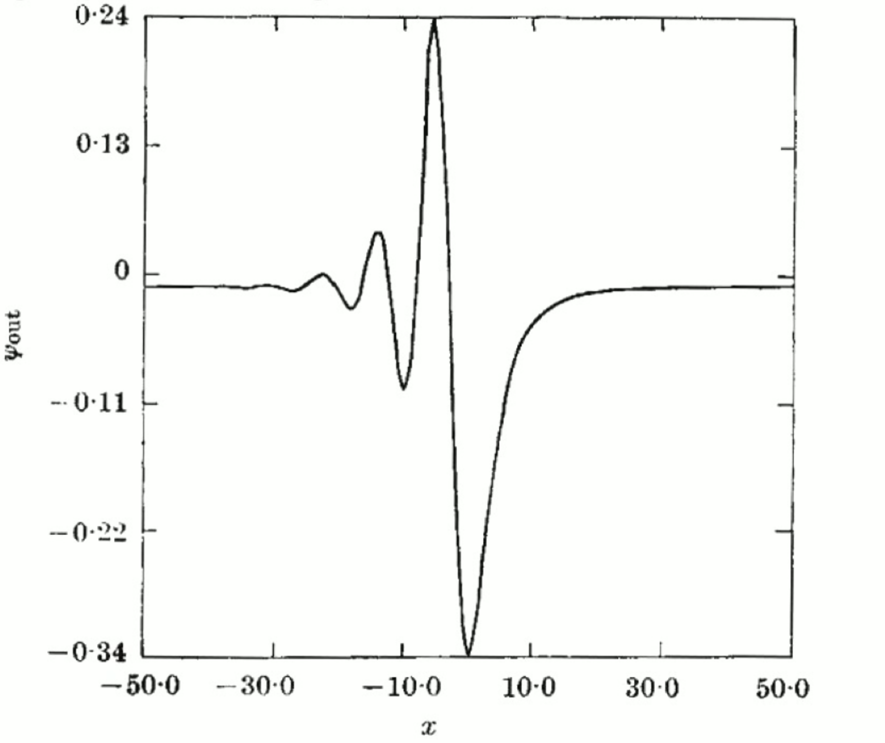
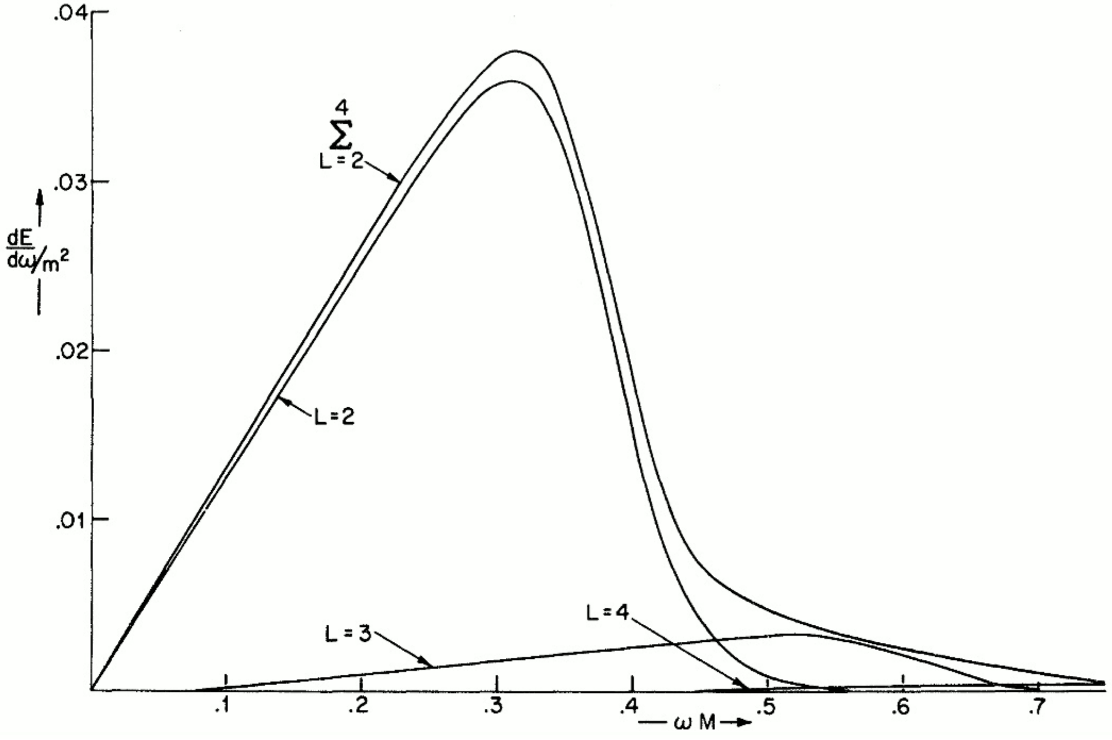
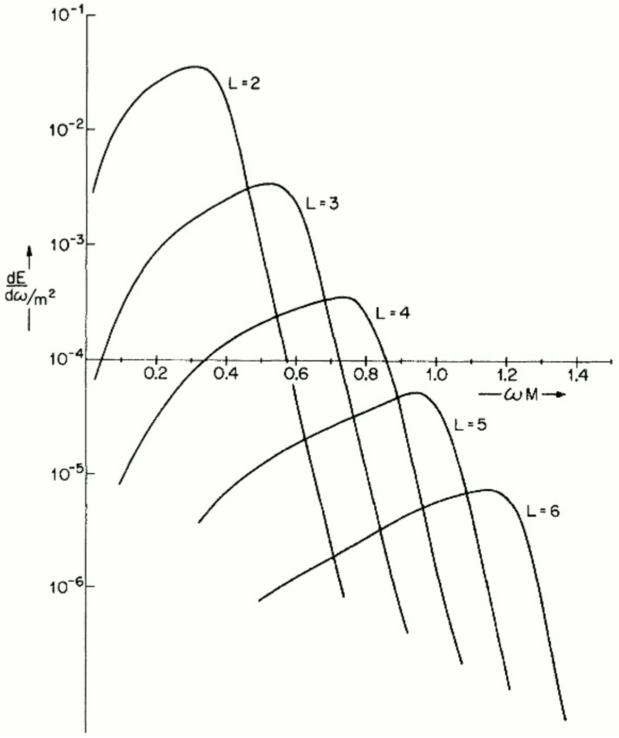


Fig. 3. The outgoing wave packet $\psi_{out}(x)$ at spatial infinity corresponding to the incident Gaussian wave packet $\psi_{in}(x) = e^{-ax^2}$ with $a = 1$.

QNMs and overtones: some milestones. Phase 1 – theory development

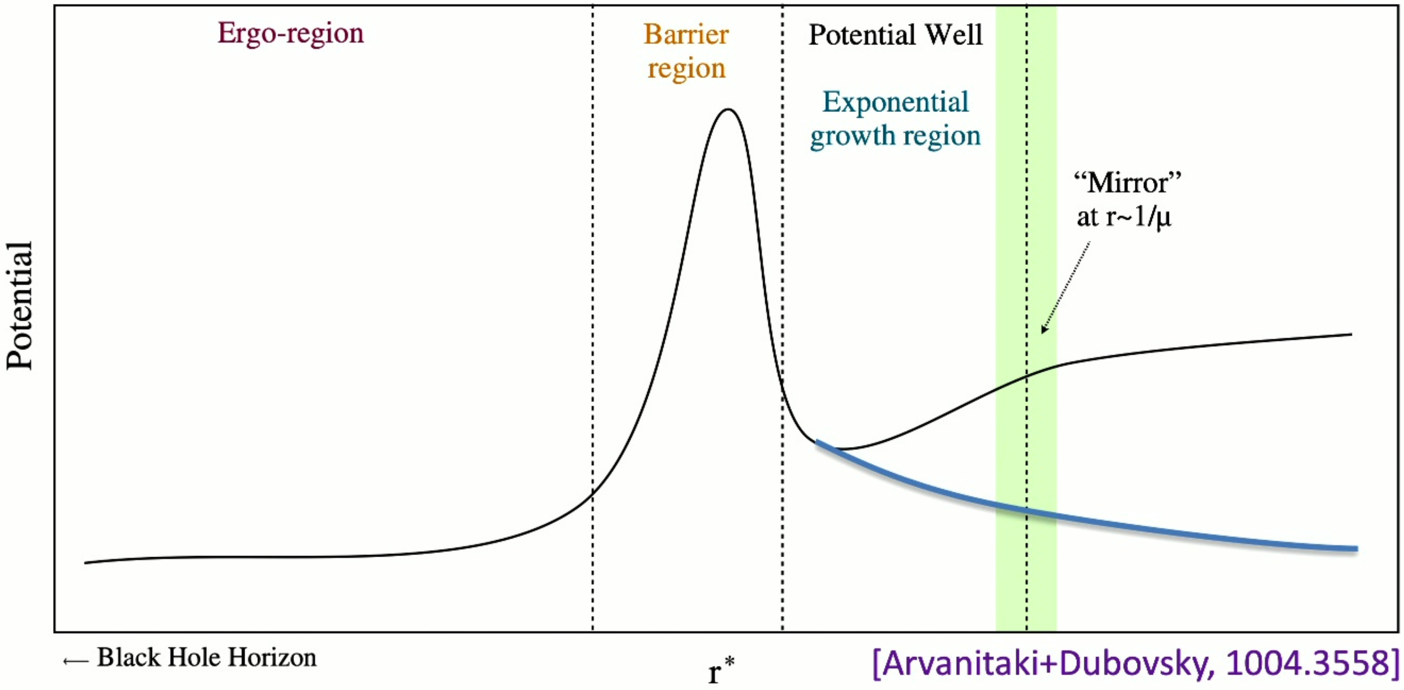
1957 – Regge-Wheeler axial (odd-parity) perturbations as a scattering problem, boundary conditions not understood
1970 – Zerilli polar (even-parity) perturbations, much harder!
1970 – Vishveshwara now boundary conditions are clear: scattering experiment, “ringdown waves”
1971 – Press ringdown waves are free oscillation modes of the black hole
1971 – Davis-Ruffini-Press-Price these modes are excited when radially falling particles cross the light ring
1973 – Teukolsky formalism for Kerr perturbations





QNMs and overtones: some milestones. Phase 2 – overtones and spectroscopy

$$\left(\frac{d^2}{dr_*^2}+\omega^2\right)\Phi = V\Phi$$



[Arvanitaki+Dubovsky, 1004.3558]

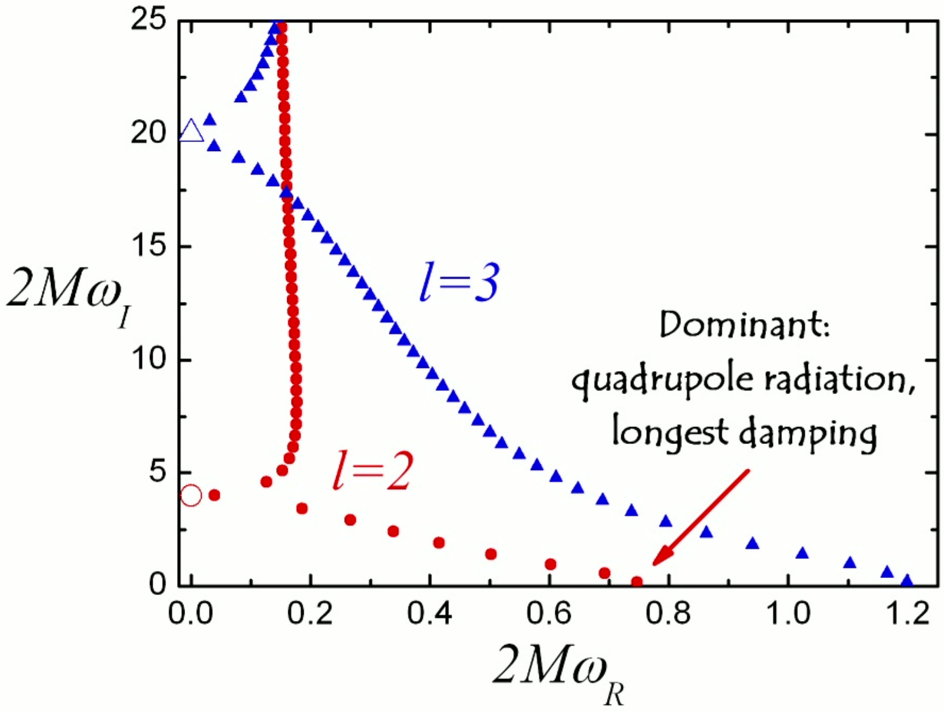
Quasinormal modes:

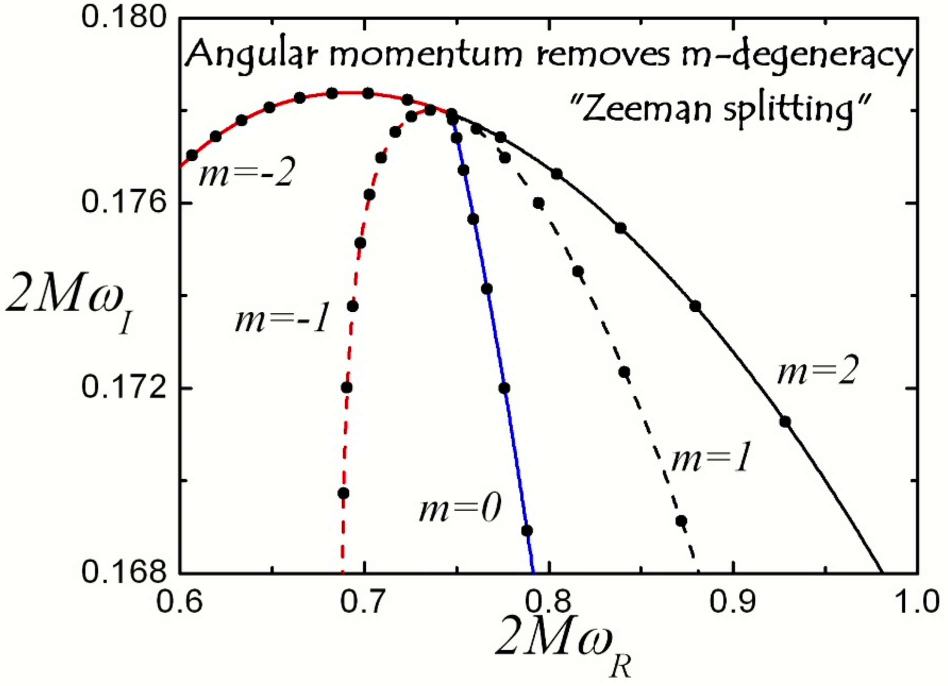
- Ingoing waves at the horizon, outgoing waves at infinity
- Spectrum of damped modes ("ringdown")

Massive scalar field:

- Superradiance: black hole bomb when $0 < \omega < m\Omega_H$ [Press-Teukolsky 1972]
- Hydrogen-like, unstable bound states [Detweiler 1980, Zouros+Eardley, Dolan…]

QNMs and overtones: some milestones. Phase 2 – overtones and spectroscopy





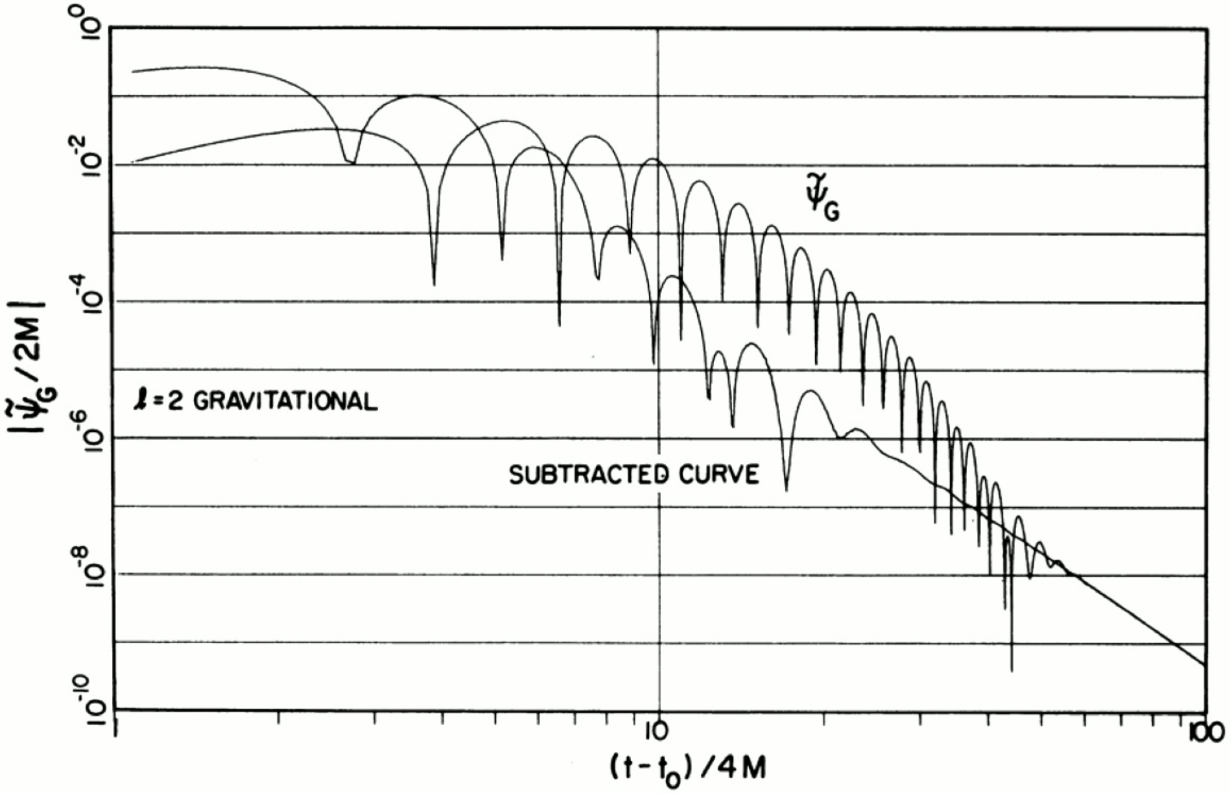
- **One mode** fixes mass and spin – and the whole spectrum!
- **N modes:** N tests of GR dynamics…**if** they can be measured
- **Measurement requires understanding of QNM excitation** (as in atomic physics!)
- **Retrograde modes, nonlinear modes** (not negligible)

[Berti-Cardoso-Will, gr-qc/0512160; EB+, gr-qc/0707.1202]

QNMs and overtones: some milestones. Phase 2 – overtones and spectroscopy

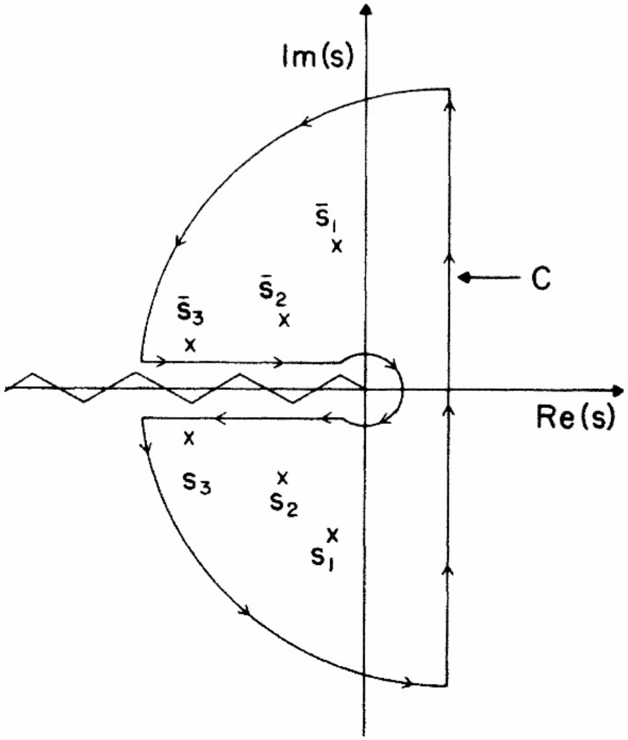
1975 – Chandrasekhar-Detweiler first numerical calculation of overtones in Schwarzschild, with limited accuracy
1978 – Cunningham-Price-Moncrief observe overtones in perturbative calculation of collapse to Schwarzschild
1979 – Detweiler first complete calculation of the Kerr spectrum, **"black hole spectroscopy"**

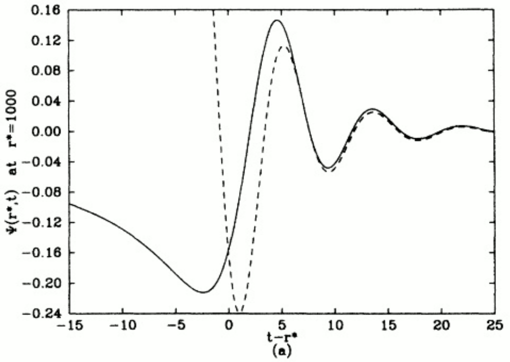
"After the advent of gravitational wave astronomy, the observation of [the black hole's] resonant frequencies might finally provide direct evidence of black holes with the same certainty as, say, the 21 cm line identifies interstellar hydrogen."



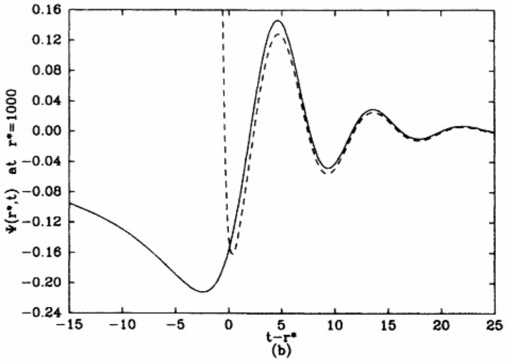
QNMs and overtones: some milestones. Phase 3 – excitation, pre-NR

1986 – Leaver Green's function, continued fractions, excitation factors (also **Andersson**) – **by analogy with H_2^+ ion!**
1989 – Echeverria quantifies how well you can measure mass and spin from a single mode
1998 – Flanagan-Hughes ringdown may have as much SNR as inspiral
2002 – Hod-Dreyer are QNMs related with Bekenstein's ideas on area quantization and LQG?
2003 – Dreyer+ revive/rebrand Detweiler's idea of "black hole spectroscopy"
2005 – Berti-Cardoso-Will SNRs, measurability, QNM frequencies+fits, **overtones vs. higher multipoles**





Radial infall vs. one mode



Radial infall vs. six modes

QNMs and overtones: some milestones. Phase 4 – excitation, post-NR

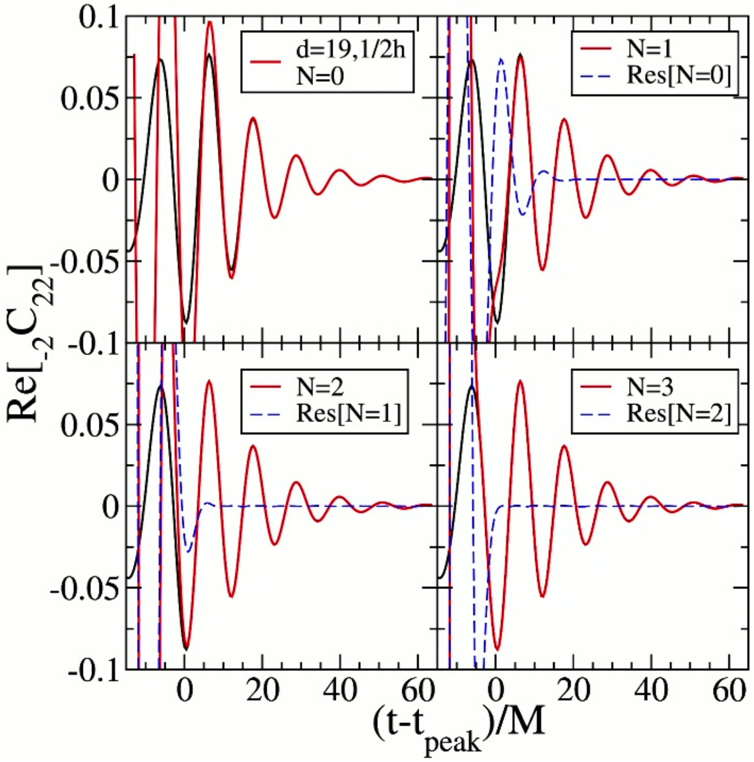
2005 – Pretorius numerical relativity breakthrough: merger simulations. Soon after Brownsville/RIT, Goddard…

2006 – Berti-Cardoso systematic calculation of Kerr excitation factors

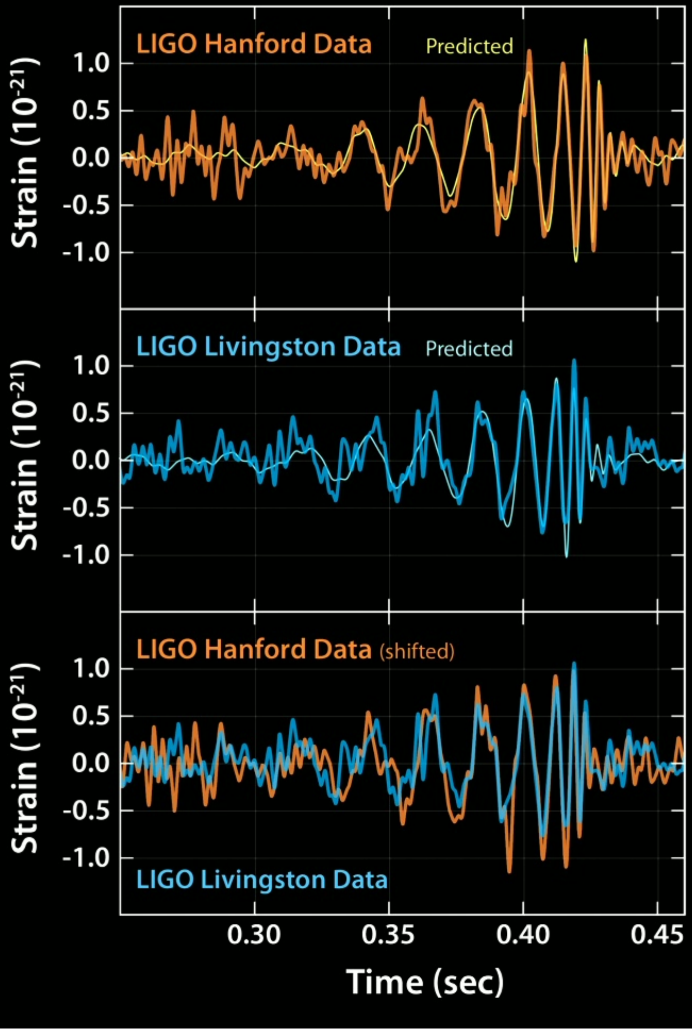
2006 – Buonanno-Cook-Pretorius fit overtones to Pretorius' equal-mass simulations – but are they physical?
Spherical-spheroidal mixing: numerical simulations use the "wrong" basis (Berti-Cardoso-Casals 2005)

2007 – Berti+ quantify excitation of higher multipole QNMs in unequal-mass, nonspinning mergers

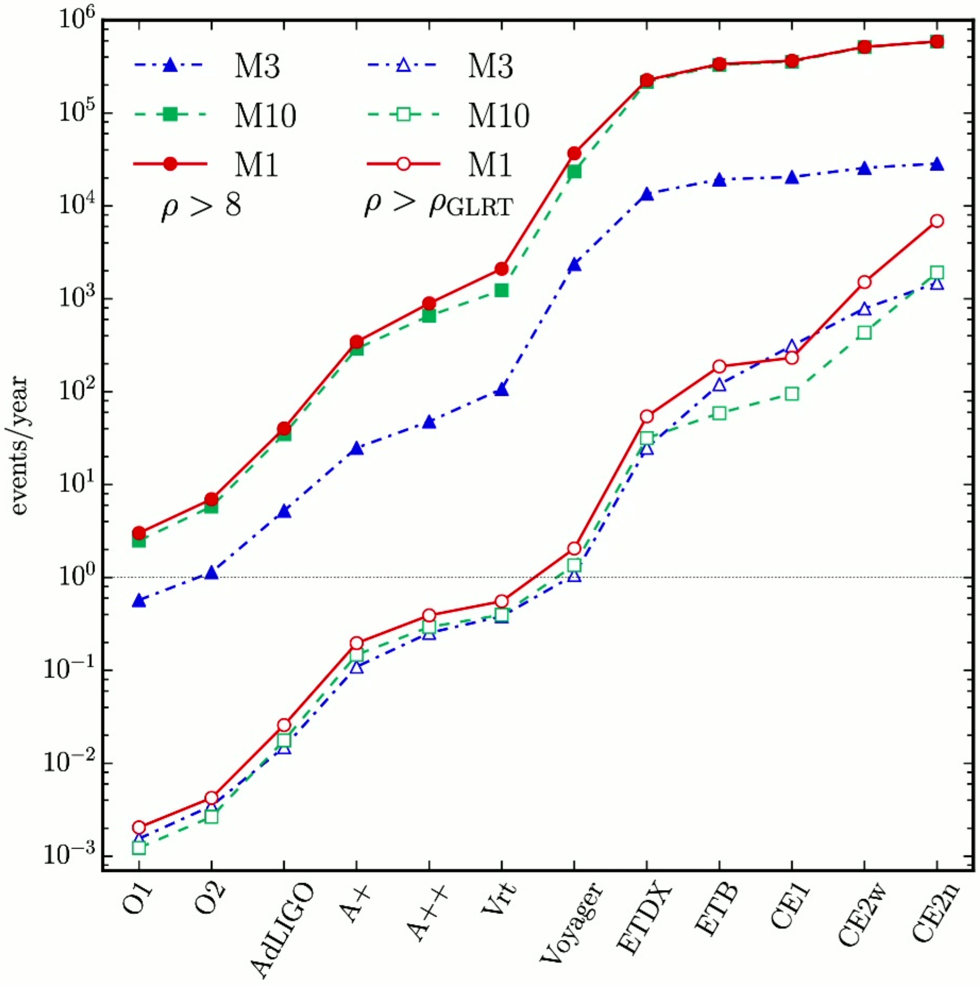
2012, 2014 – Gossan+, Meidam+ first Bayesian study of ringdown

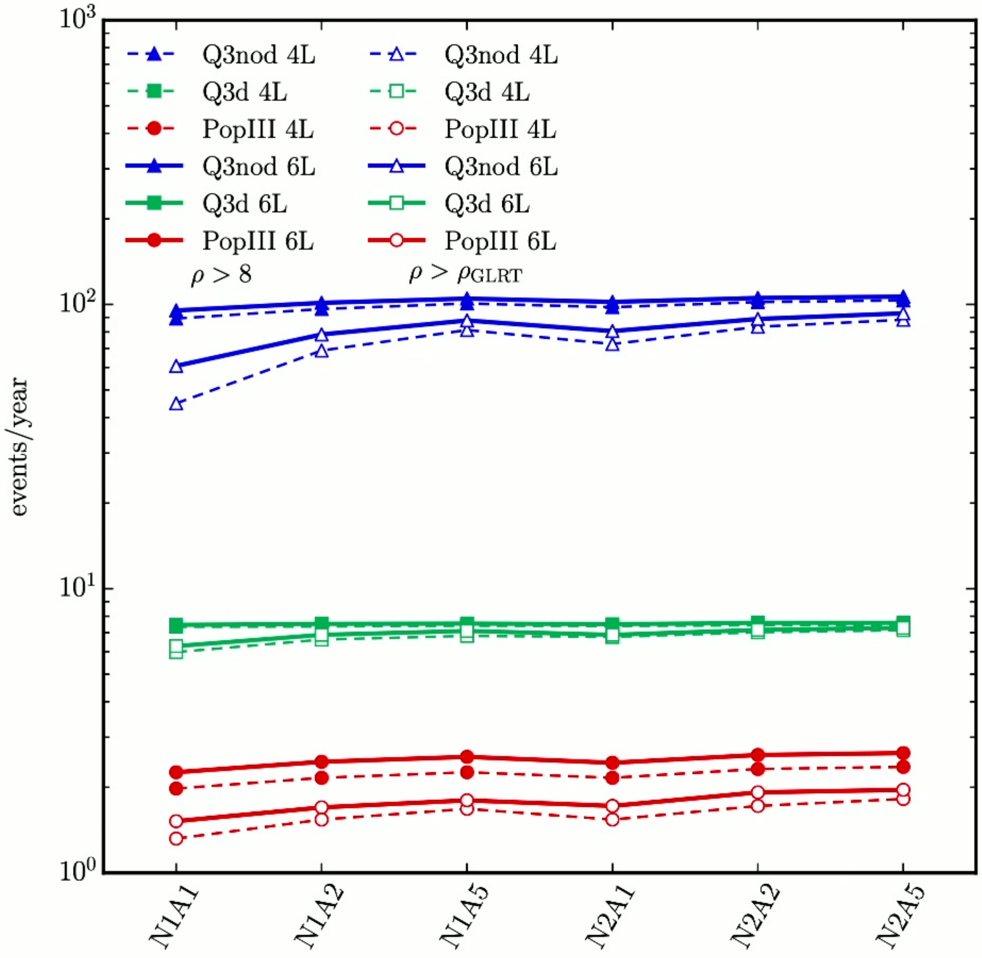


A new Golden age: GW150914 – SNR~7 in ringdown



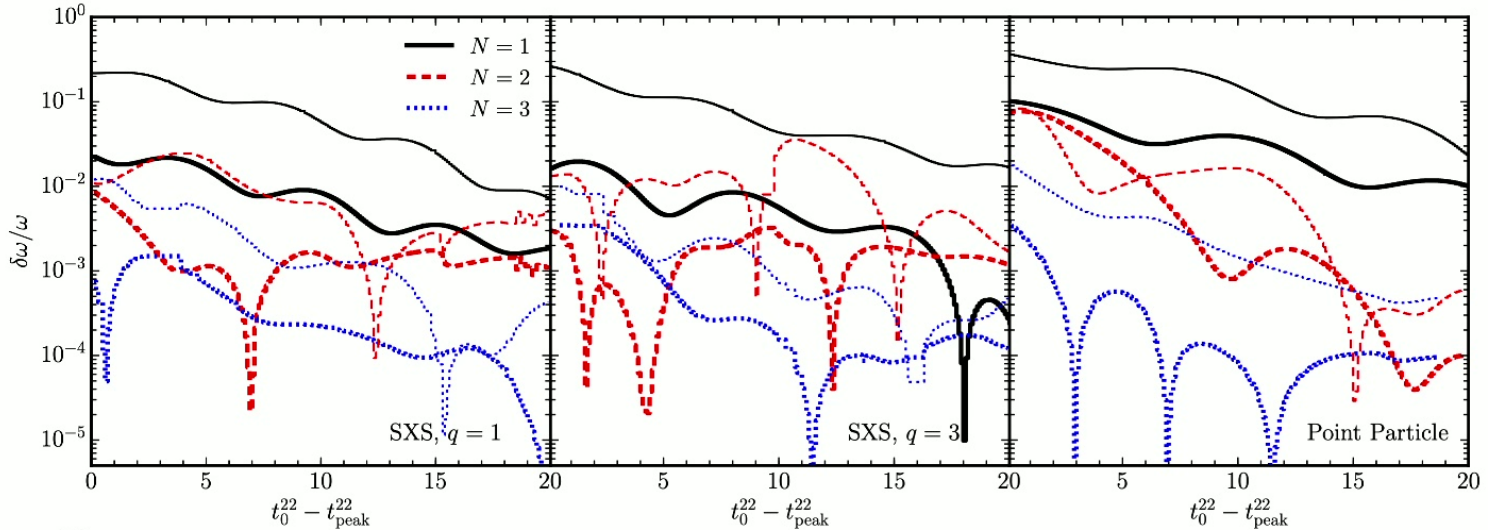
Earth vs. space-based: ringdown detections and black hole spectroscopy





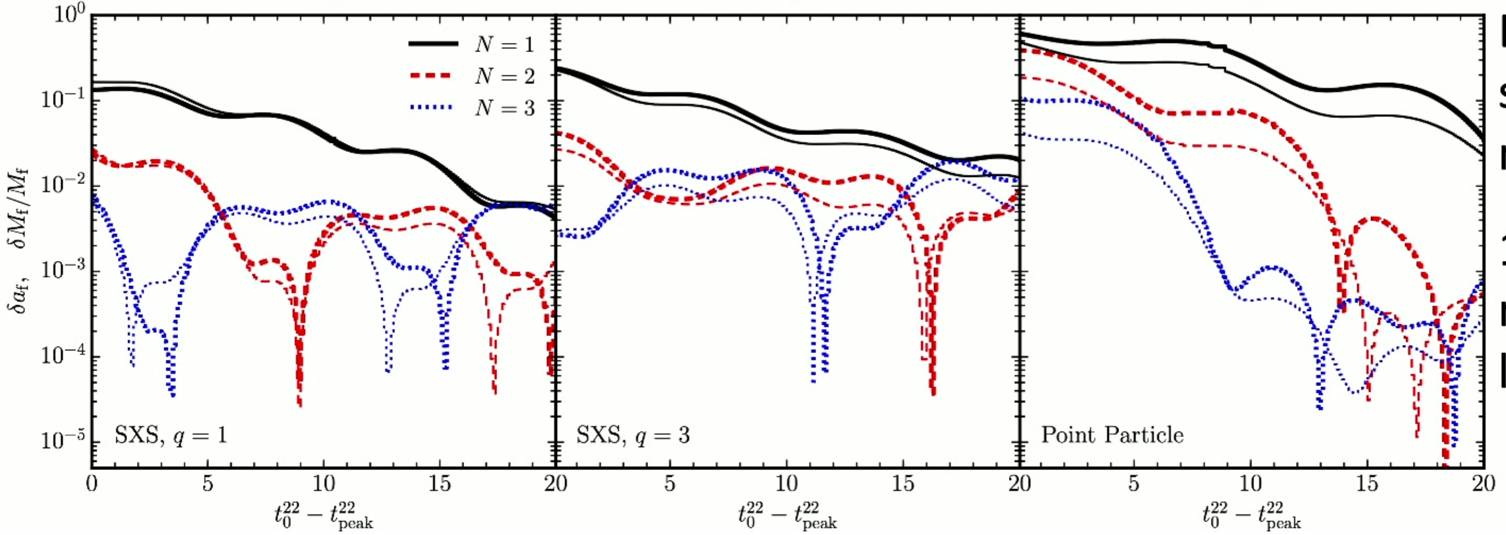
[EB+, 1605.09286]

Overtones are needed to reduce mass/spin errors



Top:
real part (thick)
imaginary part (thin)

1% determination of ω_{220} needs one overtone (better if two or three)



Bottom:
spin (thick)
mass (thin)

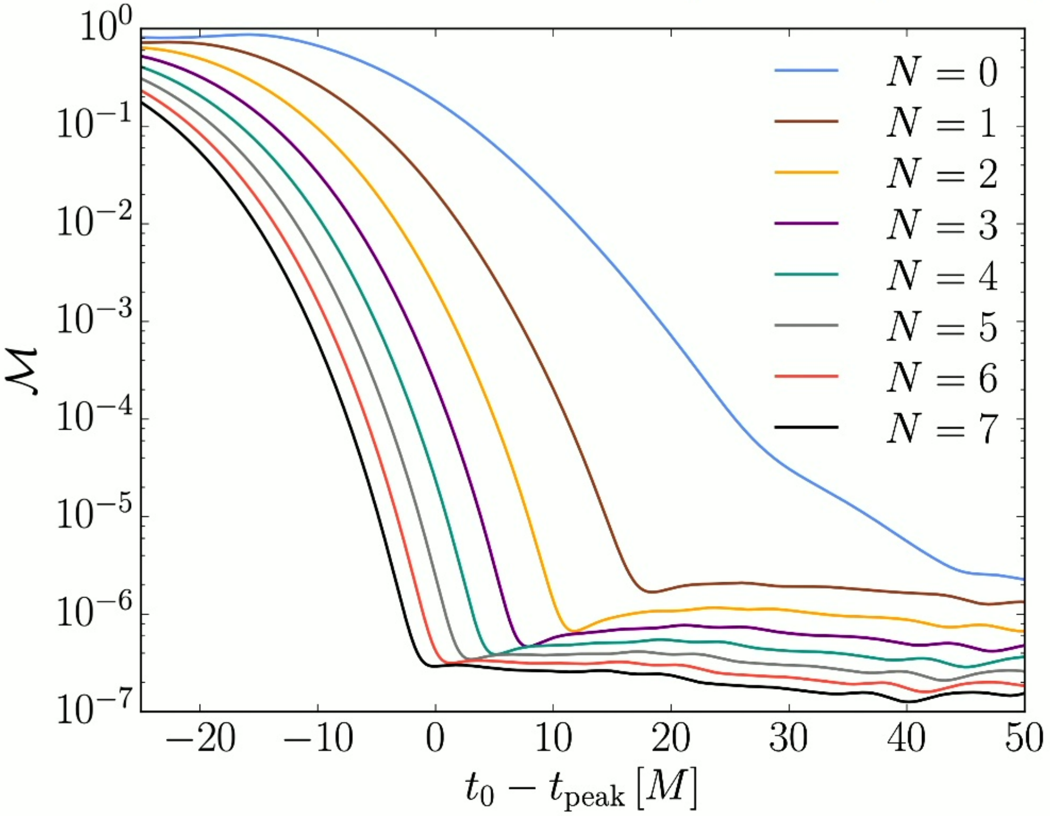
1% determination of mass and spin needs at least two modes

[Baibhav+, 1710.02156]

Nonlinear merger: is it just a superposition of linear QNMs?

“Including overtones allows for the modeling of the ringdown signal for all times beyond the peak strain amplitude, indicating that **the linear quasinormal regime starts much sooner than previously expected**. This implies that the spacetime is **well described as a linearly perturbed black hole with a fixed mass and spin as early as the peak**”

$$\mathcal{M} = 1 - \frac{\langle h_{22}^{\rm NR}, h_{22}^{N}\rangle}{\sqrt{\langle h_{22}^{\rm NR}, h_{22}^{\rm NR}\rangle\langle h_{22}^{N}, h_{22}^{N}\rangle}}$$

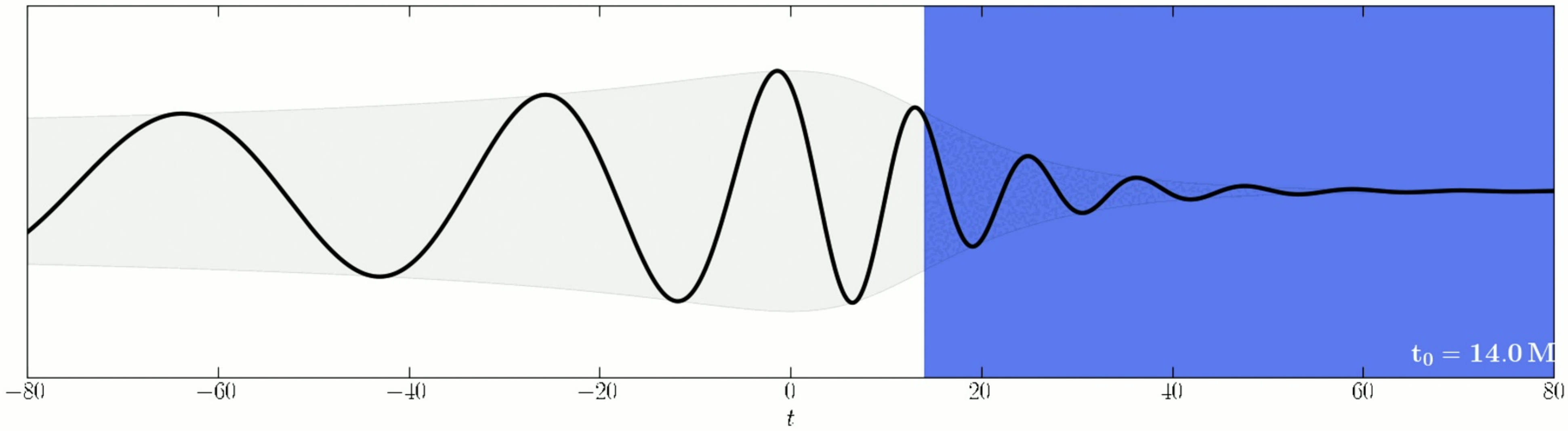


Does it?

[Giesler+, 1903.08284]

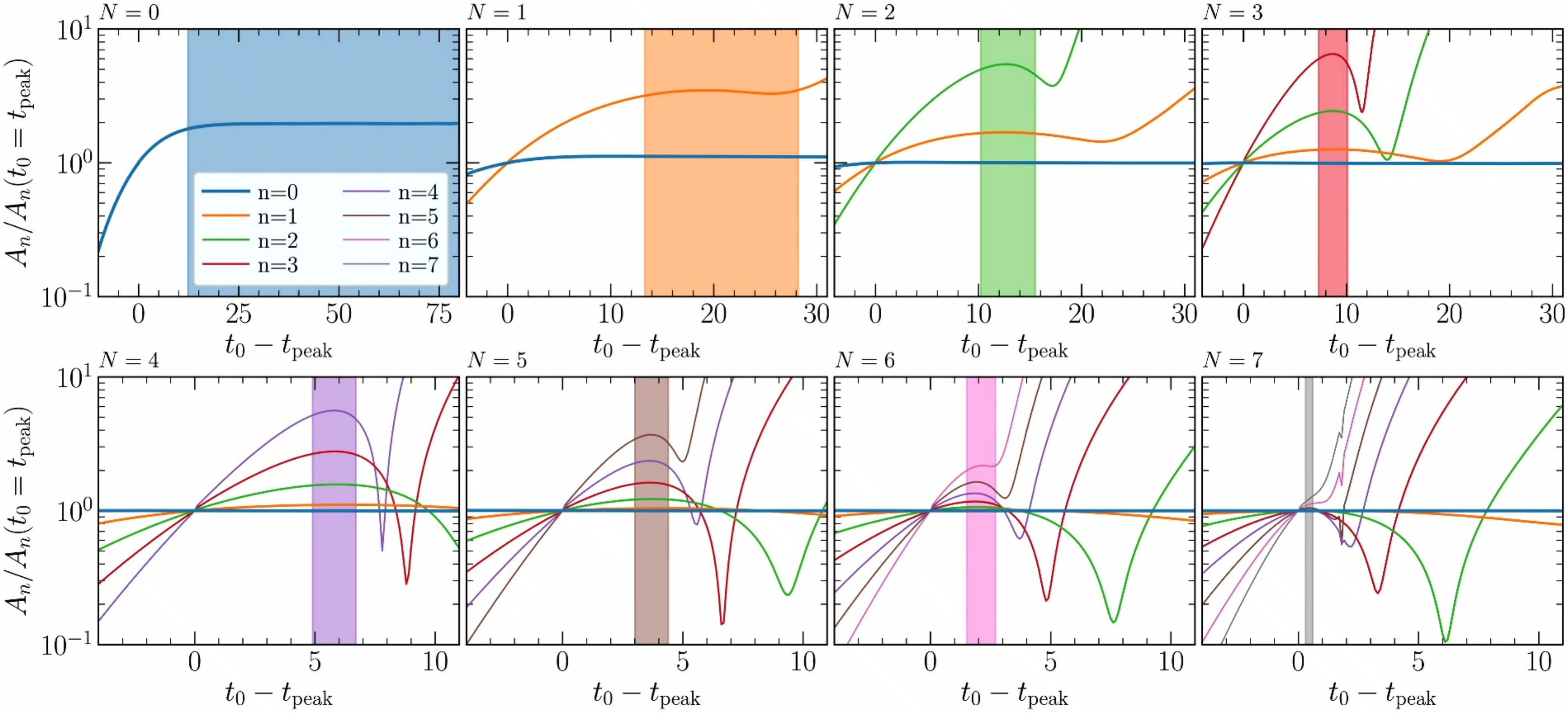
Is the linear model consistent when we change the fitting window?

$$h = \sum A e^{-\omega_i(\chi,M)t} \cos(\omega_r(\chi, M)t + \phi)$$



QNM amplitudes are not constant near the peak

"Fixed-frequency" fits as in Giesler (weaker test). Bands show regions where amplitudes are constant within 10%

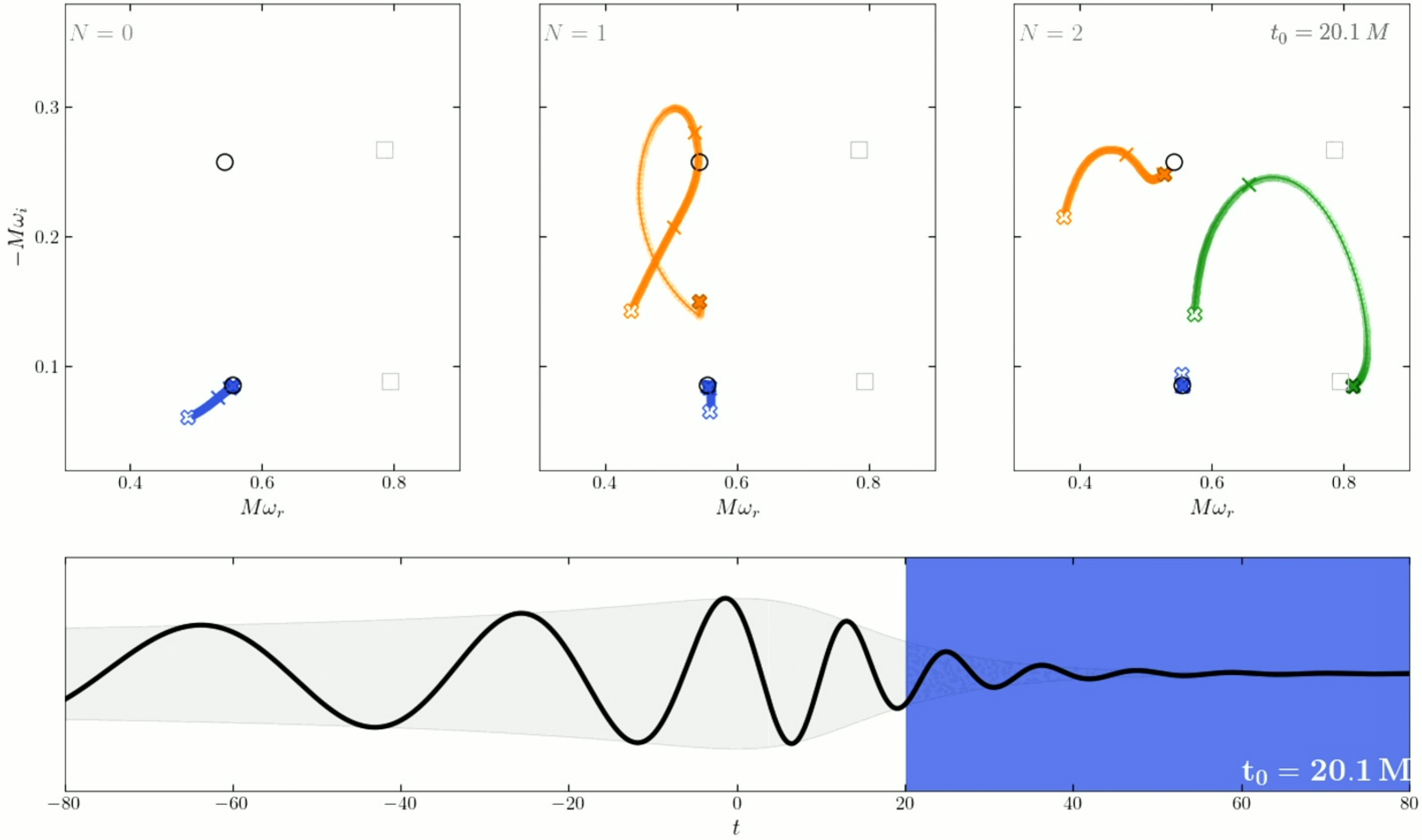


[Bhagwat+, 1910.08708; Baibhav+, 2302.03050]

My mom always said life was like a box of chocolates.
You never know what you're gonna get.

Why wrong? Spherical-spheroidal mode mixing

$$_{-2}S_{lm} = {}_{-2}Y_{lm} + j_f\,\tilde{\omega}_{lmn} \sum_{l \neq l'} {}_{-2}Y_{l'm}\, c_{l'lm}$$



Search for nonlinearities and nonlinear modes

Two stages

Before the 2005 NR breakthrough: perturbation theory to the rescue
Close limit approximation [e.g. Gleiser+ gr-qc/9609022…]
"Lazarus project", second-order Kerr [e.g. Campanelli-Lousto gr-qc/9811019]

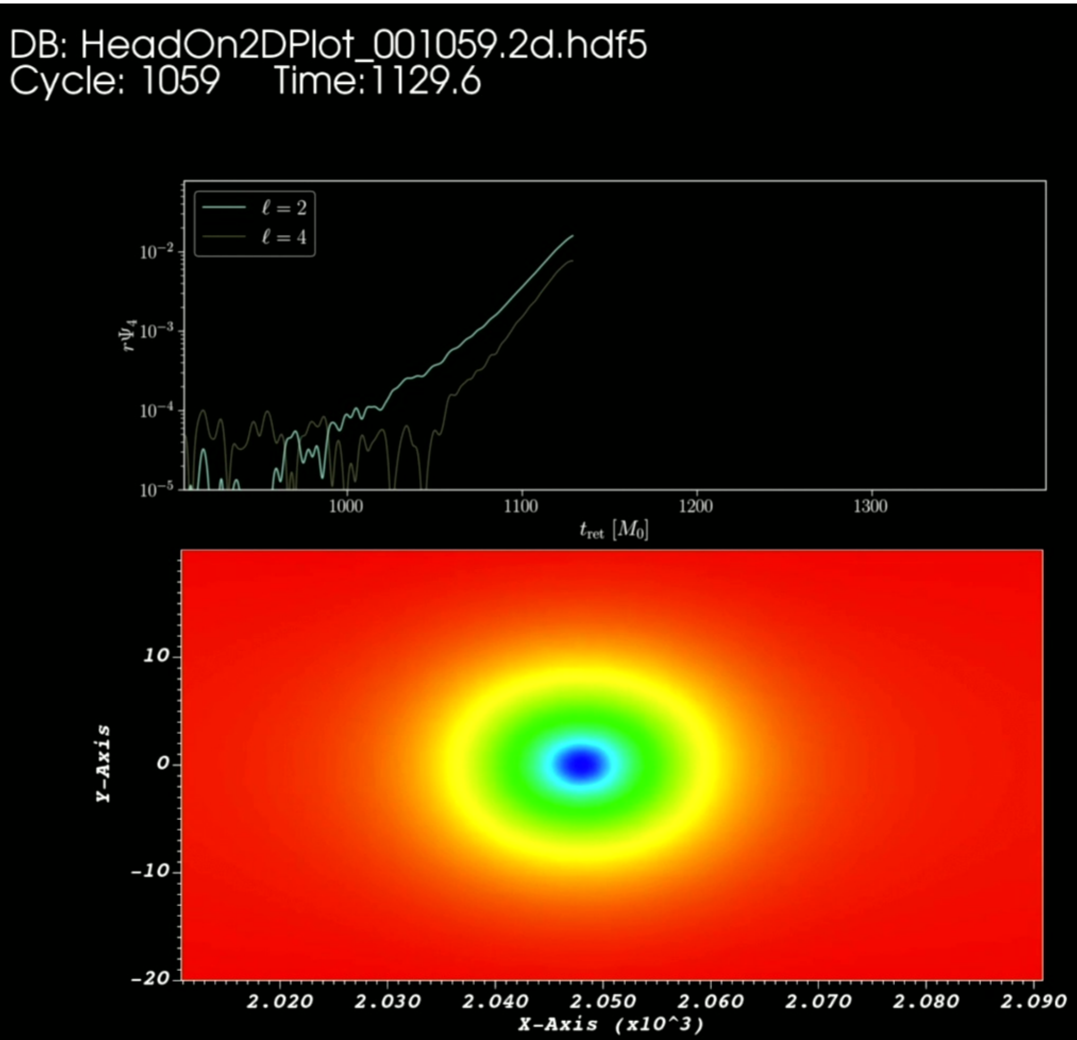
After the 2005 NR breakthrough:
Where are all the nonlinearities?
[Zlochower+, gr-qc/0306098; Ioka-Nakano, 0704.3467 + 0708.0450; Brizuela+, 0903.1134; Pazos+, 1009.4665]

$$\psi = \epsilon\psi_{(1)} + \epsilon^2\psi_{(2)}$$

$$\mathcal{L}\psi_{(2)} \propto \psi_{(1)}^2 \sim A_1 A_2 e^{i(\phi_1+\phi_2)}$$

Pioneering search for nonlinearities in the Georgia Tech NR catalog
[London+, 1404.3197]

Recent explosion of activity – analytical and numerical
[Loutrel+, 2008.11770; Ripley+, 2010.00162; Magana-Zertuche+, 2110.15922; Sberna+, 2112.11168; Ma+, 2207.10870; Lagos-Hui, 2208.07379; Cheung+, 2208.07374; Mitman+, 2208.07380; Zhu+, 2309.13204; Kehagias+, 2301.09345 + 2302.01240; Nee+, 2302.06634; Perrone+, 2308.15886; Bucciotti+, 2309.08501…]



[Cheung+, 2208.07374]

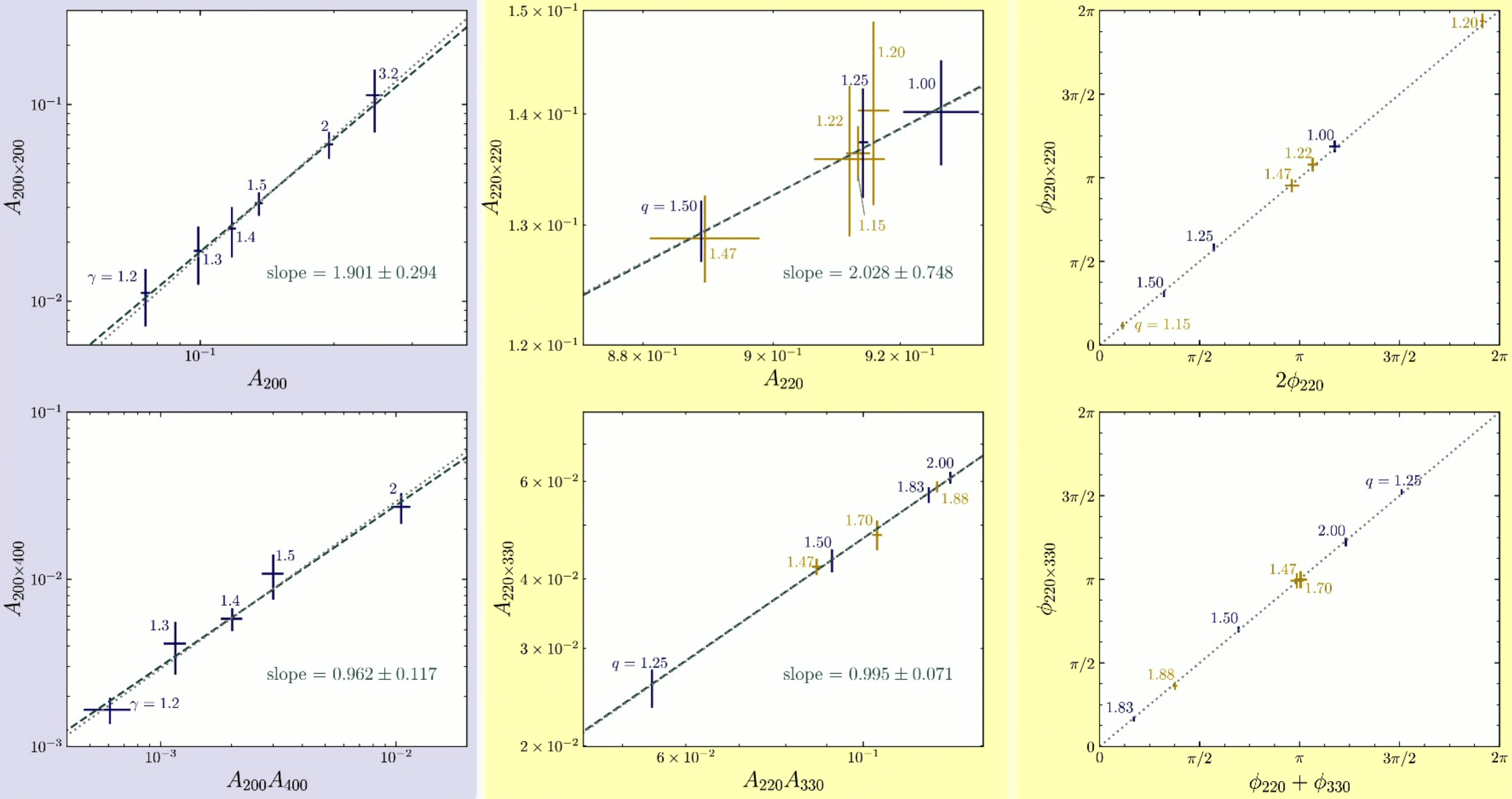
Head-on mergers

Quasicircular mergers

Amplitude dependence

Amplitude dependence

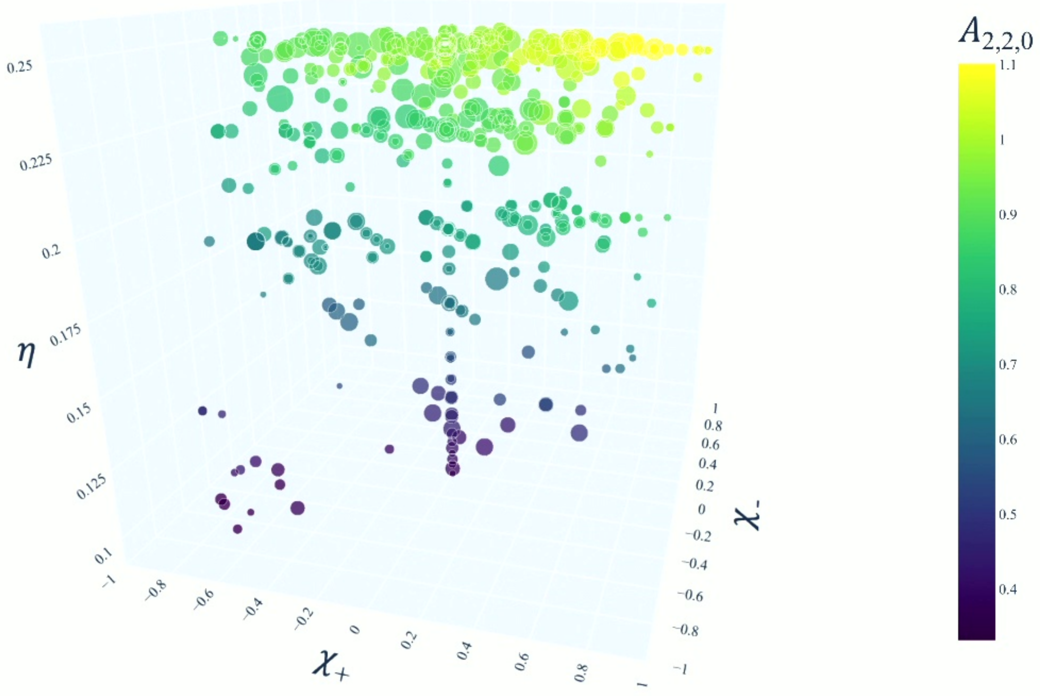
Phase dependence

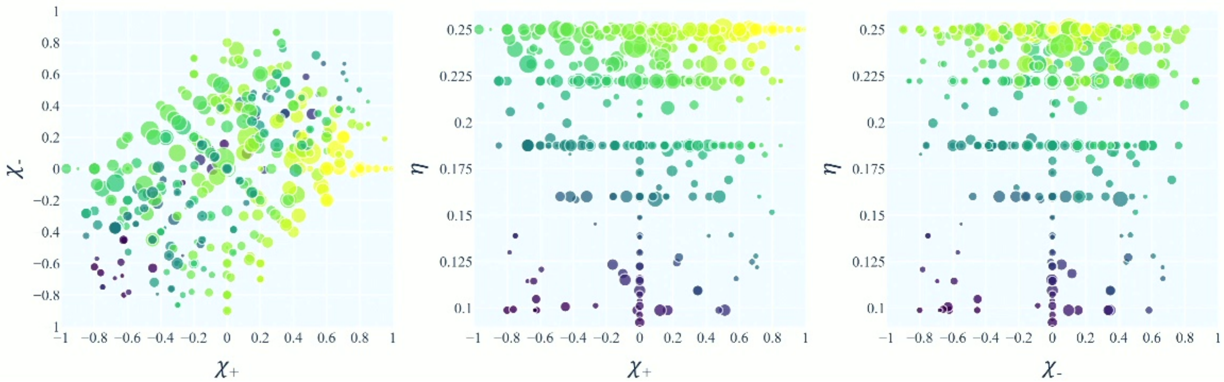


$$\eta = \frac{q}{(1+q)^2}$$

$$\chi_+ = \frac{q\chi_1 + \chi_2}{1+q}$$

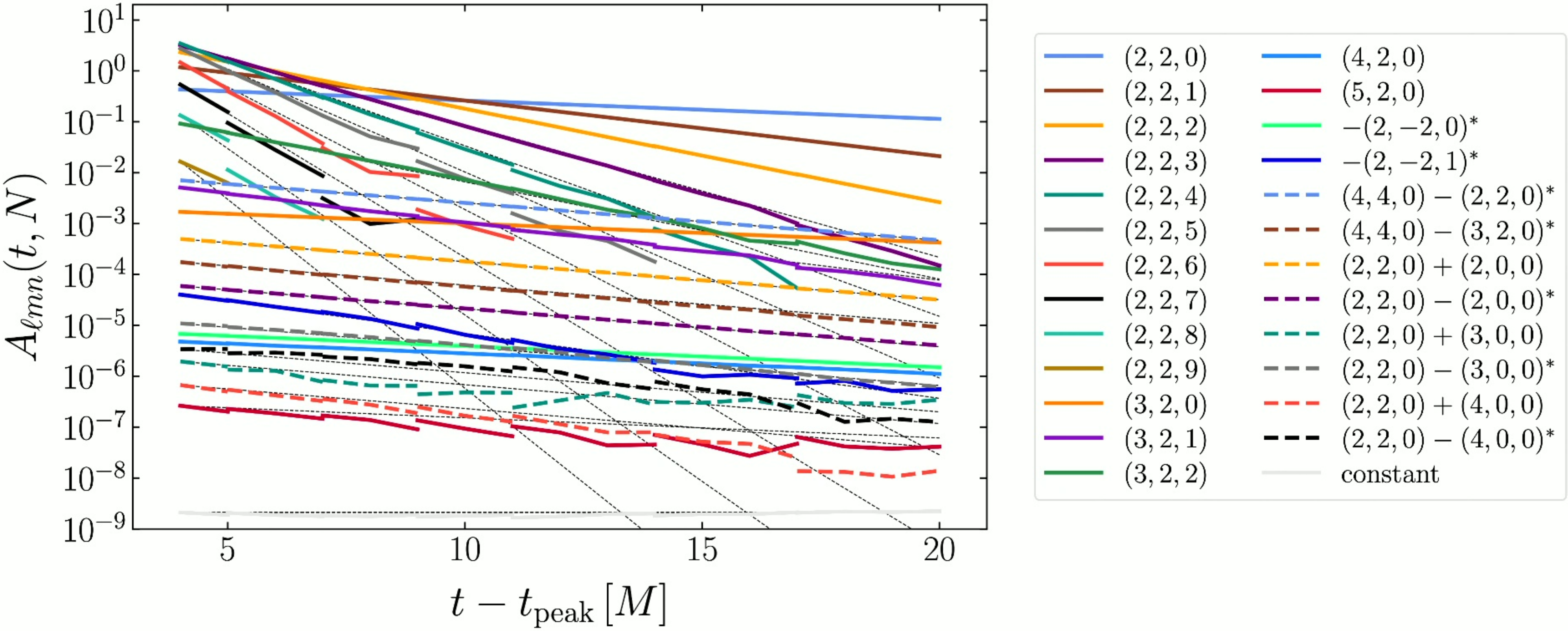
$$\chi_- = \frac{q\chi_1 - \chi_2}{1+q}$$





Agnostic, nonlinear spectroscopy reloaded: nonlinear modes in the (2,2)

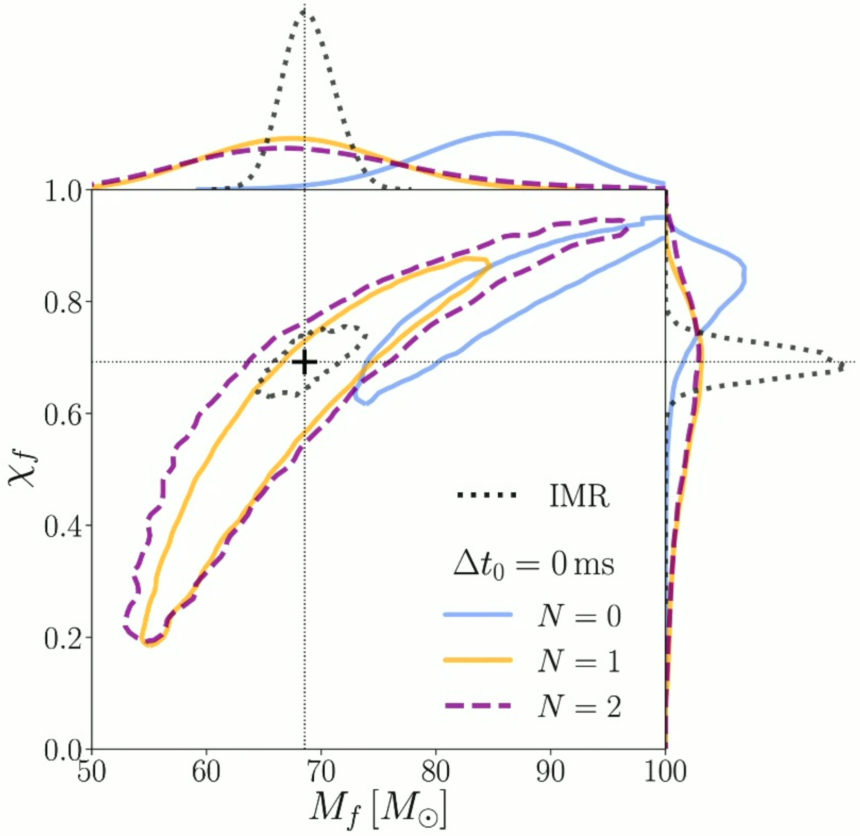
More accurate simulations (CCE crucial), agnostic fits, includes nonlinear modes, variable projection
Finds more overtones as long as nonlinear modes are included. “Stable amplitudes are not achievable until~4M after the peak in a moderately spinning case and until ~8M post-peak in a high-spin case”



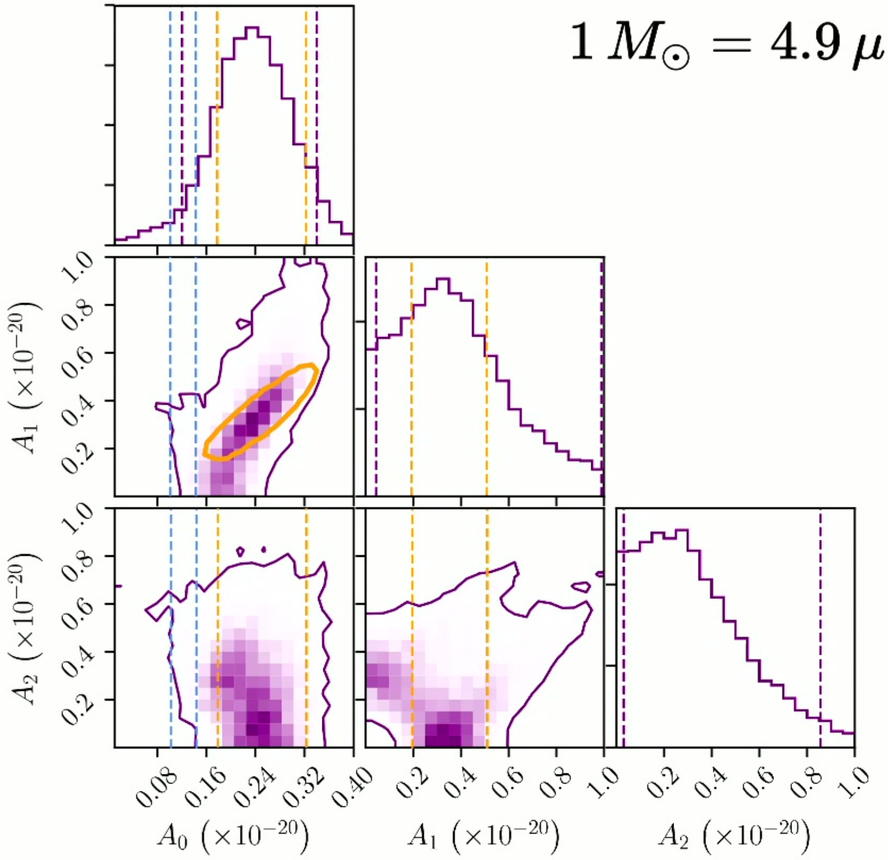
[Giesler+, 2411.11269]

GW150914 tests of the no-hair theorem with the first overtone?

Overtones improve quality of consistency tests for GW150914
Is the overtone detection robust? Assumes t_{start}=1126259462.423 s



$$1\,M_\odot = 4.9\,\mu s$$



[Isi+, 1905.00869]

Black hole spectroscopy: are we there yet? Not so fast

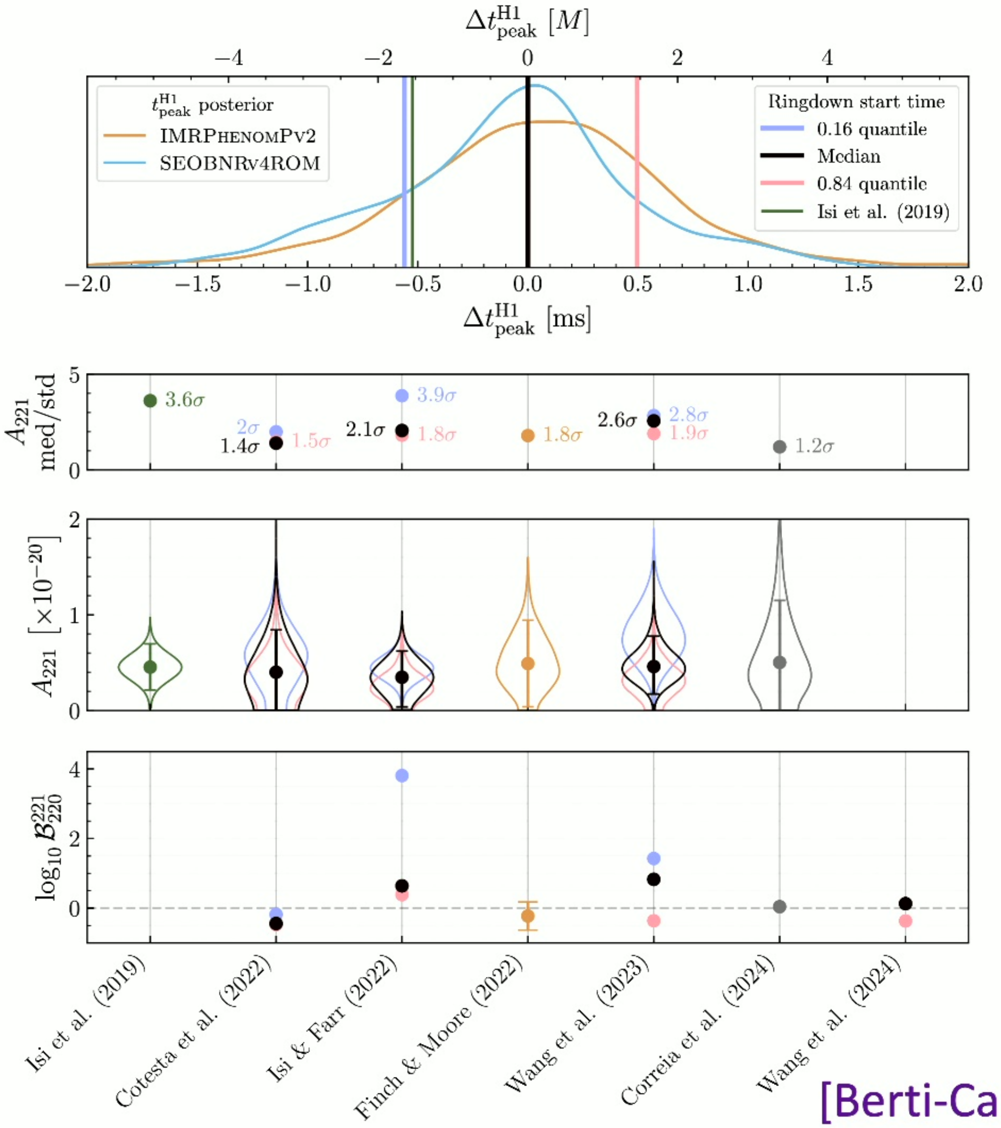
Theory: need agnostic analysis of NR waveforms including all physics, not just linear modes

Cannot just *assume* that the second mode is an overtone: must include
BMS effects (memory), tails, transients, mode mixing, counterrotating + nonlinear modes
Low-frequency QNMs (overtones) are good at fitting the inspiral: "pseudo-QNMs" in EOB
QNMs physically present only ~10M after the peak, where SNR is low
High overtones do not contribute to mass/spin estimates, can be unstable
[Baibhav+, 2302.03050; Cheung+, 2111.05415 and 2208.07374; Mitman+, 2208.07380…]

Data analysis: second mode evidence depends on assumptions

Time or frequency domain?
Ringdown only vs. modeled (e.g. pSEOBNR)/unmodeled (wavelets) pre-ringdown
Must take into account uncertainty in starting time, sampling rate, noise modeling…
Weak Bayesian evidence (if any) for a second mode in GW150914, GW190521
What does "mode detection" even mean? [Isi+, Capano+, Cotesta+, Finch-Moore, Wang+…]
XG: "golden" events with SNR~300 for CE/ET, SNR~1000 for LISA – but Ockham penalties
Amplitude/phase tests; population-based tests

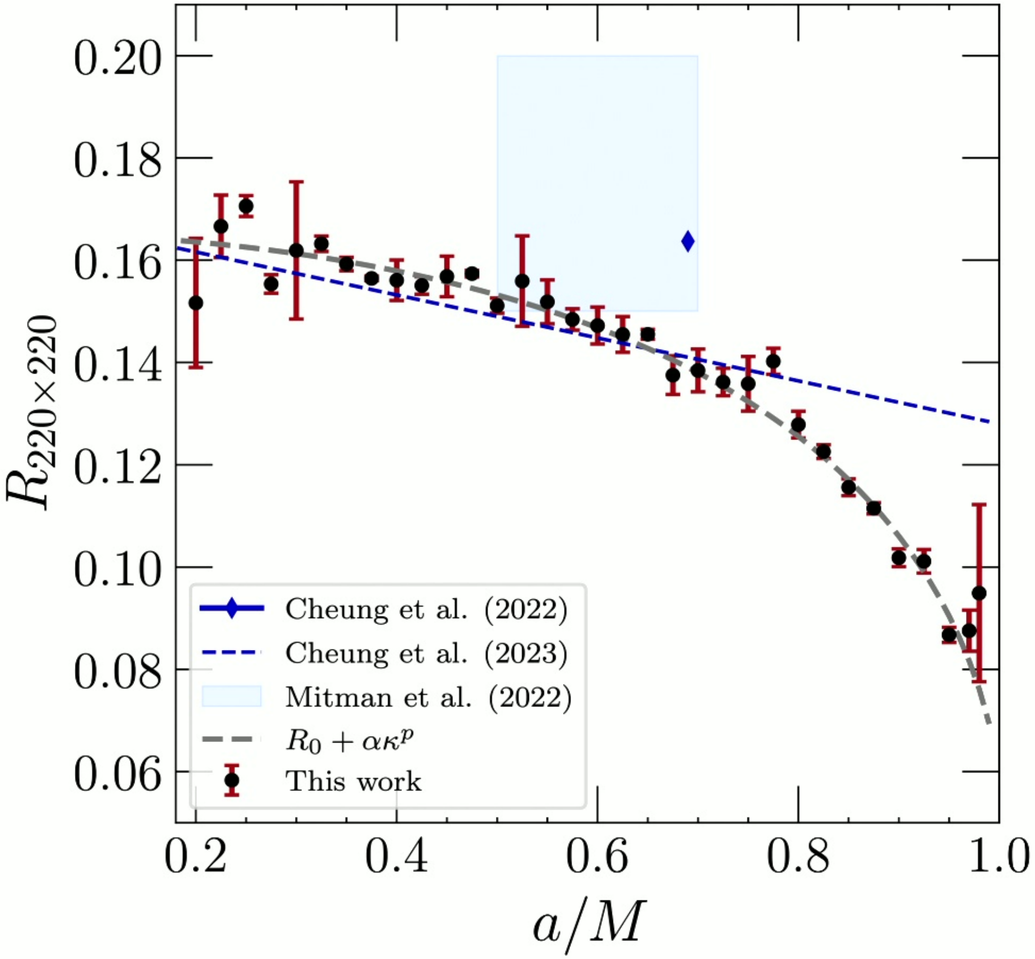
[*Ringdown Inside & Out*]



[Berti-Cardoso-Carullo+, to appear]

Tests of strong gravity with nonlinear modes

Gaussian scattering of second-order perturbations: good agreement!



[Redondo-Yuste+, arXiv:2308.14796; see also Zhu+, 2401.00805; Ma-Yang, 2401.15516]

Ongoing work and a new test: ratio of nonlinear and linear mode amplitudes

Numerical extraction of modes

Takahashi-Motohashi 2311.12762: iterative extraction of overtones
Clarke+, 2402.02819: “striking the right tone” (up to N=3)
Zhu+ 2312.08588: precessing binaries
Carullo 2406.19442: eccentric binaries
Carullo-De Amicis 2310.12968 , Islam+ 2407.04682 : tails for eccentric binaries
Carullo-De Amicis 2406.17018: perturbation theory arguments

Systematic calculation of nonlinear / linear mode amplitudes in Schwarzschild

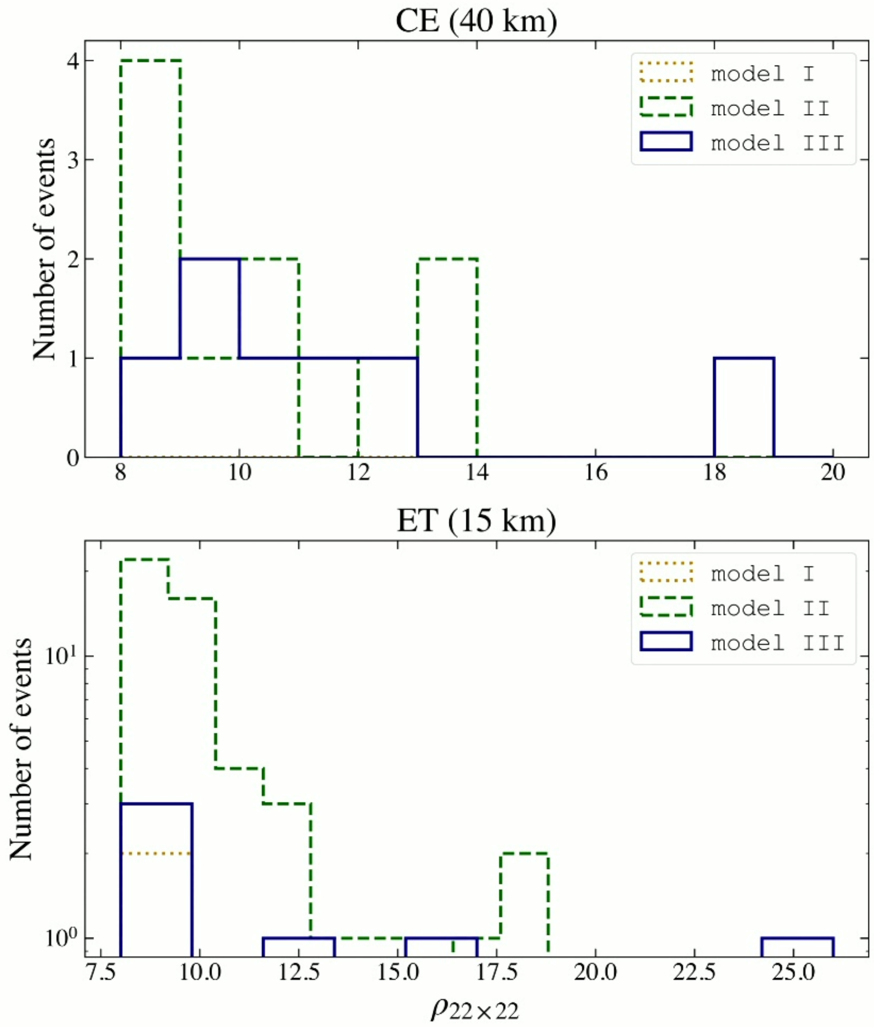
Ioka-Nakano 0704.3467, 0708.0450: first estimate
Lagos-Hui 2208.07379: Green’s function
Kehagias-Riotto 2301.09345 + 2302.01240, Perrone+ 2308.15886: Kerr/CFT, gauge invariance, light ring
Bucciotti+ 2309.08501, 2405.06012, 2406.14611: generic quadratic/linear mode ratios
Bourg+ 2405.10270: dependence on parity

Calculation of nonlinear / linear mode amplitudes in Kerr

Redondo-Yuste+ 2308.14796, Zhu+ 2309.13204 + 2401.00805: time-domain fits with quadratic code
Redondo-Yuste+ 2312.04633, Zhu+ 2404.12424: changing mass and spin (Vaidya/nonlinear evolutions)
May+ 2405.18303: absorption
Ma-Yang 2401.15516, Khera+ 2410.14529: quadratic/linear mode ratios for dominant modes

…plus work on greybody factors, spectral stability, etcetera

XG ground-based detectors, quadratic $(220)^2$ mode in the (44) multipole



[Yi+, 2403.09767]

LISA rates for quadratic mode detectability

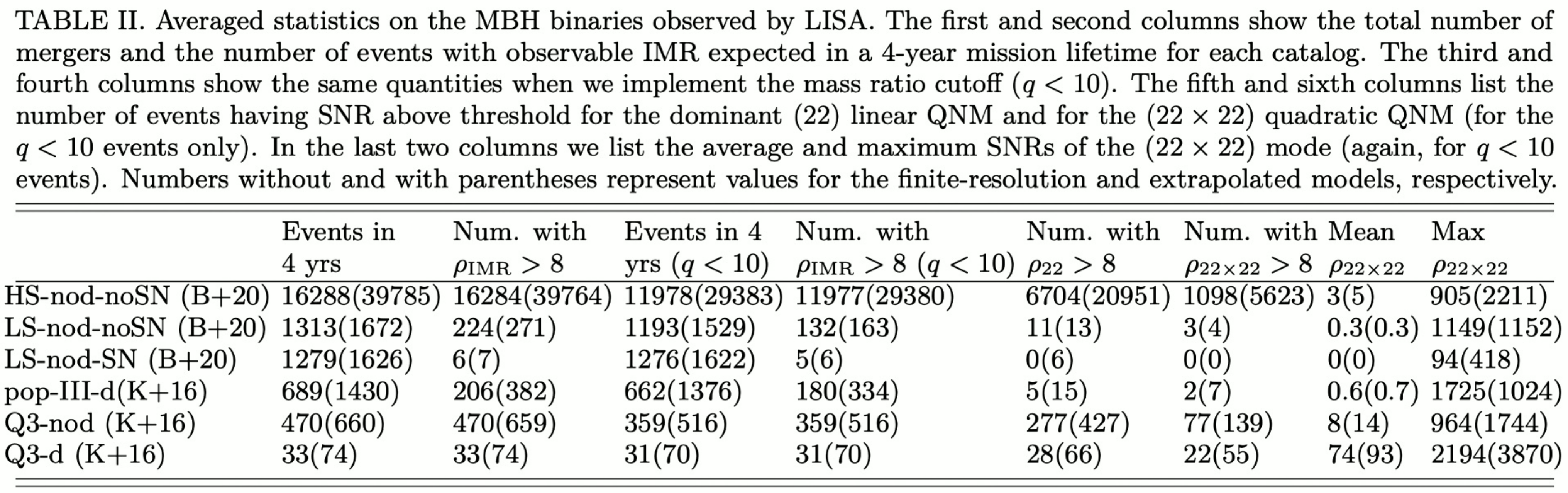


TABLE II. Averaged statistics on the MBH binaries observed by LISA. The first and second columns show the total number of mergers and the number of events with observable IMR expected in a 4-year mission lifetime for each catalog. The third and fourth columns show the same quantities when we implement the mass ratio cutoff ($q < 10$). The fifth and sixth columns list the number of events having SNR above threshold for the dominant (22) linear QNM and for the (22×22) quadratic QNM (for the $q < 10$ events only). In the last two columns we list the average and maximum SNRs of the (22×22) mode (again, for $q < 10$ events). Numbers without and with parentheses represent values for the finite-resolution and extrapolated models, respectively.

	Events in 4 yrs	Num. with $\rho_{\rm IMR} > 8$	Events in 4 yrs ($q < 10$)	Num. with $\rho_{\rm IMR} > 8$ ($q < 10$)	Num. with $\rho_{22} > 8$	Num. with $\rho_{22\times22} > 8$	Mean $\rho_{22\times22}$	Max $\rho_{22\times22}$
HS-nod-noSN (B+20)	16288(39785)	16284(39764)	11978(29383)	11977(29380)	6704(20951)	1098(5623)	3(5)	905(2211)
LS-nod-noSN (B+20)	1313(1672)	224(271)	1193(1529)	132(163)	11(13)	3(4)	0.3(0.3)	1149(1152)
LS-nod-SN (B+20)	1279(1626)	6(7)	1276(1622)	5(6)	0(6)	0(0)	0(0)	94(418)
pop-III-d(K+16)	689(1430)	206(382)	662(1376)	180(334)	5(15)	2(7)	0.6(0.7)	1725(1024)
Q3-nod (K+16)	470(660)	470(659)	359(516)	359(516)	277(427)	77(139)	8(14)	964(1744)
Q3-d (K+16)	33(74)	33(74)	31(70)	31(70)	28(66)	22(55)	74(93)	2194(3870)

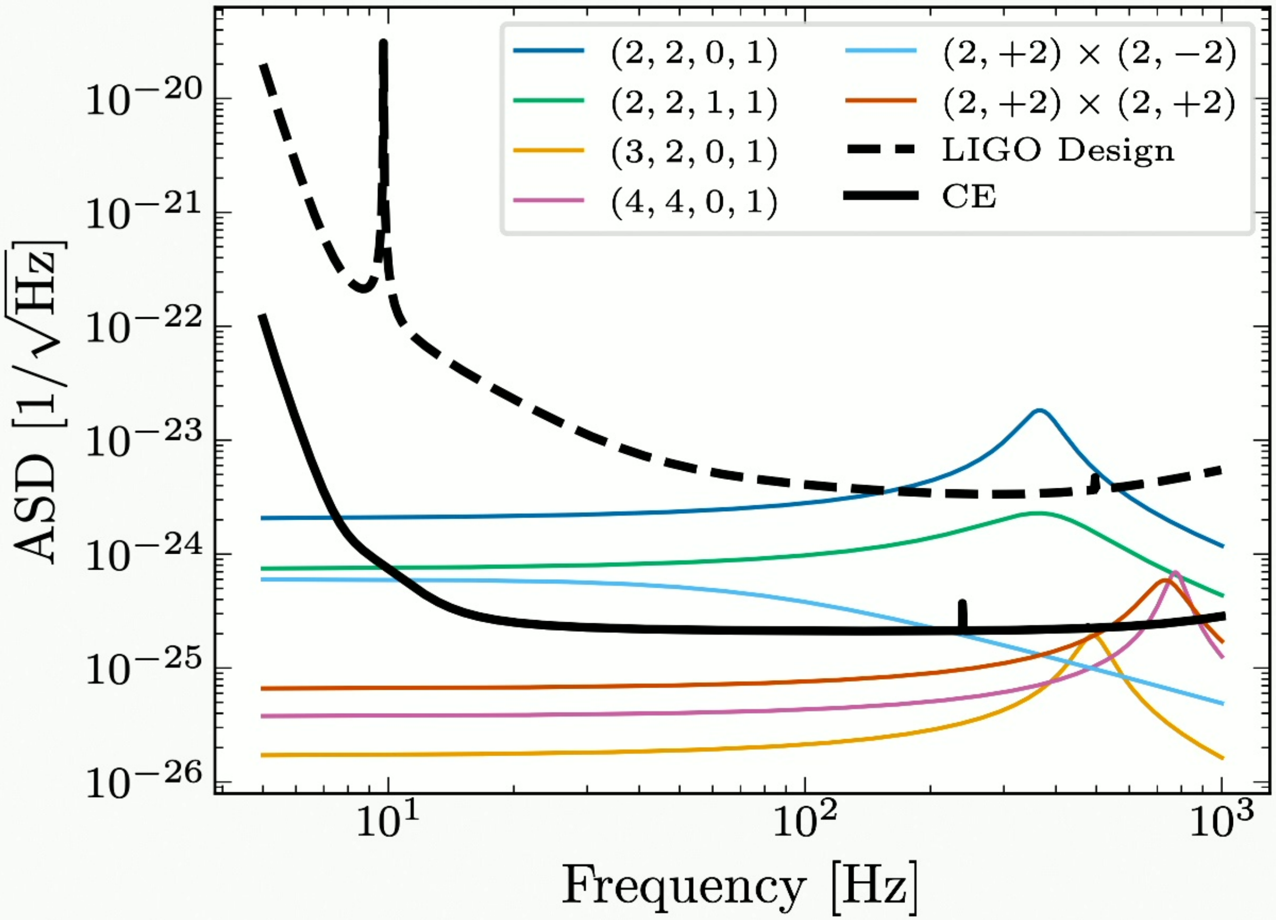
[Yi+, 2403.09767]

Observing Memory as a 2nd-order BHPT Excitation

- **NR simulation:**
  - $q = 1$
  - $\vec{\chi}_{1,2} = \chi^{(z)}_{1,2} = 0.6$
  - $M_f = 60M_\odot$
  - $R = 400\text{Mpc}$
- **QNM Lorentzian:**

$$h_{\text{QNM}} = A_{(\ell,m,n,p)}e^{-i\omega_{(\ell,m,n,p)}t}$$

$$\tilde{h}_{\text{QNM}} = \frac{i}{\sqrt{2\pi}}\frac{A_{(\ell,m,n,p)}}{\omega - \omega_{(\ell,m,n,p)}}$$



[Mitman, *Ringdown Inside & Out*]

Deviations from GR in the ringdown?

Rotating BH QNMs in modified gravity: the EFT viewpoint

QNM calculations:
Significant progress in the past few years

Theories: sum over curvature invariants with scalar-dependent coefficients

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R+\sum_{n=2}^{\infty}\ell^{2n-2}\mathcal{L}_{(n)}\right]$$

and more specifically, at order ℓ^4

EsGB dCS (dilaton+axion)

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{|g|}\Big\{R+\alpha_1\phi_1\ell^2R_{\rm GB}+\alpha_2(\phi_2\cos\theta_m+\phi_1\sin\theta_m)\ell^2R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$$
$$+\lambda_{\rm ev}\ell^4R_{\mu\nu}^{\rho\sigma}R_{\rho\sigma}^{\delta\gamma}R_{\delta\gamma}^{\mu\nu}+\lambda_{\rm odd}\ell^4R_{\mu\nu}^{\rho\sigma}R_{\rho\sigma}^{\delta\gamma}\tilde{R}_{\delta\gamma}^{\mu\nu}-\frac{1}{2}(\partial\phi_1)^2-\frac{1}{2}(\partial\phi_2)^2\Big\}$$

Einsteinian cubic gravity (+parity-breaking) - causality constraints [Camanho+ 1407.5597]
Next order, no new DOFs [Endlich-Gorbenko-Huang-Senatore, 1704.01590]

$$S_{(4)}=\frac{\ell^6}{16\pi G}\int d^4x\sqrt{|g|}\left\{\epsilon_1\mathcal{C}^2+\epsilon_2\tilde{\mathcal{C}}^2+\epsilon_3\mathcal{C}\tilde{\mathcal{C}}\right\}\qquad \mathcal{C}=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma},\quad \tilde{\mathcal{C}}=R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$$

[Cano, *Ringdown Inside & Out*]

Great progress in calculations of rotating BH QNMs in modified gravity

Teukolsky equation separability is special: Petrov Type D, hidden symmetries (Killing tensor)
Beyond GR: no analytical background, no Petrov Type D, non-separability, higher-order EOMs

Metric perturbations

Slowly rotating BHs in specific theories (EdGB/EsGB, dCS), not restricted to small coupling
[Molina+ (dCS), Blazquez-Salcedo+ (EdGB/EsGB), Pierini-Gualtieri (EsGB)…]

Generalized Teukolsky equations

Linear shift in QNM frequencies can be computed (Leaver, eigenvalue perturbation techniques)
Algorithm to compute small-coupling corrections to the frequencies, up to order 18 in rotation
[Li-Wagle-Chen-Yunes, Hussain-Zimmerman, Cano-Fransen-Hertog-Maenaut, Cano+]

Spectral methods

Arbitrary coupling, in principle (but not in practice) arbitrary rotation
[Chung-Wagle-Yunes, Blazquez-Salcedo-Scen Koo-Kleihaus-Kunz]

[Cano, *Ringdown Inside & Out*]

Parametrized ringdown (in the ppE spirit) for small coupling

Modifications to the gravity sector and/or beyond Standard Model physics: expect

- small modifications to the functional form of the potentials – parametrize
- coupling between the wave equations

$$V = V_\pm + \delta V_\pm \qquad \delta V_\pm = \frac{1}{r_H^2} \sum_{j=0}^{\infty} \alpha_j^\pm \left(\frac{r_H}{r}\right)^j \qquad \omega_{\rm QNM}^\pm = \omega_0^\pm + \sum_{j=0}^{\infty} \alpha_j^\pm e_j^\pm$$

$$V = V_s + \delta V_s \qquad \delta V_s = \frac{1}{r_H^2} \sum_{j=0}^{\infty} \beta_j^s \left(\frac{r_H}{r}\right)^j \qquad \omega_{\rm QNM}^s = \omega_0^s + \sum_{j=0}^{\infty} \beta_j^s d_j^s$$

Maximum of $f(r)\alpha_j^\pm \left(\frac{r_H}{r}\right)^j$ is $\alpha_j^\pm \frac{(1+1/j)^{-j}}{j+1}$, so corrections are small if:

$$(\alpha_j^\pm, \beta_j^s) \ll (1+1/j)^j (j+1)$$

Can map to specific theories like ppE, now extended to rotating black holes

[Cardoso+, 1901.01265; McManus+, 1906.05155; Kimura+; Cano+ 2407.15947]

Parametrized spectroscopy (ParSpec): how many observations do we need?

Use a small-spin expansion and add parametric deviations to frequency and damping time
Assume you detect **N** sources, and **q** QNM frequencies for each source

$J = 1, 2, \dots, q$ modes/source

Order in the spin expansion: need at least 4 or 5 in GR

$i = 1, \dots, N$ sources

$$\omega_i^{(J)} = \frac{1}{M_i}\sum_{n=0}^{D}\chi_i^n w_J^{(n)}\left(1+\gamma_i\delta w_J^{(n)}\right)$$

$$\tau_i^{(J)} = M_i\sum_{n=0}^{D}\chi_i^n t_J^{(n)}\left(1+\gamma_i\delta t_J^{(n)}\right)$$

Expansion coefficients in GR

Small, universal non-GR corrections

How many parameters?
If $\gamma_i = \alpha$ for all sources ,
reabsorb $\gamma_i \delta w^{(n)} \to \delta w^{(n)}$

$\mathcal{P} = 2(D+1)q \longrightarrow D = 4$

$q = 1$, $\ell = m = 2 \longrightarrow \mathcal{P} = 10$

$q = 2$, $\ell = m = 2$, $\ell = m = 3 \longrightarrow \mathcal{P} = 20$

How many observables? $\mathcal{O} = 2N \times q$

$\boxed{\text{Need only } N \geq D+1}$

[Maselli+, 1910.12893]

Parametrized spectroscopy (ParSpec): full population study for ET/CE

Construct astrophysical populations based on LVK observations, different assumptions on spins

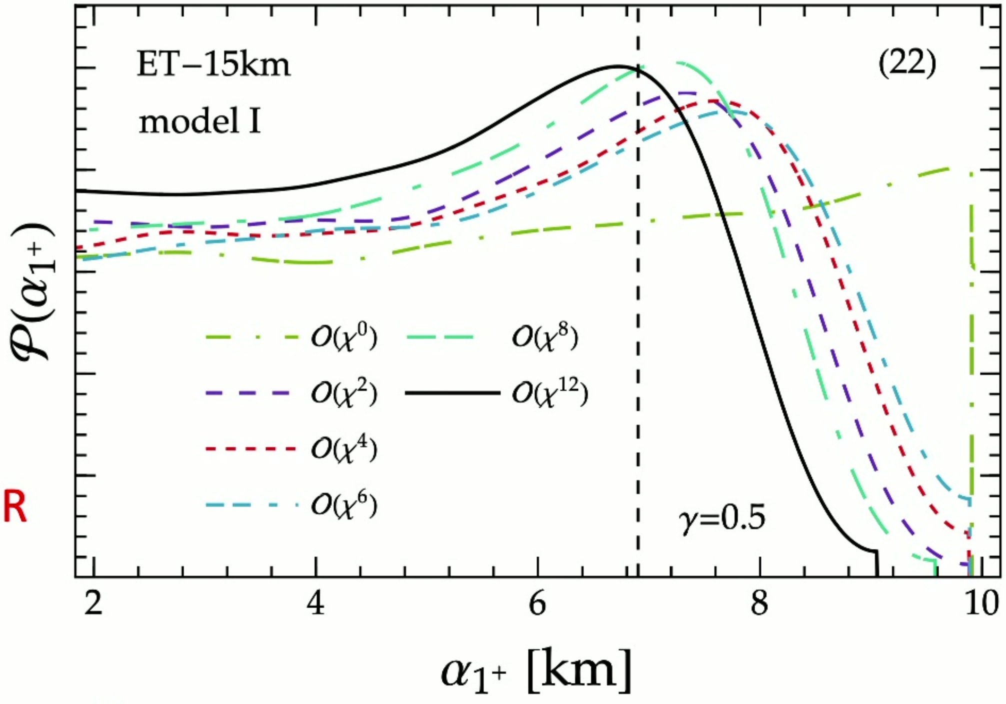
Theory-agnostic tests (with/without knowledge of mass and spin)
Theory-specific bounds for EsGB, dCS, various classes of EFTs

Convergence of EsGB and dCS bounds limited by low order of the spin expansion (but this can be improved with recent results)

EFT posteriors converge with spin (peak tends to injected value)

Bad news: bounds generally compatible with GR
Why?

low-mass binaries have low SNR
high mass binaries have small curvature corrections

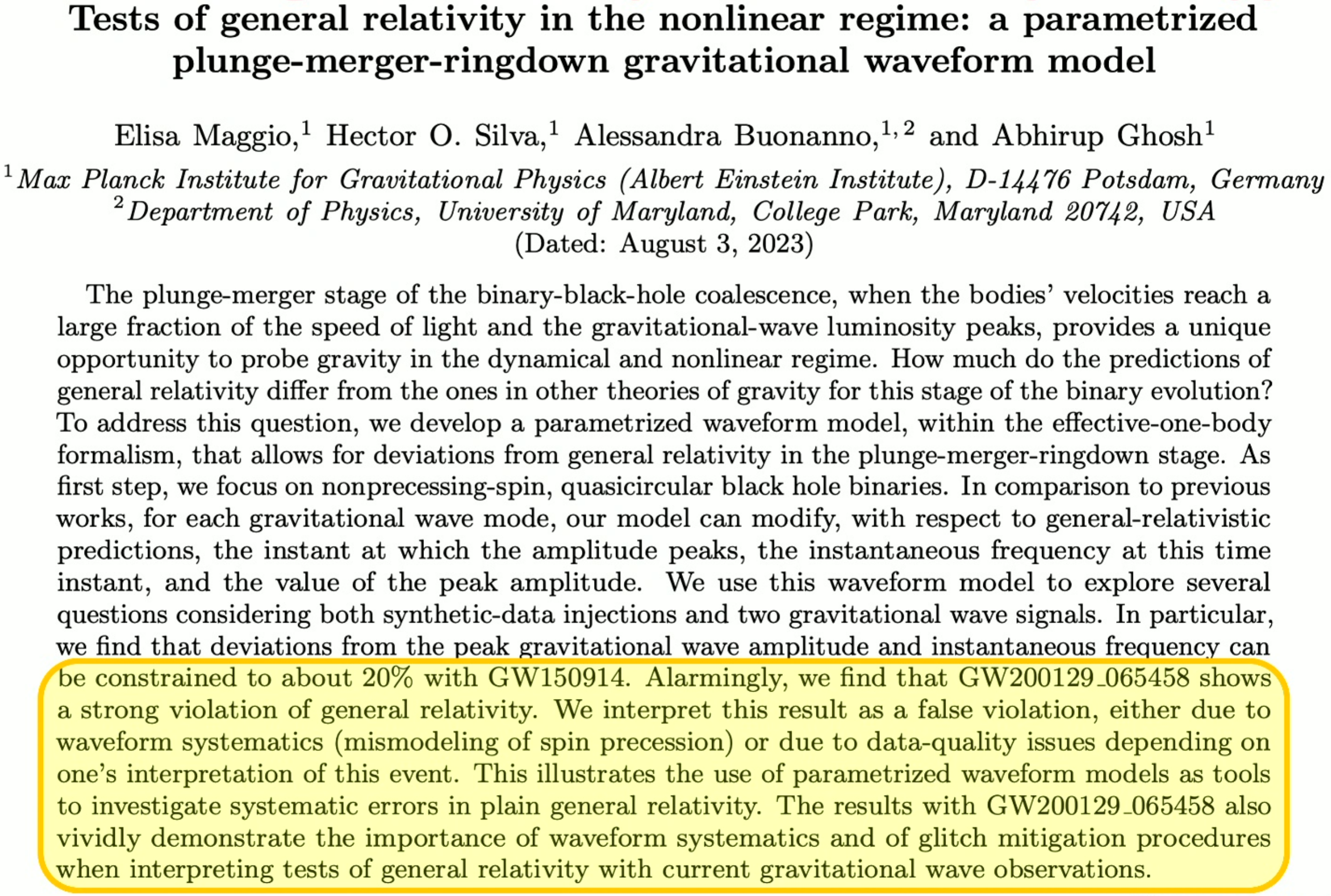


[Maselli+, 2311.14803]

WITH GREAT SNR
COMES GREAT
RESPONSIBILITY.

SPIDERMAN

Beware! False general relativity violations already in binary pulsar



Tests of general relativity in the nonlinear regime: a parametrized plunge-merger-ringdown gravitational waveform model

Elisa Maggio,[1] Hector O. Silva,[1] Alessandra Buonanno,[1,2] and Abhirup Ghosh[1]

[1]*Max Planck Institute for Gravitational Physics (Albert Einstein Institute), D-14476 Potsdam, Germany*
[2]*Department of Physics, University of Maryland, College Park, Maryland 20742, USA*
(Dated: August 3, 2023)

The plunge-merger stage of the binary-black-hole coalescence, when the bodies' velocities reach a large fraction of the speed of light and the gravitational-wave luminosity peaks, provides a unique opportunity to probe gravity in the dynamical and nonlinear regime. How much do the predictions of general relativity differ from the ones in other theories of gravity for this stage of the binary evolution? To address this question, we develop a parametrized waveform model, within the effective-one-body formalism, that allows for deviations from general relativity in the plunge-merger-ringdown stage. As first step, we focus on nonprecessing-spin, quasicircular black hole binaries. In comparison to previous works, for each gravitational wave mode, our model can modify, with respect to general-relativistic predictions, the instant at which the amplitude peaks, the instantaneous frequency at this time instant, and the value of the peak amplitude. We use this waveform model to explore several questions considering both synthetic-data injections and two gravitational wave signals. In particular, we find that deviations from the peak gravitational wave amplitude and instantaneous frequency can be constrained to about 20% with GW150914. Alarmingly, we find that GW200129_065458 shows a strong violation of general relativity. We interpret this result as a false violation, either due to waveform systematics (mismodeling of spin precession) or due to data-quality issues depending on one's interpretation of this event. This illustrates the use of parametrized waveform models as tools to investigate systematic errors in plain general relativity. The results with GW200129_065458 also vividly demonstrate the importance of waveform systematics and of glitch mitigation procedures when interpreting tests of general relativity with current gravitational wave observations.

[Maggio+, 2212.09655]

Take-home messages

Addition of overtones long known to provide a better fit to:

point-particle waveforms, nonrotating (1970s) and rotating (1980s) collapse
head-on black hole collisions (1990s), quasicircular mergers (circa 2005)

Can a linear superposition of overtones describe nonlinear mergers up to the peak? No

Clear evidence (now from multiple groups) of nonlinear modes in numerical waveforms
jaxqualin, variable projection: systematic extraction of linear and nonlinear modes from NR simulations
Need more modeling of nonlinear modes (merger simulations, high-order perturbation theory)

Have we observed overtones in GW150914?

At best inconclusive
Analysis only **at or before** the peak, where the linear model is definitely not applicable
Injections show that noise can induce artificial evidence for an overtone
My best bet for O4/O5: higher multipole observation in unequal-mass events

Future

LISA (and less likely, XG detectors on the ground) may observe nonlinear modes
…if rates are high and we control systematics
Ringdown bounds on theories with mass-dependent scales severely limited by SNR/curvature interplay
Beware of false general relativity violations! Systematics
Complementarity with ngEHT, BHEX: imaging & ringdown probe same physics (light ring)