Title: Lecture - Numerical Methods, PHYS 777

Speakers: Erik Schnetter, Dustin Lang

Collection/Series: Numerical Methods (Core), PHYS 777-, January 6 - February 5, 2025

Subject: Other

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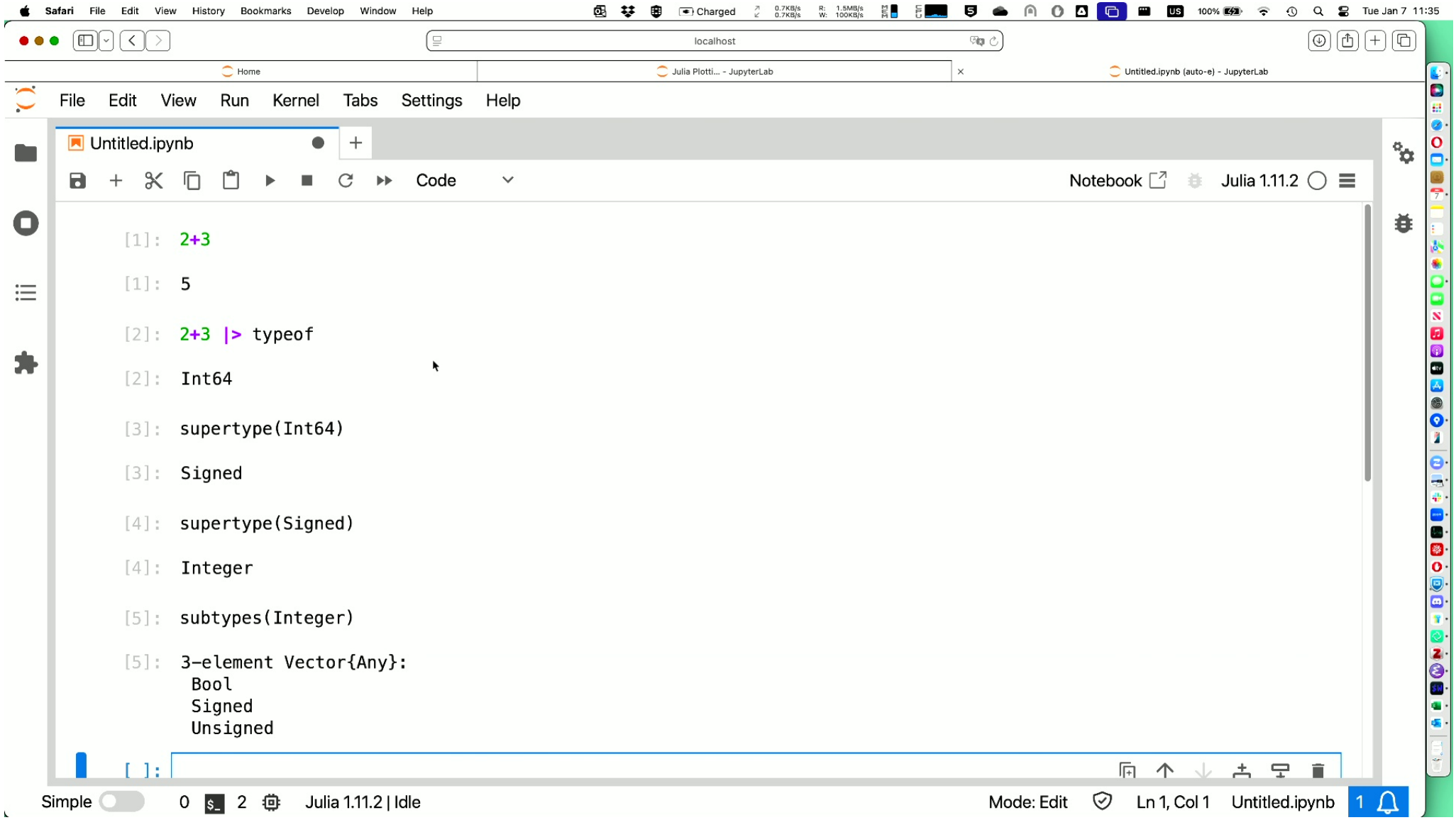
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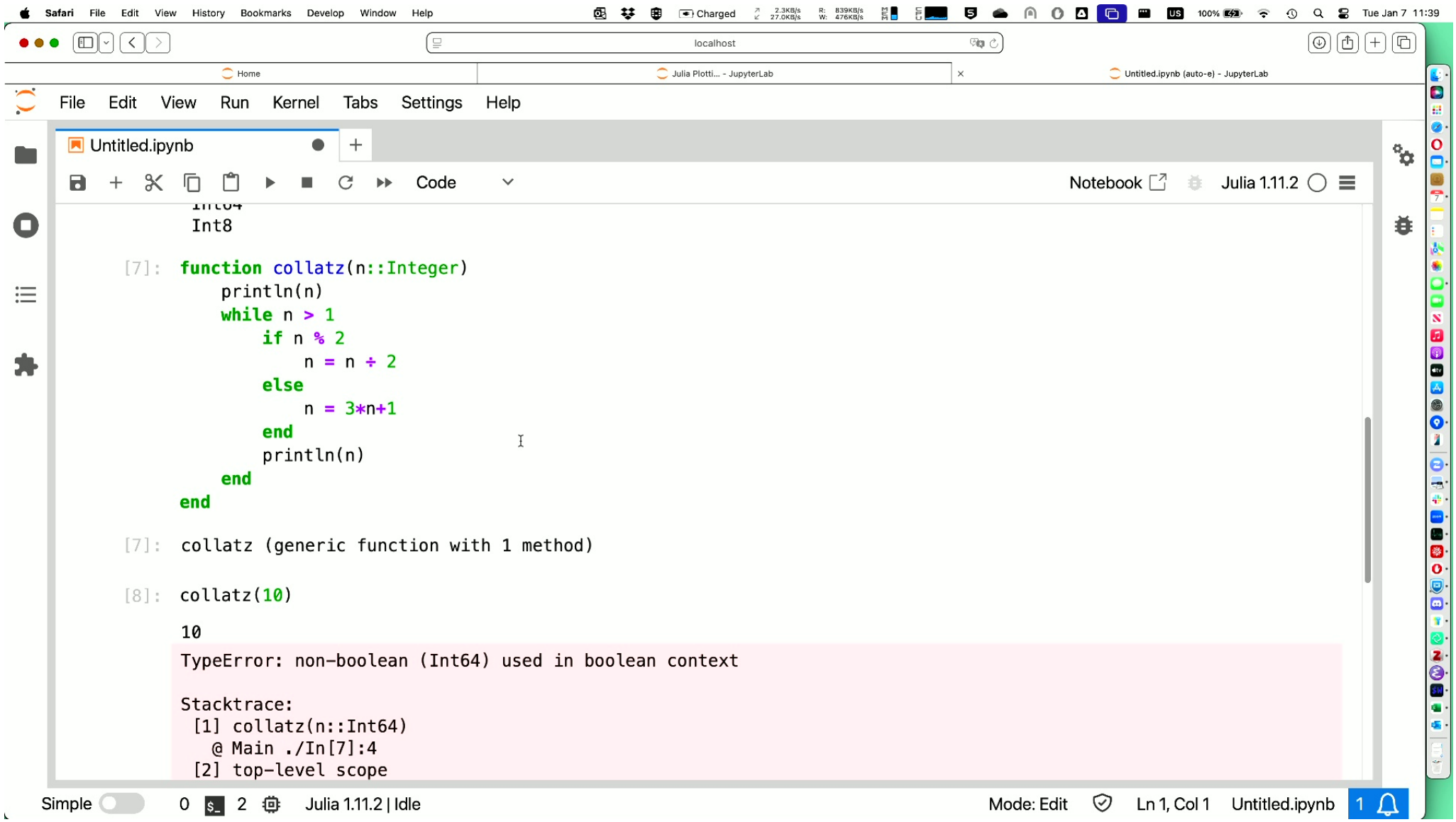
Code
Notebook
Julia 1.11.2
[1]: 2+3
[1]: 5
[2]: 2+3 |> typeof
[2]: Int64
[3]: supertype(Int64)
[3]: Signed
[4]: supertype(Signed)
[4]: Integer
[5]: subtypes(Integer)
[5]: 3-element Vector{Any}:
Bool
Signed
Unsigned
[ ]:
Simple
0
2
Julia 1.11.2 | Idle
Mode: Edit
Ln 1, Col 1
Untitled.ipynb
1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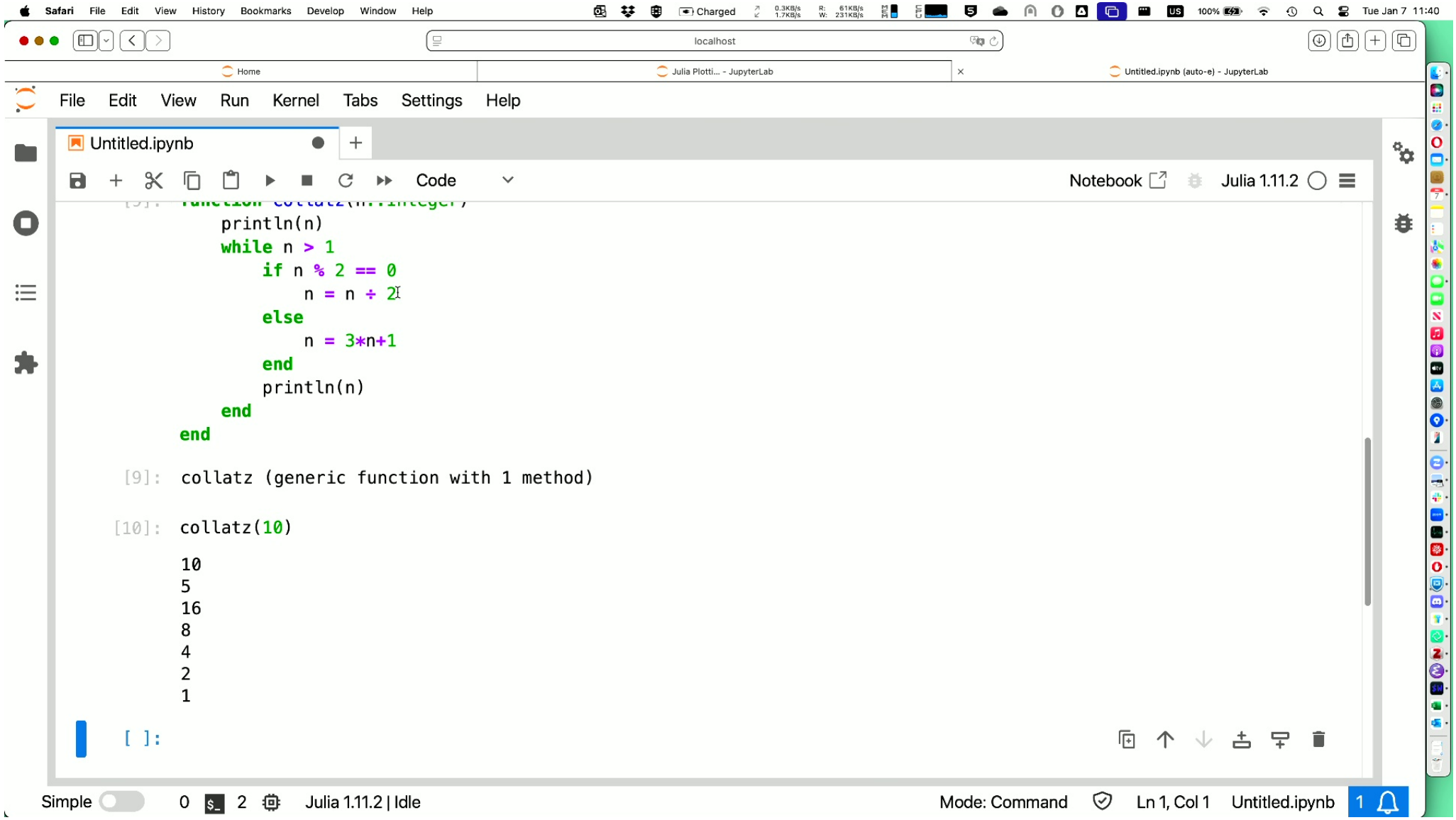
Code
Notebook
Julia 1.11.2
Int8
[7]:
function collatz(n::Integer)
println(n)
while n > 1
if n % 2
n = n ÷ 2
else
n = 3*n+1
end
println(n)
end
end
[7]:
collatz (generic function with 1 method)
[8]:
collatz(10)
10
TypeError: non-boolean (Int64) used in boolean context
Stacktrace:
[1] collatz(n::Int64)
@ Main ./In[7]:4
[2] top-level scope
Simple
0
2
Julia 1.11.2 | Idle
Mode: Edit
Ln 1, Col 1
Untitled.ipynb
1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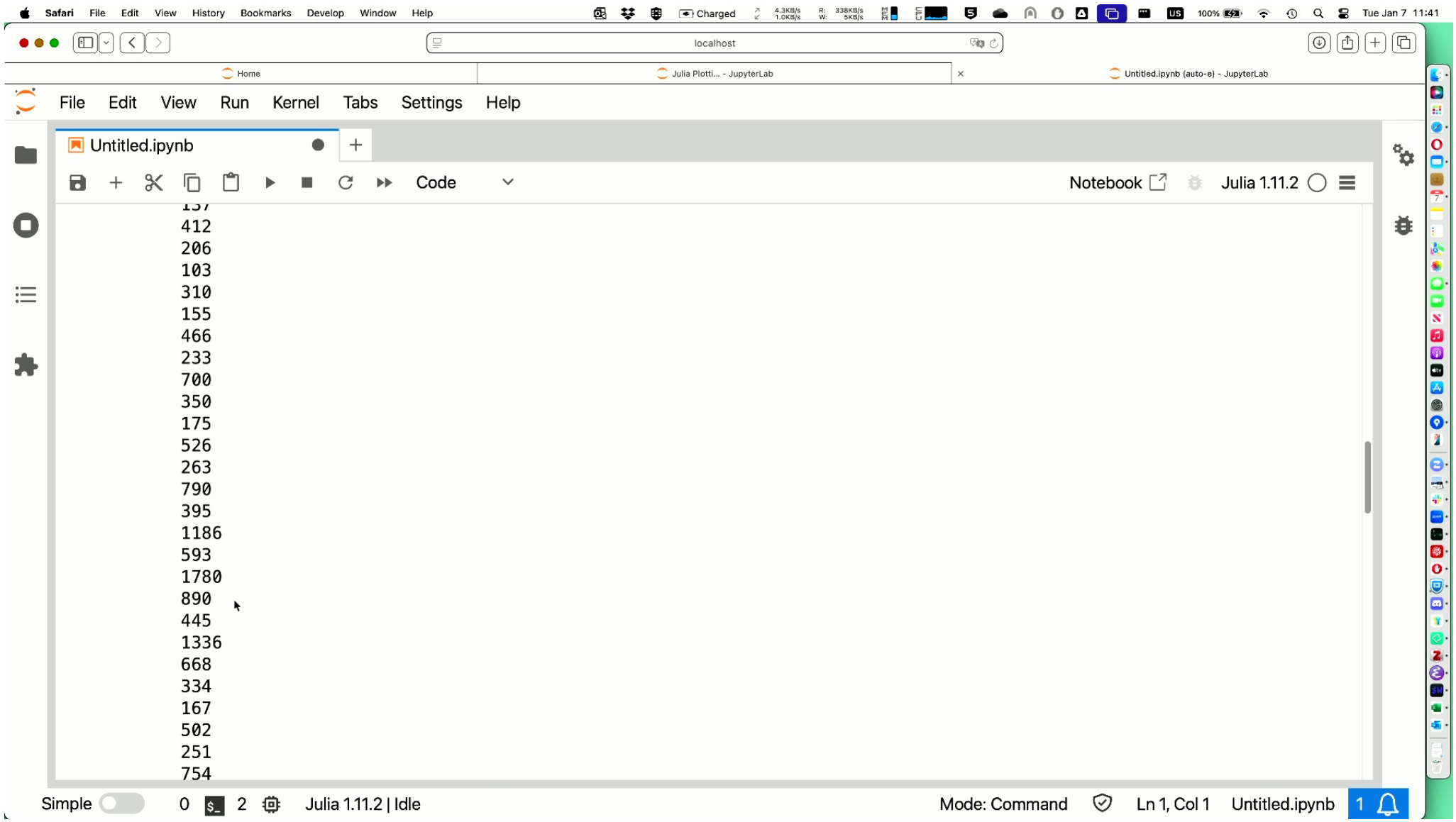
Code
Notebook
Julia 1.11.2
println(n)
while n > 1
if n % 2 == 0
n = n ÷ 2
else
n = 3*n+1
end
println(n)
end
end
[9]: collatz (generic function with 1 method)
[10]: collatz(10)
10
5
16
8
4
2
1
[ ]:
Simple
0
2
Julia 1.11.2 | Idle
Mode: Command
Ln 1, Col 1
Untitled.ipynb
1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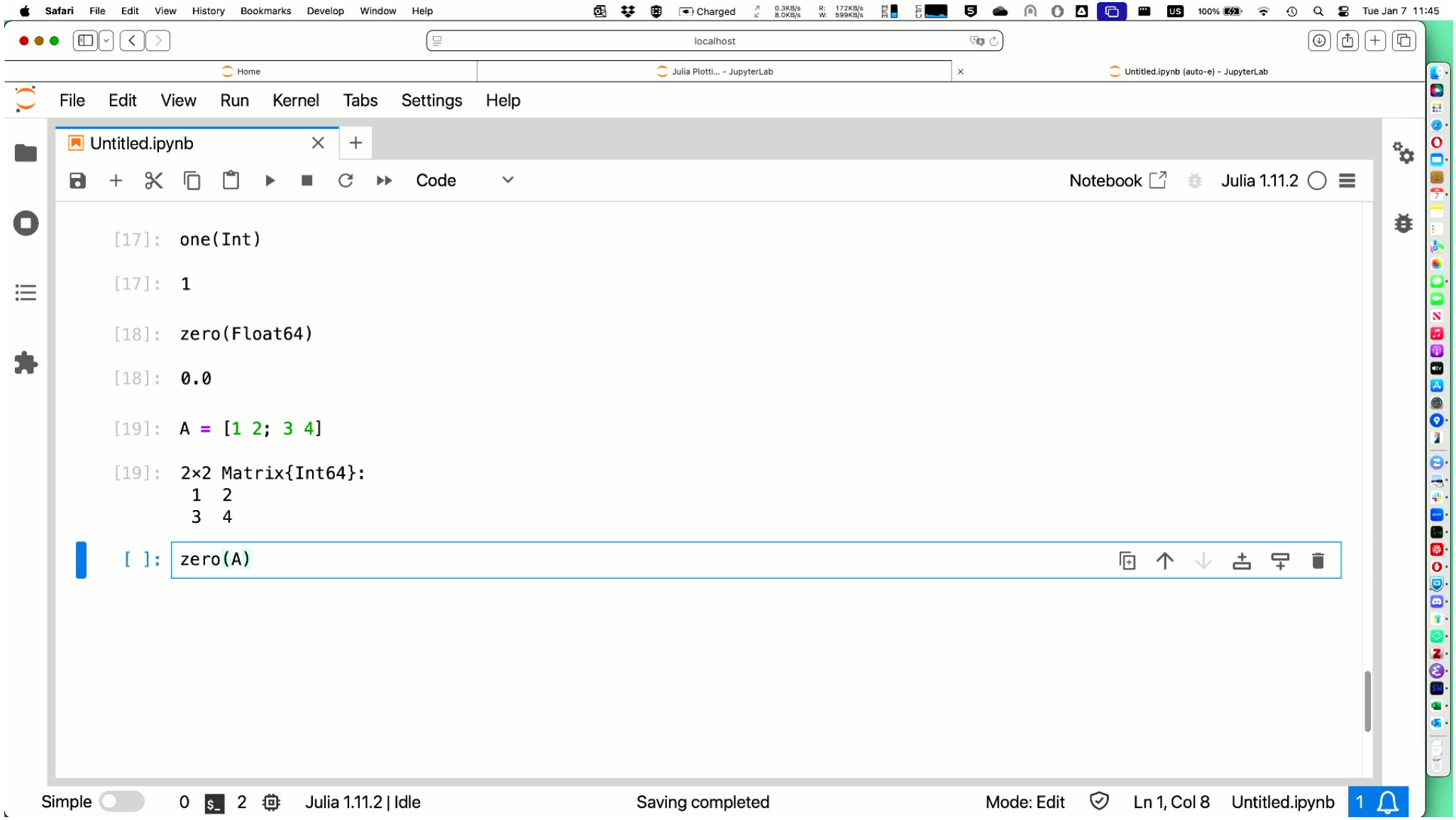
Code
Notebook
Julia 1.11.2
137
412
206
103
310
155
466
233
700
350
175
526
263
790
395
1186
593
1780
890
445
1336
668
334
167
502
251
754
Simple
0
2
Julia 1.11.2 | Idle
Mode: Command
Ln 1, Col 1
Untitled.ipynb
1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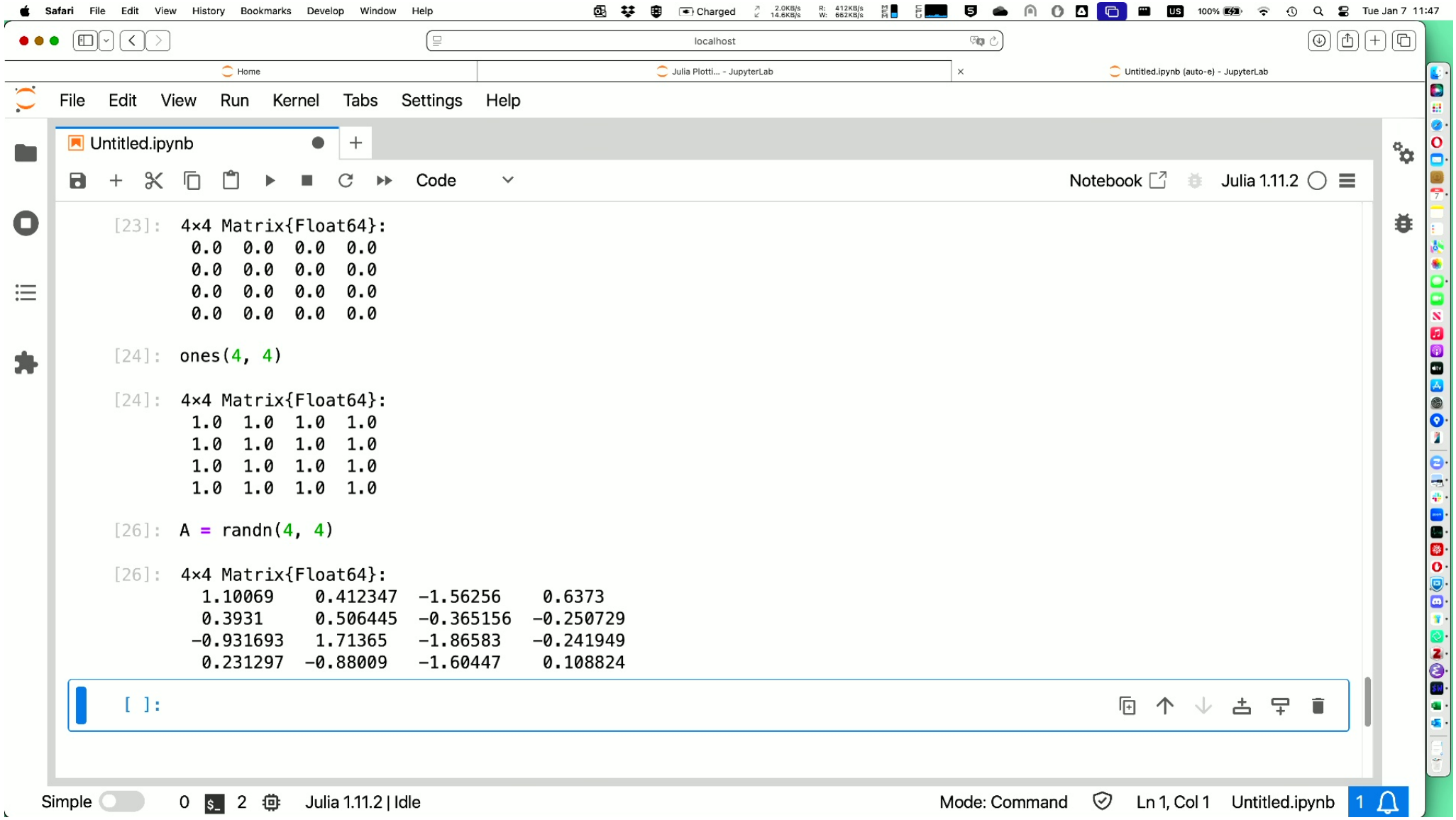
Untitled.ipynb
Code
Notebook
Julia 1.11.2
[17]: one(Int)
[17]: 1
[18]: zero(Float64)
[18]: 0.0
[19]: A = [1 2; 3 4]
[19]: 2×2 Matrix{Int64}:
1 2
3 4
[ ]: zero(A)
Simple
0
2
Julia 1.11.2 | Idle
Saving completed
Mode: Edit
Ln 1, Col 8
Untitled.ipynb
1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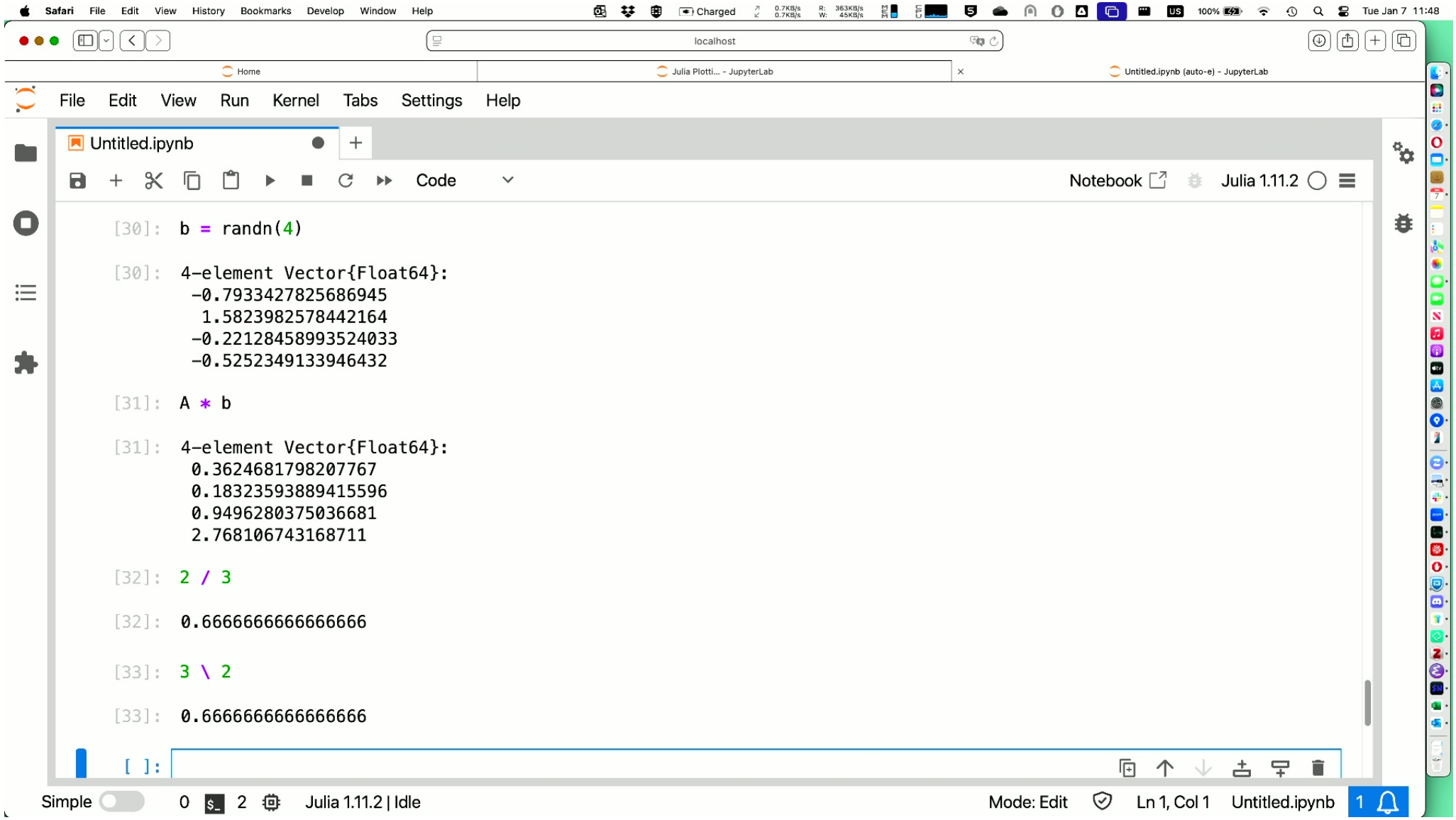
Code
Notebook
Julia 1.11.2
[23]: 4×4 Matrix{Float64}:
0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0
[24]: ones(4, 4)
[24]: 4×4 Matrix{Float64}:
1.0 1.0 1.0 1.0
1.0 1.0 1.0 1.0
1.0 1.0 1.0 1.0
1.0 1.0 1.0 1.0
[26]: A = randn(4, 4)
[26]: 4×4 Matrix{Float64}:
1.10069 0.412347 -1.56256 0.6373
0.3931 0.506445 -0.365156 -0.250729
-0.931693 1.71365 -1.86583 -0.241949
0.231297 -0.88009 -1.60447 0.108824
[ ]:
Simple
0
2
Julia 1.11.2 | Idle
Mode: Command
Ln 1, Col 1
Untitled.ipynb
1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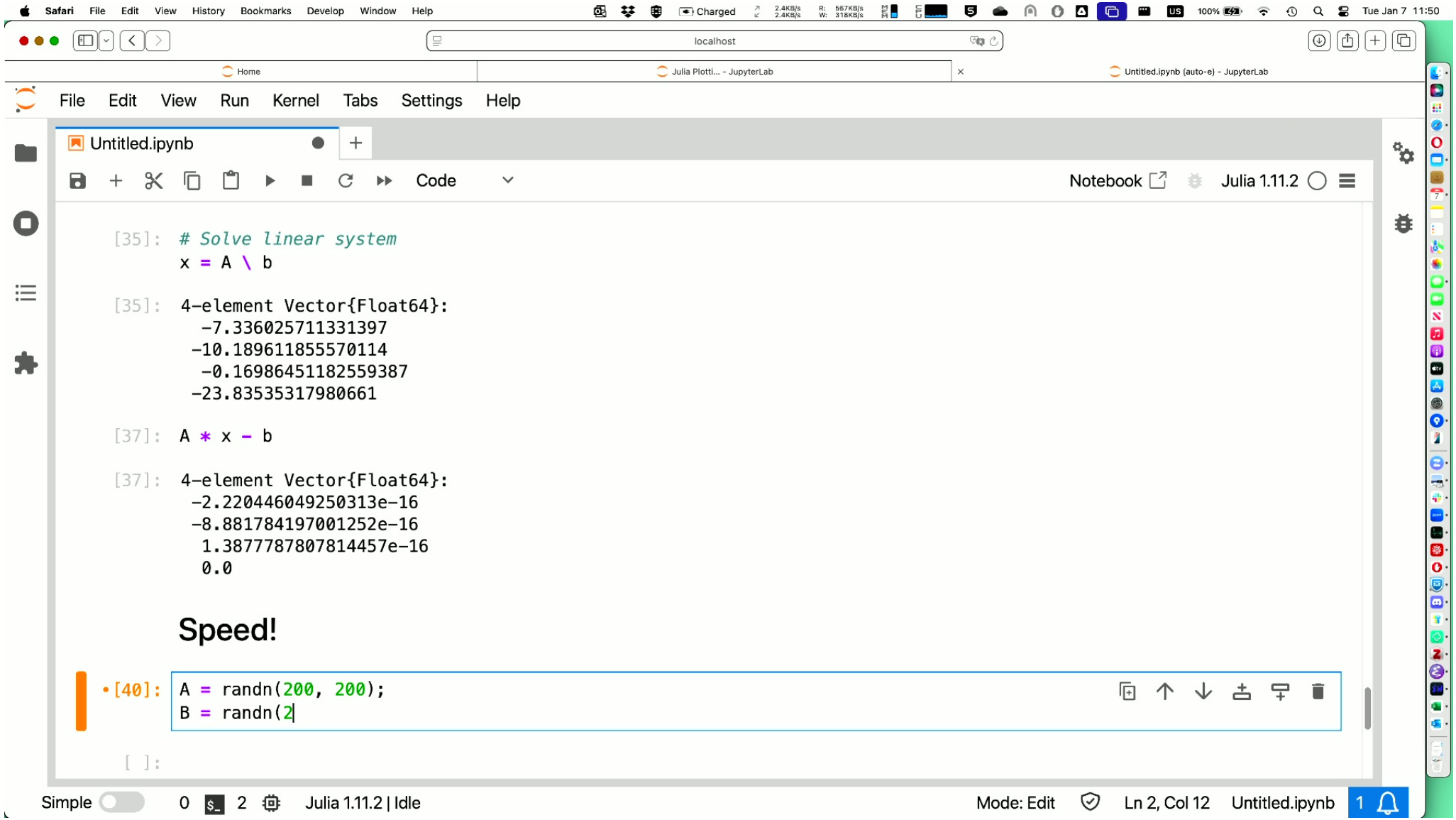
Code
Notebook
Julia 1.11.2
[30]: b = randn(4)
[30]: 4-element Vector{Float64}:
-0.7933427825686945
1.5823982578442164
-0.22128458993524033
-0.5252349133946432
[31]: A * b
[31]: 4-element Vector{Float64}:
0.3624681798207767
0.18323593889415596
0.9496280375036681
2.768106743168711
[32]: 2 / 3
[32]: 0.6666666666666666
[33]: 3 \ 2
[33]: 0.6666666666666666
[ ]:
Simple
0
2
Julia 1.11.2 | Idle
Mode: Edit
Ln 1, Col 1
Untitled.ipynb
1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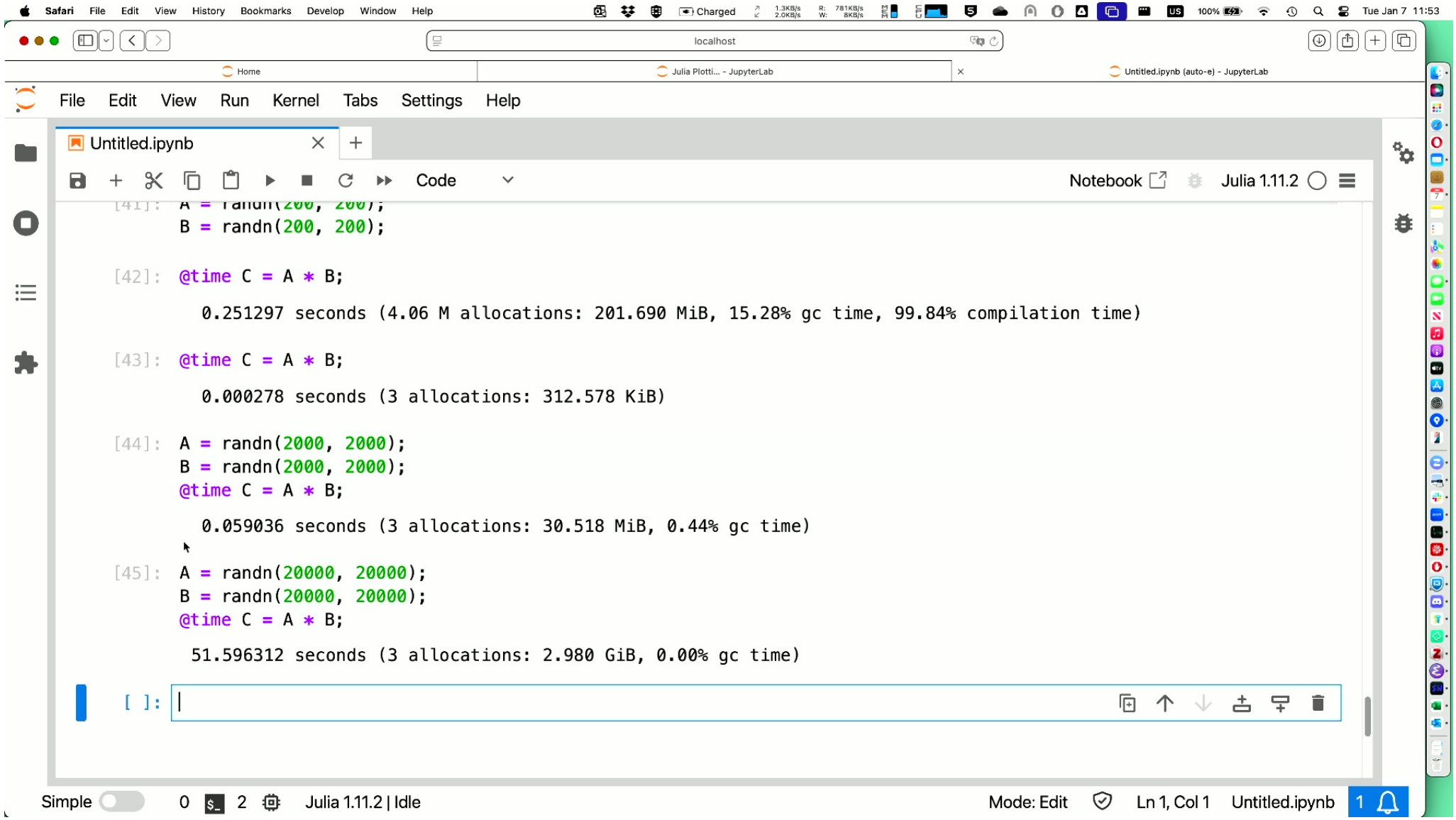
Code
Notebook
Julia 1.11.2
[35]: # Solve linear system
x = A \ b
[35]: 4-element Vector{Float64}:
-7.336025711331397
-10.189611855570114
-0.16986451182559387
-23.83535317980661
[37]: A * x - b
[37]: 4-element Vector{Float64}:
-2.220446049250313e-16
-8.881784197001252e-16
1.3877787807814457e-16
0.0
Speed!
[40]: A = randn(200, 200);
B = randn(2
[ ]:
Simple
0
2
Julia 1.11.2 | Idle
Mode: Edit
Ln 2, Col 12
Untitled.ipynb
1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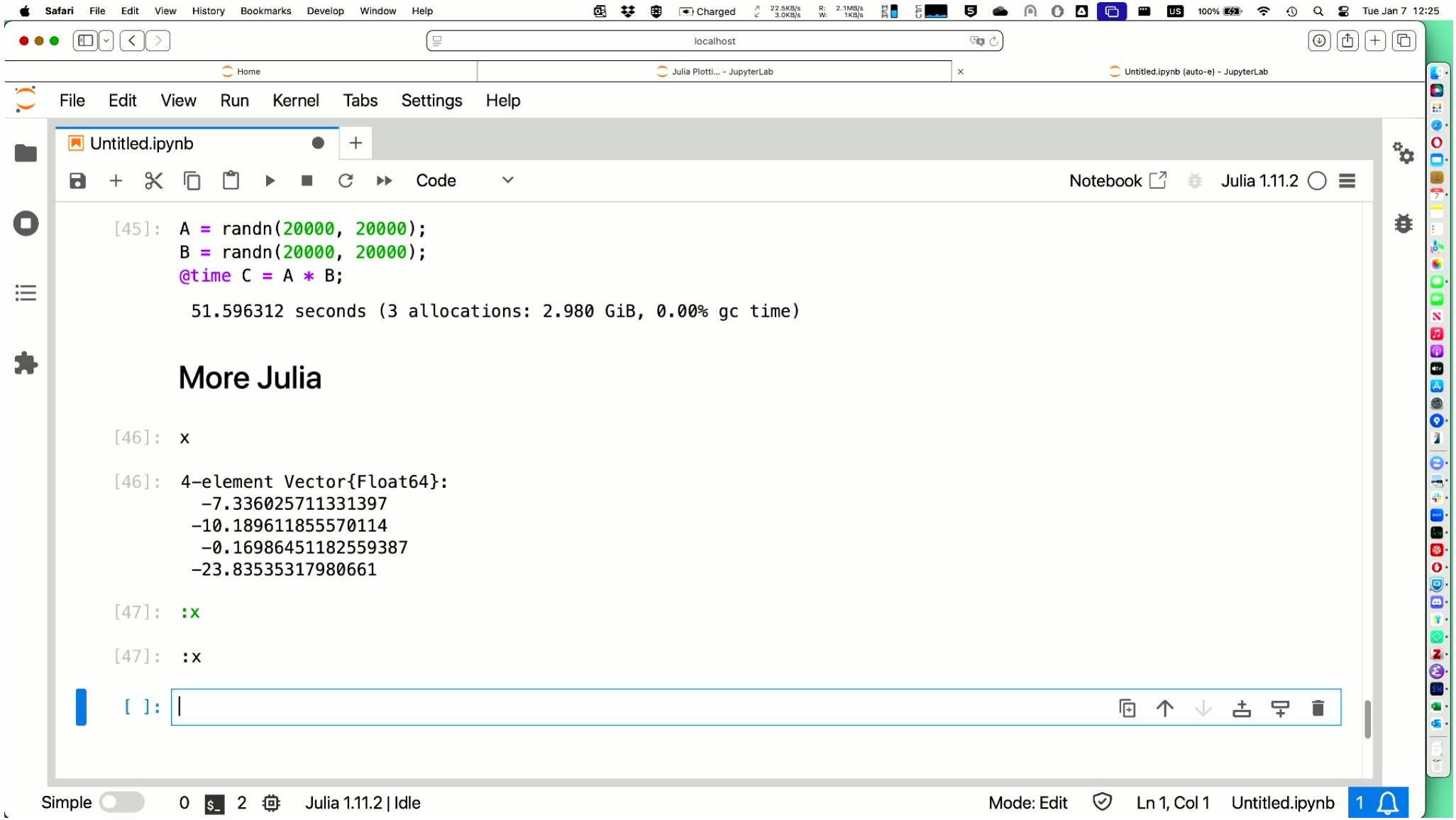
Code
Notebook
Julia 1.11.2
B = randn(200, 200);
[42]: @time C = A * B;
0.251297 seconds (4.06 M allocations: 201.690 MiB, 15.28% gc time, 99.84% compilation time)
[43]: @time C = A * B;
0.000278 seconds (3 allocations: 312.578 KiB)
[44]: A = randn(2000, 2000);
B = randn(2000, 2000);
@time C = A * B;
0.059036 seconds (3 allocations: 30.518 MiB, 0.44% gc time)
[45]: A = randn(20000, 20000);
B = randn(20000, 20000);
@time C = A * B;
51.596312 seconds (3 allocations: 2.980 GiB, 0.00% gc time)
[ ]:
Simple
0
2
Julia 1.11.2 | Idle
Mode: Edit
Ln 1, Col 1
Untitled.ipynb
1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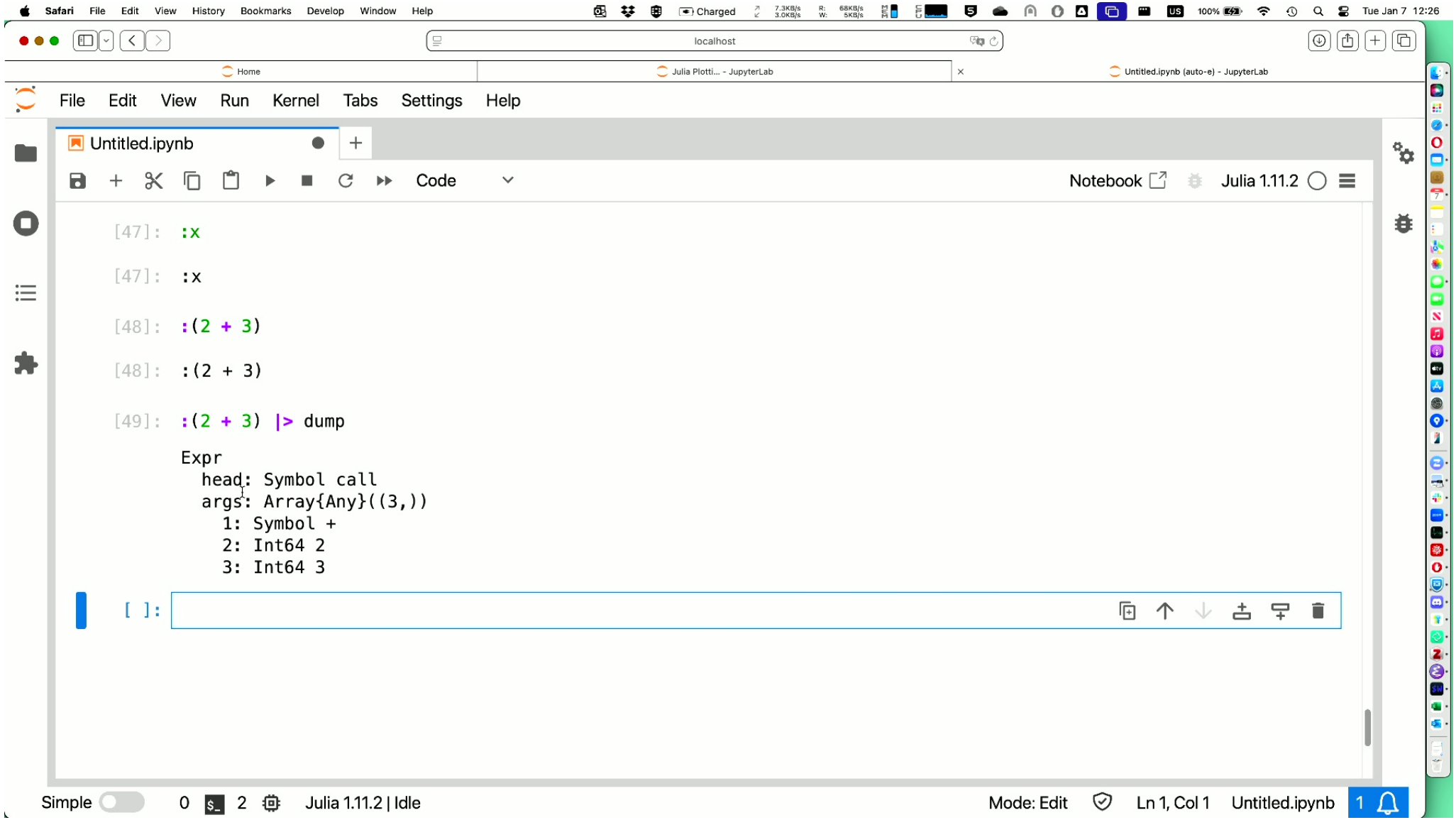
Code
Notebook
Julia 1.11.2
[45]: A = randn(20000, 20000);
B = randn(20000, 20000);
@time C = A * B;
51.596312 seconds (3 allocations: 2.980 GiB, 0.00% gc time)
More Julia
[46]: x
[46]: 4-element Vector{Float64}:
-7.336025711331397
-10.189611855570114
-0.16986451182559387
-23.83535317980661
[47]: :x
[47]: :x
[ ]:
Simple
0
2
Julia 1.11.2 | Idle
Mode: Edit
Ln 1, Col 1
Untitled.ipynb
1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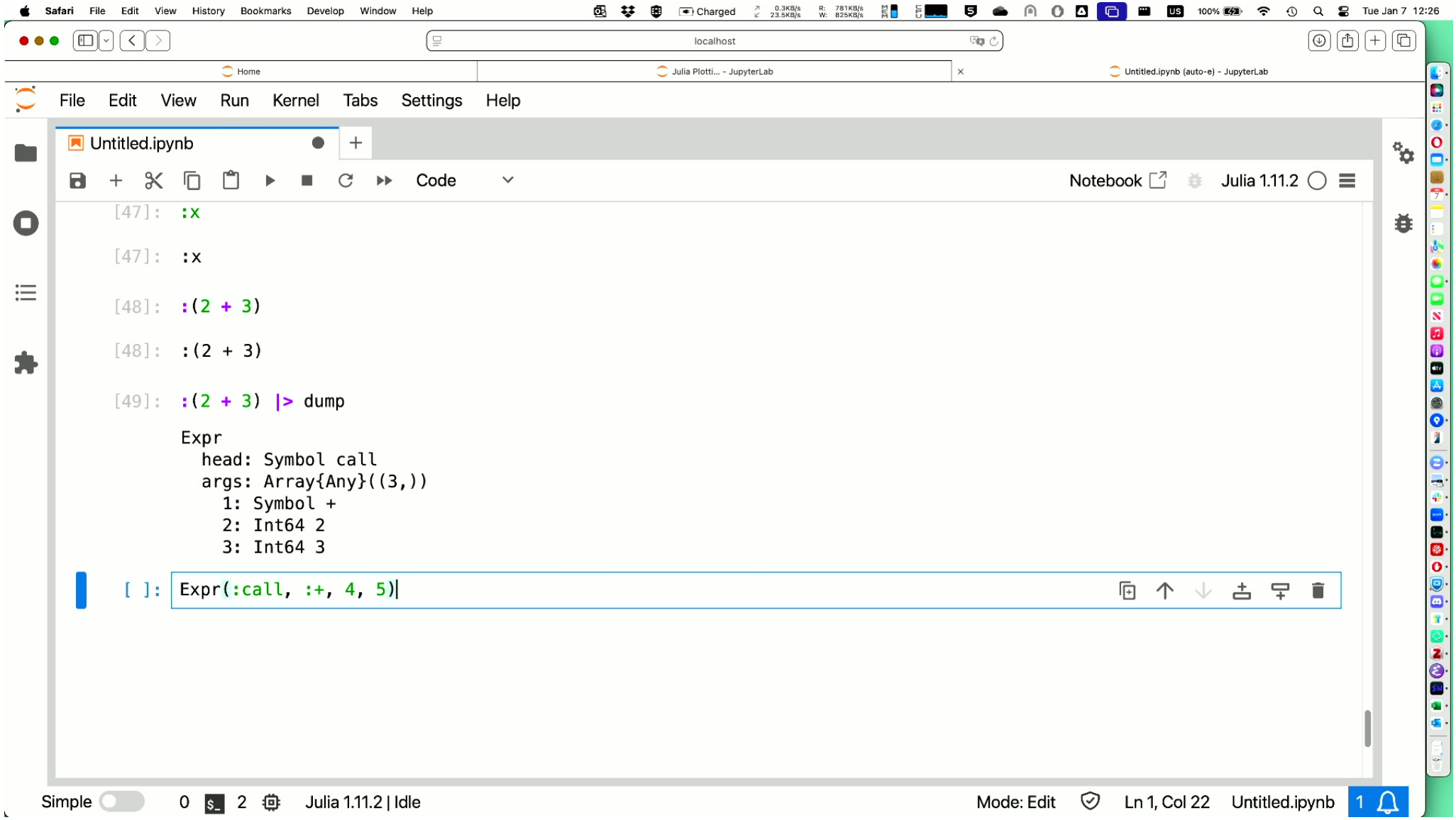
Untitled.ipynb
Code
Notebook
Julia 1.11.2
[47]: :x
[47]: :x
[48]: :(2 + 3)
[48]: :(2 + 3)
[49]: :(2 + 3) |> dump
Expr
head: Symbol call
args: Array{Any}((3,))
1: Symbol +
2: Int64 2
3: Int64 3
[ ]:
Simple
0
2
Julia 1.11.2 | Idle
Mode: Edit
Ln 1, Col 1
Untitled.ipynb
1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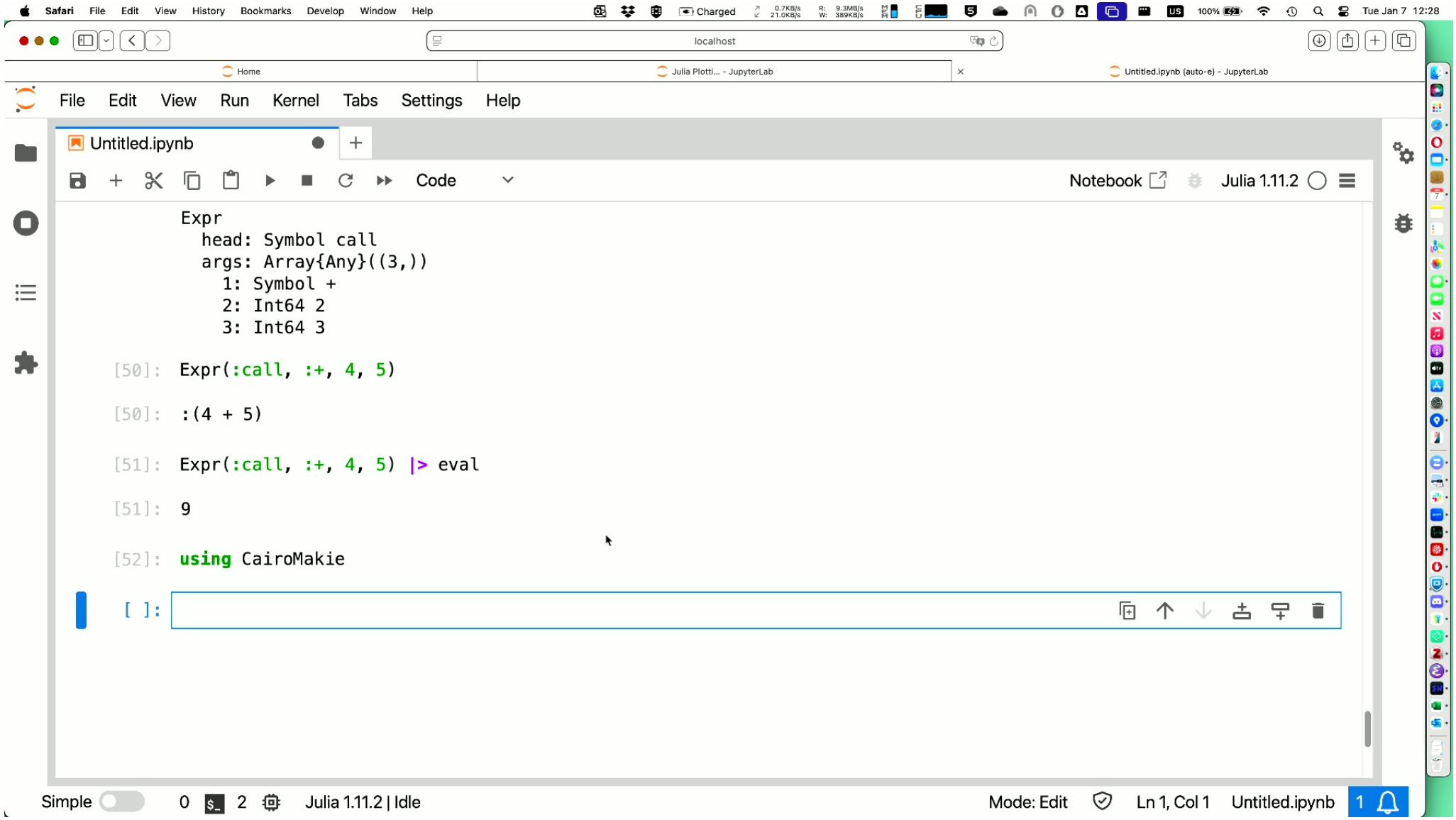
Untitled.ipynb
Code
Notebook
Julia 1.11.2
[47]: :x
[47]: :x
[48]: :(2 + 3)
[48]: :(2 + 3)
[49]: :(2 + 3) |> dump
Expr
head: Symbol call
args: Array{Any}((3,))
1: Symbol +
2: Int64 2
3: Int64 3
[ ]: Expr(:call, :+, 4, 5)
Simple
0
2
Julia 1.11.2 | Idle
Mode: Edit
Ln 1, Col 22
Untitled.ipynb
1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Code
Notebook
Julia 1.11.2
Expr
head: Symbol call
args: Array{Any}((3,))
1: Symbol +
2: Int64 2
3: Int64 3
[50]: Expr(:call, :+, 4, 5)
[50]: :(4 + 5)
[51]: Expr(:call, :+, 4, 5) |> eval
[51]: 9
[52]: using CairoMakie
[ ]:
Simple
0
2
Julia 1.11.2 | Idle
Mode: Edit
Ln 1, Col 1
Untitled.ipynb
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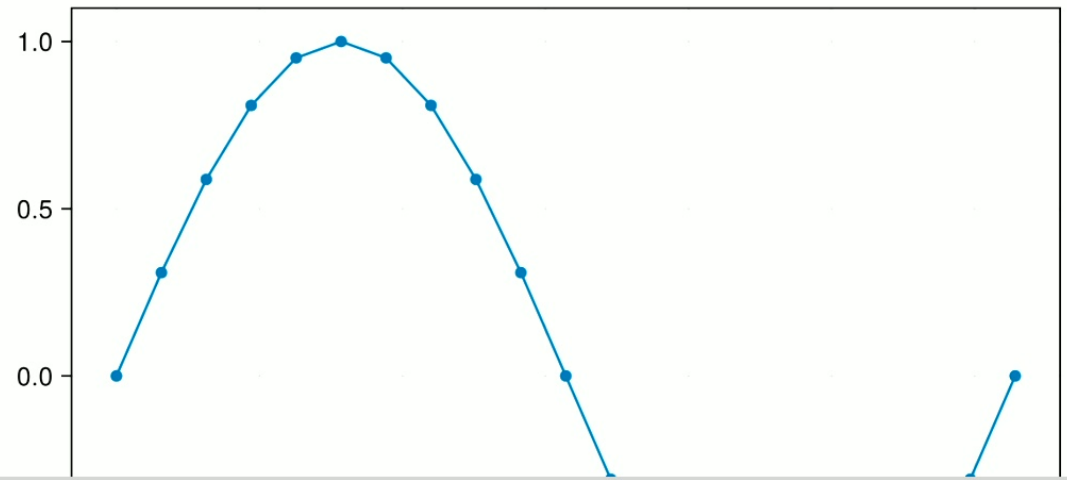
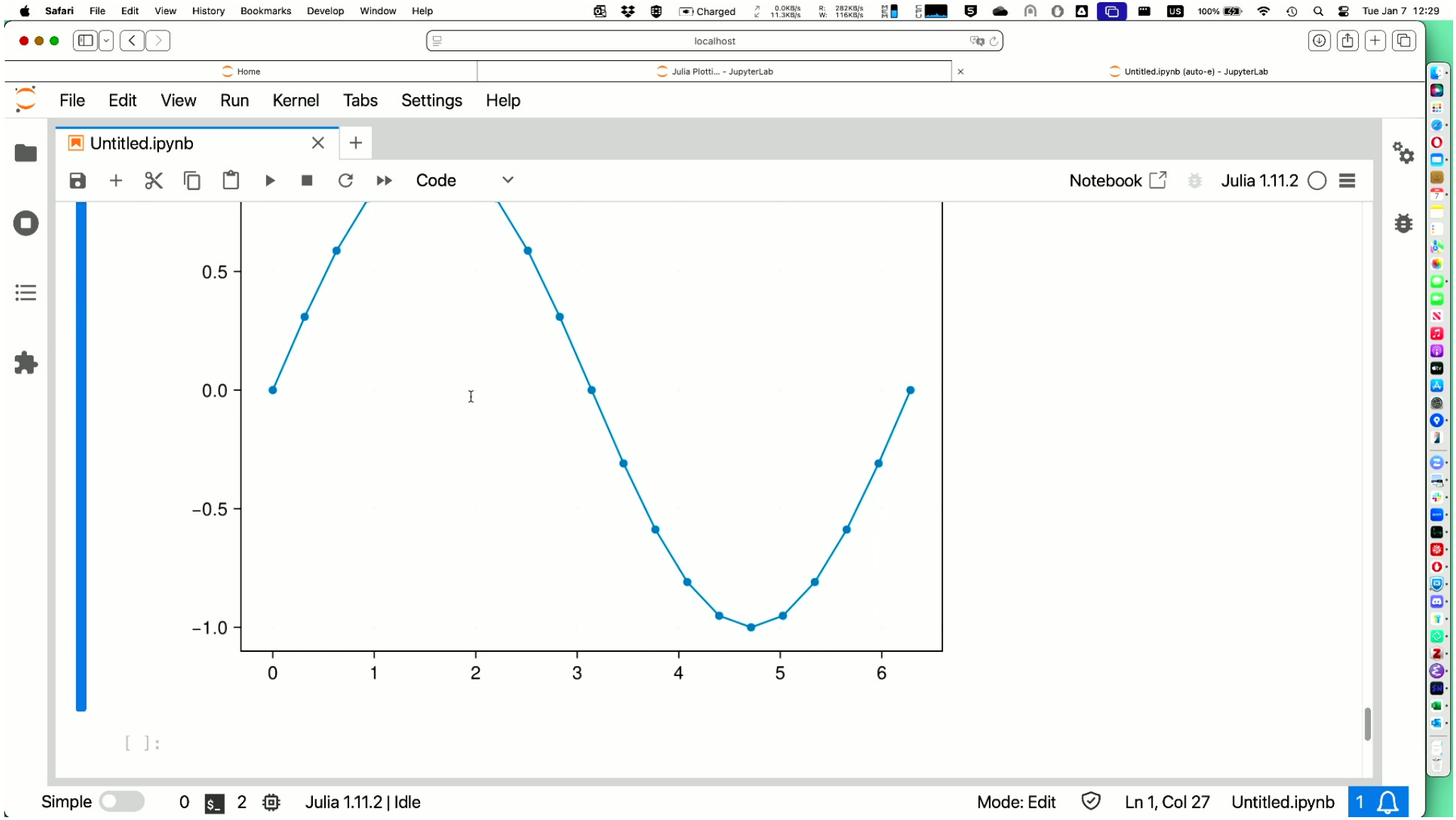
Code Notebook Julia 1.11.2

```
[52]: using CairoMakie

[53]: xs = LinRange(0, 2pi, 21)

[53]: 21-element LinRange{Float64, Int64}:
 0.0, 0.314159, 0.628319, 0.942478, …, 5.34071, 5.65487, 5.96903, 6.28319

[54]: scatterlines(xs, sin.(xs))
```

[54]:



Simple 0 2 Julia 1.11.2 | Idle Mode: Edit Ln 1, Col 27 Untitled.ipynb 1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Code
Notebook
Julia 1.11.2
0.5
0.0
−0.5
−1.0
0
1
2
3
4
5
6
[ ]:
Simple
0
2
Julia 1.11.2 | Idle
Mode: Edit
Ln 1, Col 27
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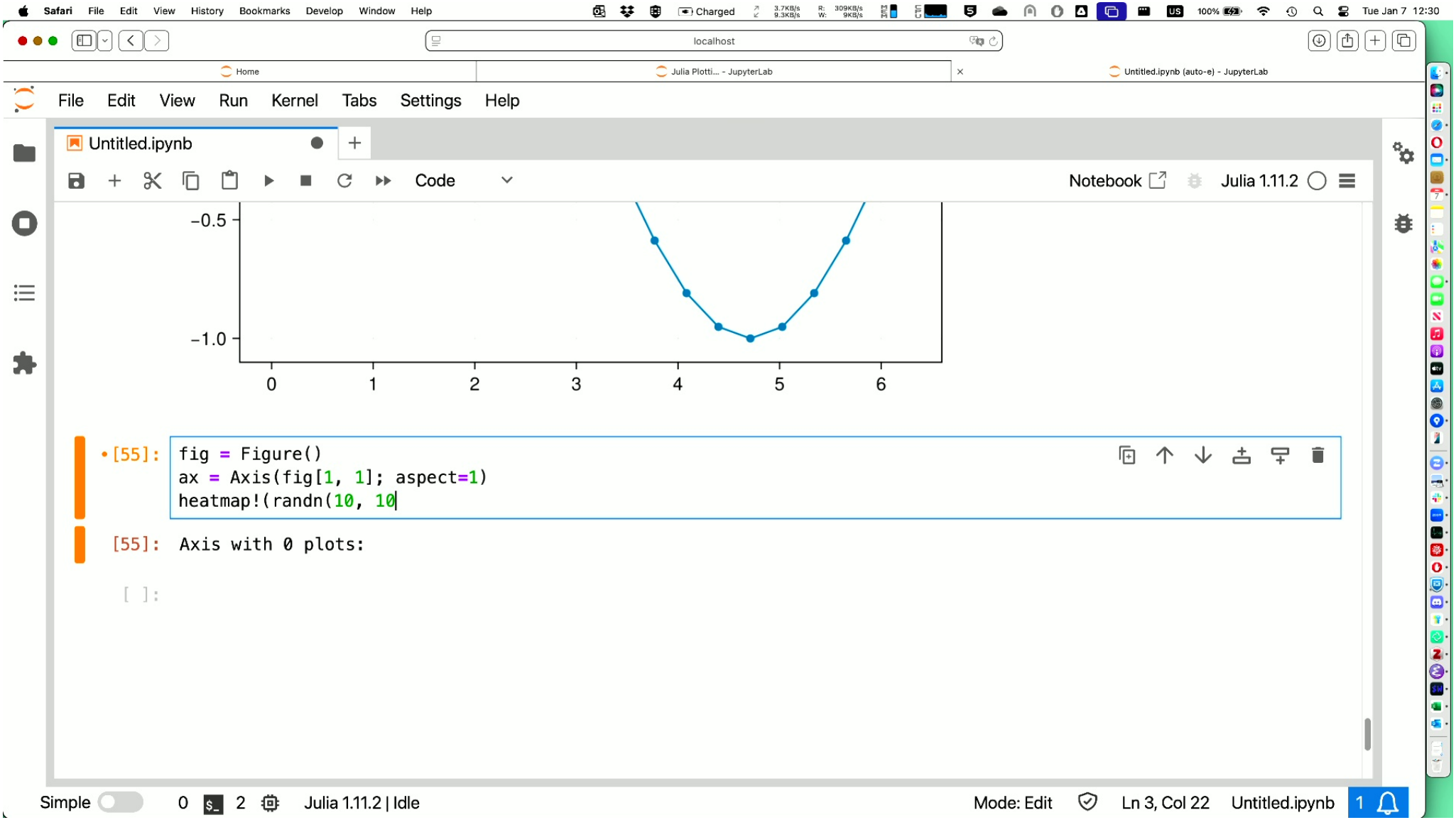
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Untitled.ipynb
Code
Notebook
Julia 1.11.2
−0.5
−1.0
0 1 2 3 4 5 6
[55]:
fig = Figure()
ax = Axis(fig[1, 1]; aspect=1)
heatmap!(randn(10, 10
[55]:
Axis with 0 plots:
[ ]:
Simple
0
2
Julia 1.11.2 | Idle
Mode: Edit
Ln 3, Col 22
Untitled.ipynb
1

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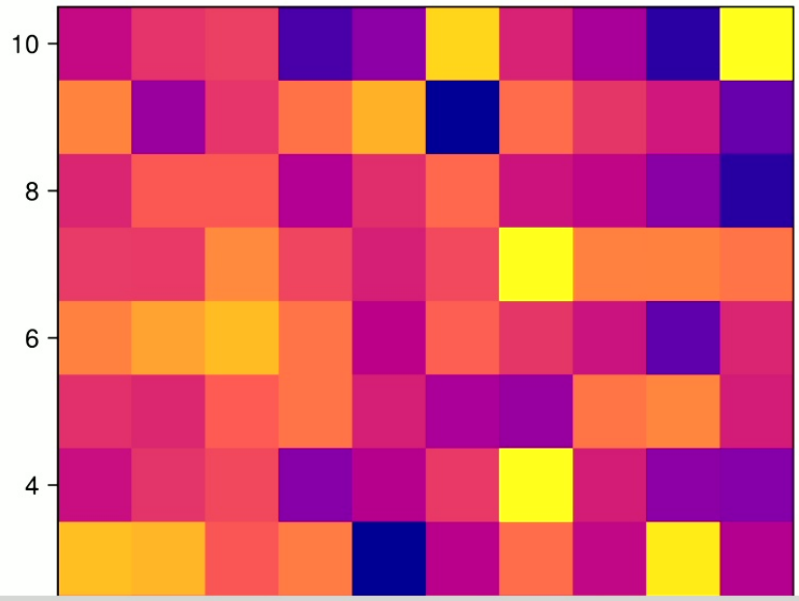
Code | Notebook | Julia 1.11.2

[56]:
```
fig = Figure()
ax = Axis(fig[1, 1]; aspect=1)
heatmap!(randn(10, 10); colormap=:plasma)
fig
```

[56]:



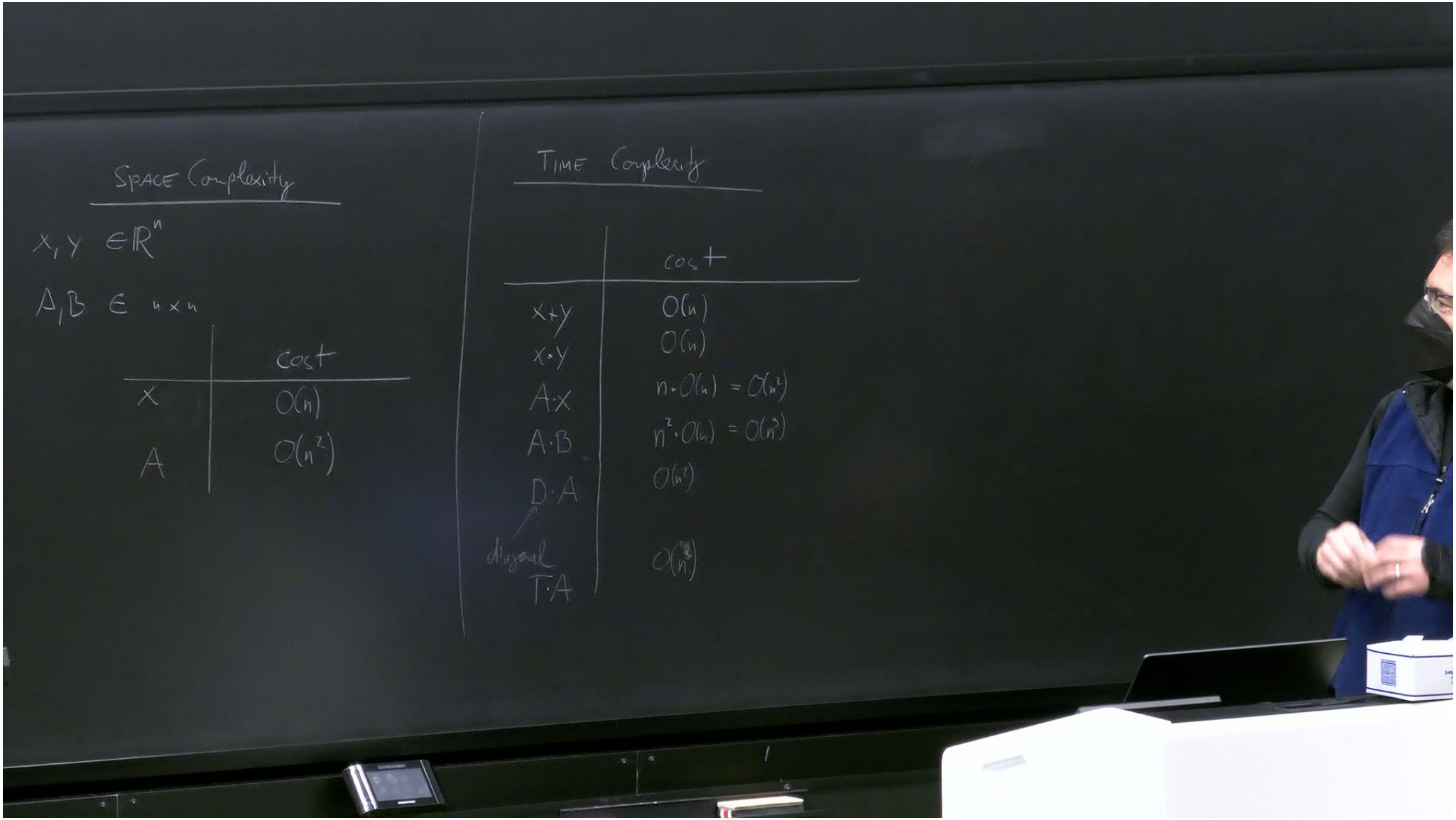
Simple 0 2 Julia 1.11.2 | Idle Mode: Command Ln 1, Col 1 Untitled.ipynb 1

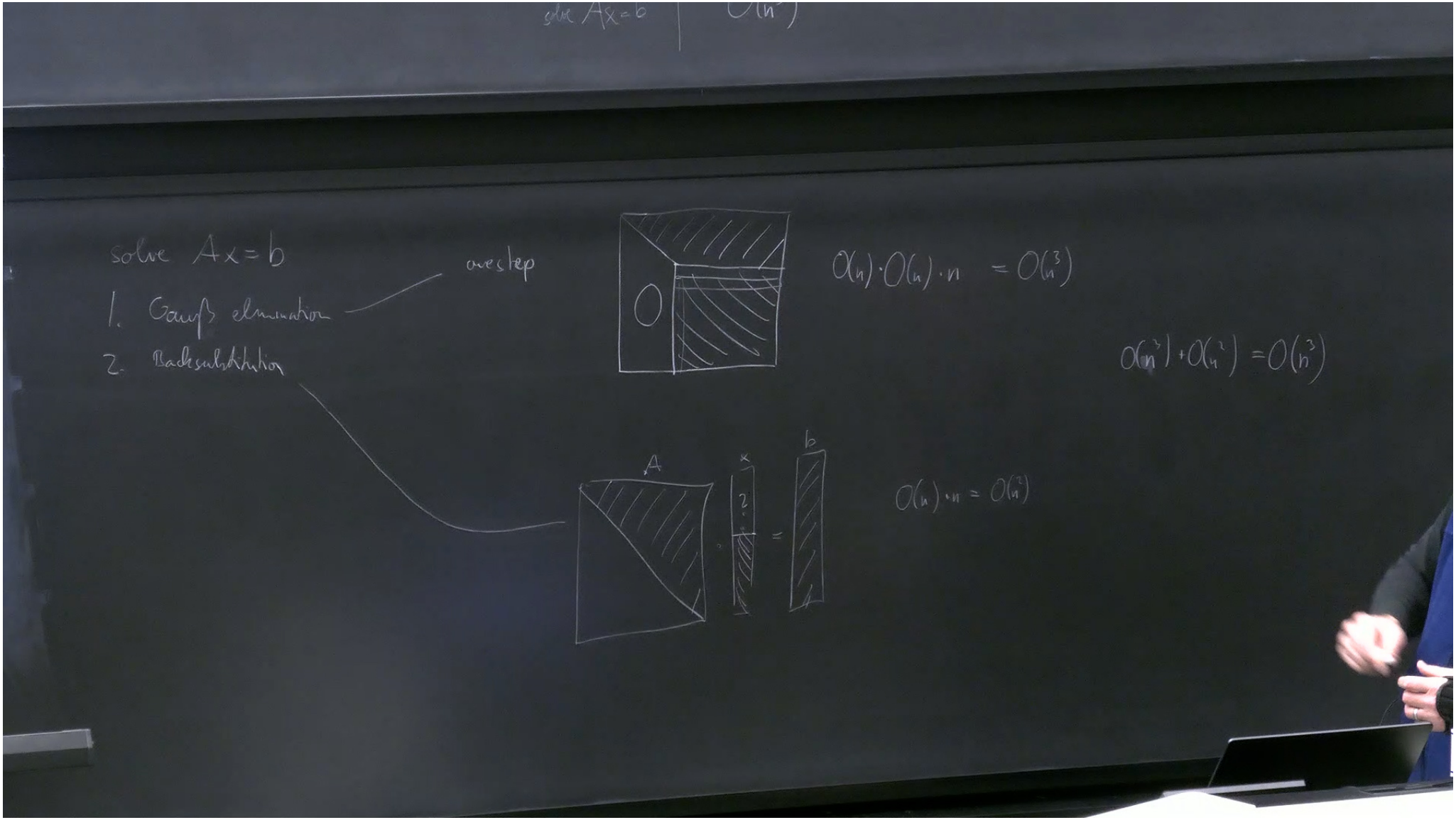
Big-O Notation

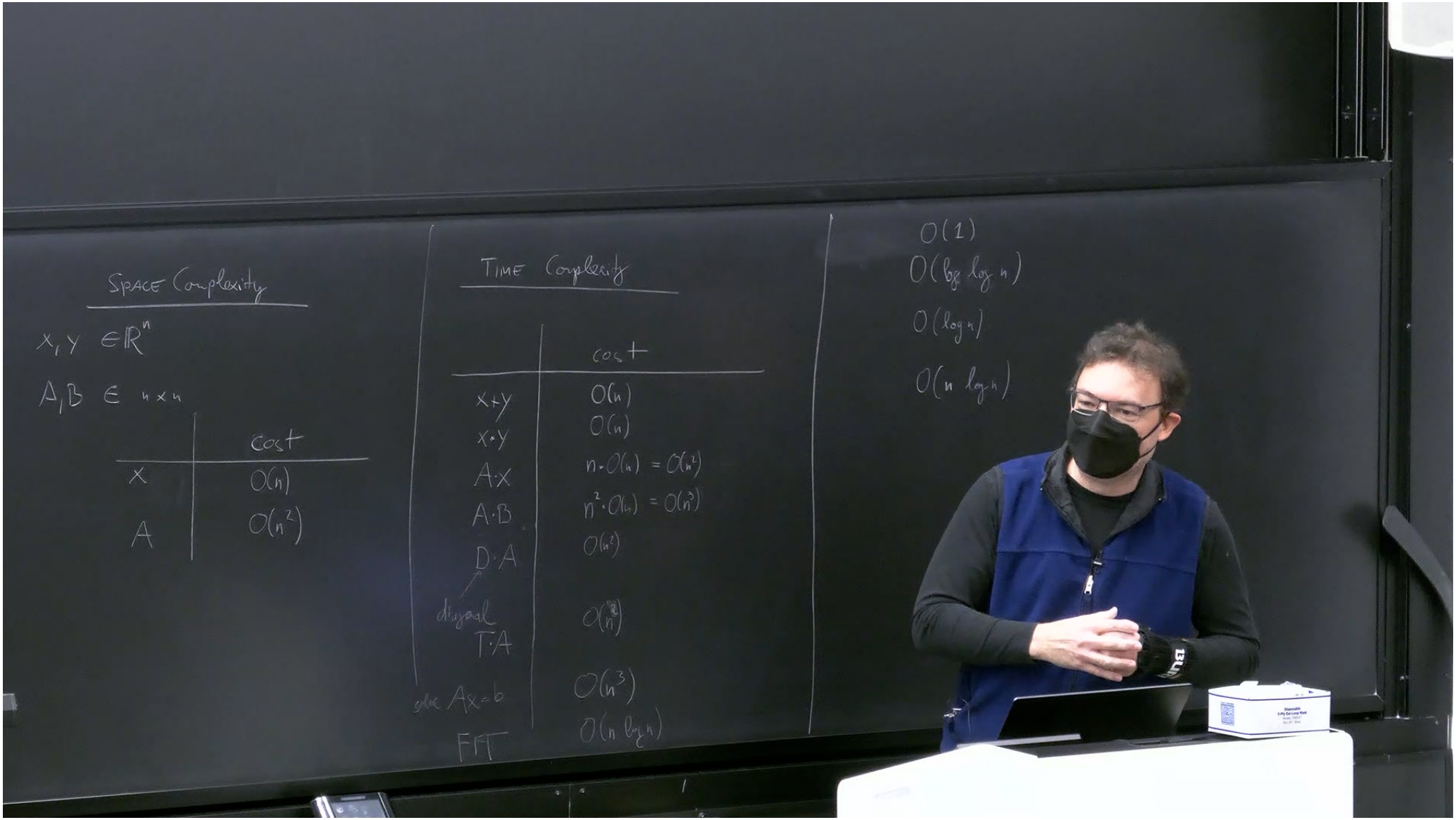
$$O(n^3) = \{ g(n) : g(n) \leq C \cdot n^3 \quad \text{for all } n > n_0 \}$$

$$O(n^3) = O(n^3 + n) = O(2n^3) \neq O(n^4)$$

$$O(n + 1000) = O(n)$$


Space Complexity
$x, y \in \mathbb{R}^n$
$A, B \in n \times n$
	cost
x	$O(n)$
A	$O(n^2)$
Time Complexity
	cost
$x+y$	$O(n)$
$x \cdot y$	$O(n)$
$A \cdot x$	$n \cdot O(n) = O(n^2)$
$A \cdot B$	$n^2 \cdot O(n) = O(n^3)$
$D \cdot A$	$O(n^2)$
$T \cdot A$ (diagonal)	$O(n^2)$


solve $Ax=b$
overstep
1. Gauß elimination
2. Backsubstitution
$O(n)\cdot O(n)\cdot n = O(n^3)$
$O(n^3)+O(n^2)=O(n^3)$
A
x
b
$O(n)\cdot n = O(n^2)$


Space Complexity
$x, y \in \mathbb{R}^n$
$A, B \in n \times n$
	cost
x	$O(n)$
A	$O(n^2)$
Time Complexity
	cost
$x+y$	$O(n)$
$x \cdot y$	$O(n)$
$A \cdot x$	$n \cdot O(n) = O(n^2)$
$A \cdot B$	$n^2 \cdot O(n) = O(n^3)$
$D \cdot A$	$O(n^2)$
diagonal $T \cdot A$	$O(n^2)$
solve $Ax=b$	$O(n^3)$
FFT	$O(n \log n)$
$O(1)$
$O(\log \log n)$
$O(\log n)$
$O(n \log n)$