

Title: Lecture - Beautiful Papers

Speakers: Pedro Vieira

Collection/Series: Beautiful Papers - October 7, 2024 - January 31, 2025

Subject: Other

Date: January 24, 2025 - 9:15 AM

URL: <https://pirsa.org/25010051>

1 + 1 dim

Entanglement Entropy $S_A(l)$ Volume Law

power law cor. fn $\left\{ \begin{array}{l} \text{CFT} \\ \sum = \infty \\ (m = \sum^i = 0) \end{array} \right\}$

QFT $\left\{ \begin{array}{l} \sum < \infty \\ (m > 0) \end{array} \right\}$ exp decay cor. fn

log l s in CFT

QFT $l \gg \sum$ $2 \log \sum$ s

Key object Φ_n twist operator $\Phi_n^{(1)} \sim \Phi_n^{(2)}$

put QFT on Riemann Surfaces

Area Law $\phi \rightarrow \phi^{(1)}$ which sheet we are

$\text{Tr} \rho^n$

CFT (3)

central charge of CFT $-\partial_n X^n /_{n=1} = -x \log x$

CFT @ temperature β^{-1}

$S_A = \frac{c}{3} \log \left(\frac{\beta}{\pi a} \sinh \frac{\pi(\tau-u)}{\beta} \right)$

lattice spacing

time periodic

space ∞

$\rho_A = \text{tr}_B \rho = e^{-\hat{P}_A}$

$-\text{tr}_A \rho_A \log \rho_A$ n times

$-\partial_n \text{tr}_A \rho_A^n$

1 + 1 dim

Entanglement Entropy $S_A(l)$ (Volume Law)

power law cor. fn $\left\{ \begin{array}{l} \text{CFT} \\ \sum = \infty \\ (m = \sum = 0) \end{array} \right\}$

QFT $\left\{ \begin{array}{l} \sum < \infty \\ (m > 0) \end{array} \right\}$ exp decay cor. fn

Key object Φ_n twist operator

put QFT on Riemann surface

Area Law

$\phi \rightarrow \phi^{(n)}$ which sheet we are

lattice

space ∞

$$\rho_A = \text{tr}_B \rho = e^{-\hat{P}H}$$

$$-\text{tr}_A \rho_A \log \rho_A$$

n times

$$-\partial_n \text{tr}_A \rho_A^n$$

$n=1$

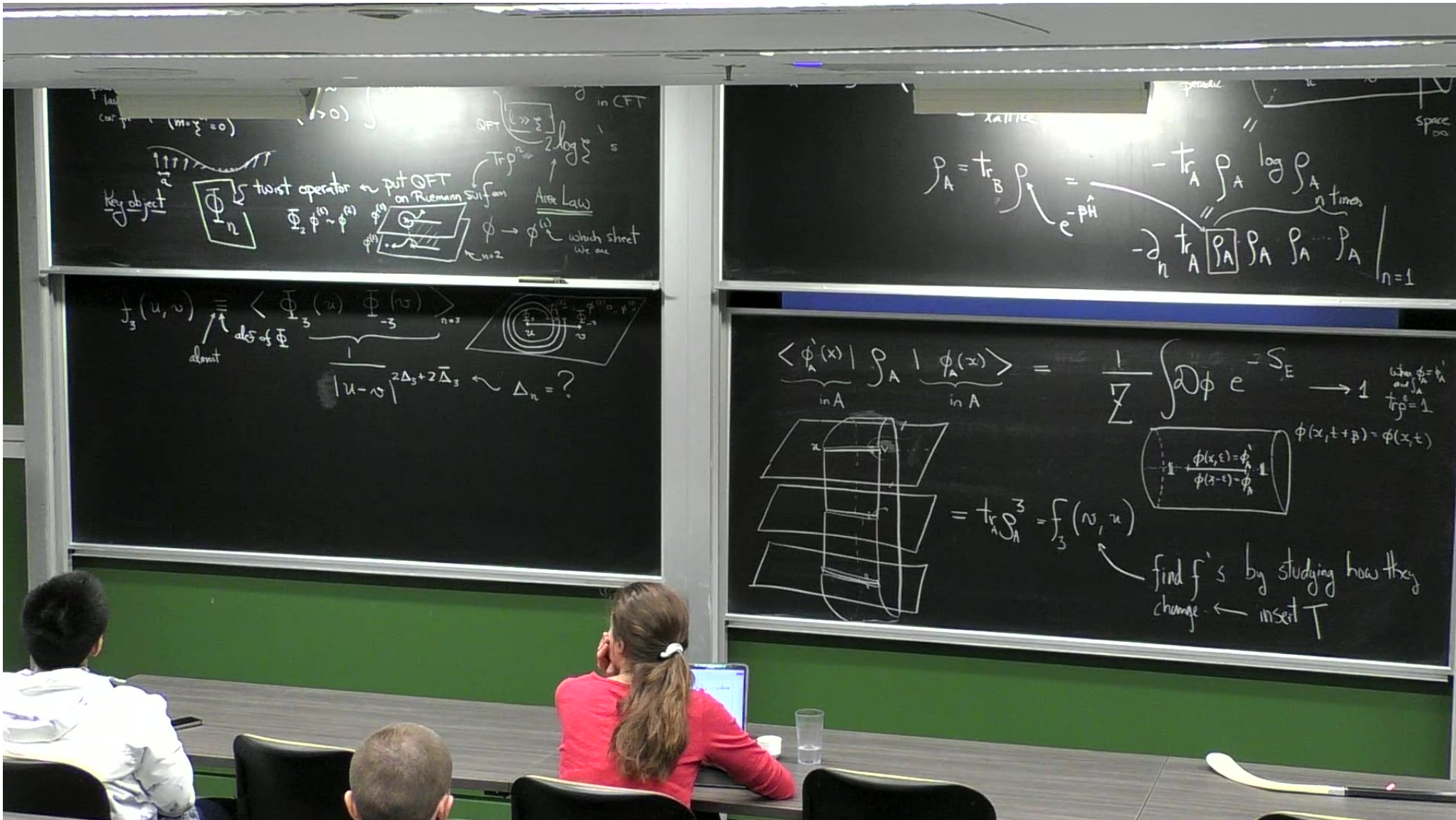
$$\langle \phi'_A(x) | \rho_A | \phi_A(x) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\phi e^{-S_E} \rightarrow 1$$

when $\phi'_A = \phi_A$ and $\mathcal{Z} = \text{tr} \rho = 1$

$\phi(x, t+\beta) = \phi(x, t)$

$$= \text{tr}_A \rho_A^3 = f_3(n, u)$$

find f's by studying how they change ← insert T



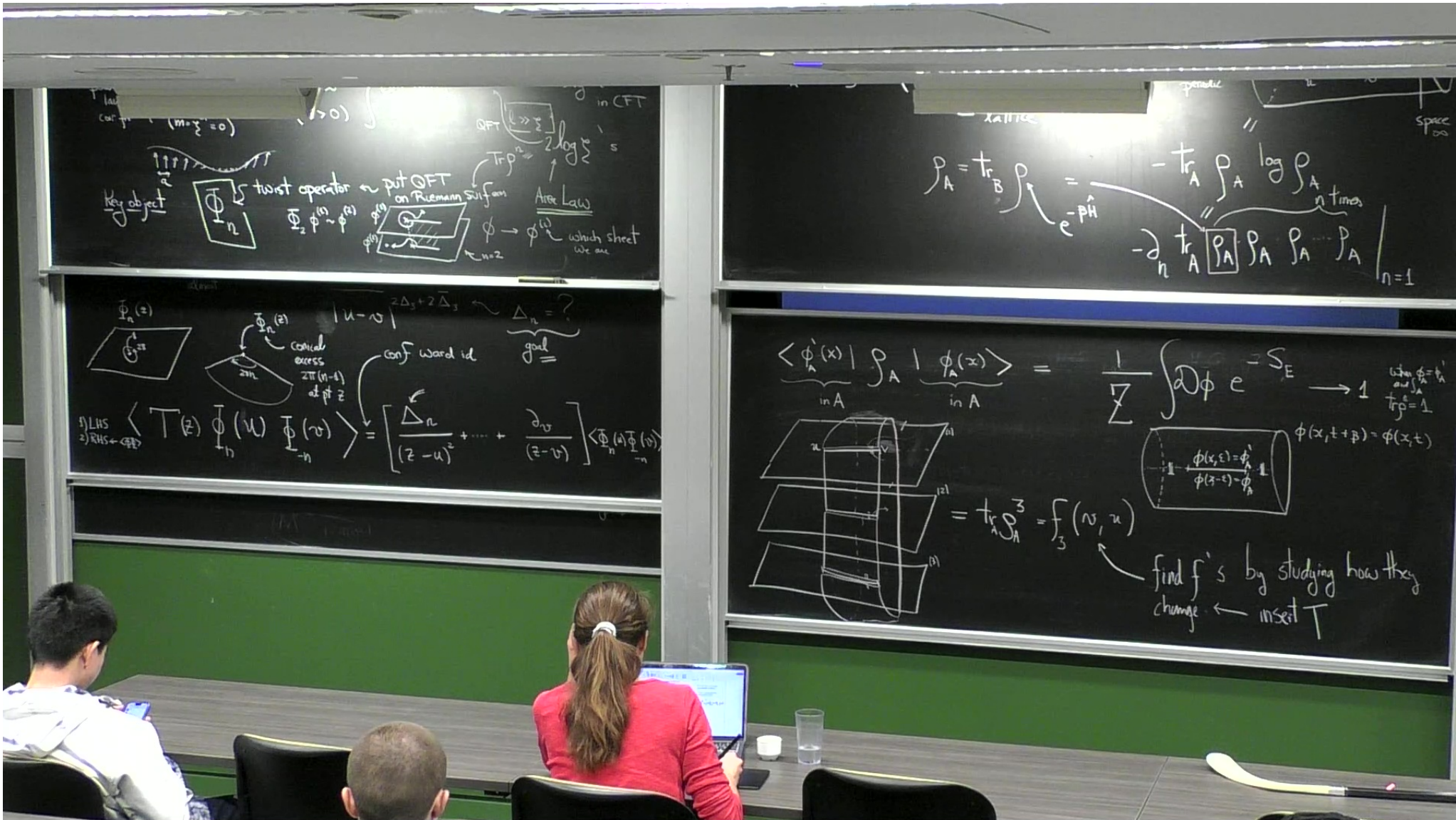
$\text{key object } \Phi_n$ twist operator \sim put QFT on Riemann surface
 $\Phi_2 \phi^{(1)} \sim \phi^{(2)}$
 $\text{Tr } \rho^n = 2 \log \dots$
 Affine Law
 $\phi \rightarrow \phi^{(n)}$ which sheet we are on
 $n=2$

$\frac{1}{3}(u, v) \equiv \langle \Phi_3(u) \Phi_3(v) \rangle_{n=3}$
 element
 $\frac{1}{|u-v|^{2\Delta_3 + 2\bar{\Delta}_3}} \sim \Delta_n = ?$

$\rho_A = \text{tr}_B \rho = e^{-\rho \hat{H}}$
 $-\text{tr}_A \rho_A \log \rho_A$ n times
 $-\partial_n \text{tr}_A \rho_A$
 $\rho_A \rho_A \rho_A \dots \rho_A$
 $n=1$

$\langle \phi_A^i(x) | \rho_A | \phi_A^j(x) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\phi e^{-S_E} \rightarrow 1$
 when $\phi_A^i = \phi_A^j$
 $\text{tr } \rho^0 = 1$
 $\phi(x, t+\beta) = \phi(x, t)$

 $= \text{tr}_A \rho_A^3 = f_3(n, u)$
 find f's by studying how they change \leftarrow insert T

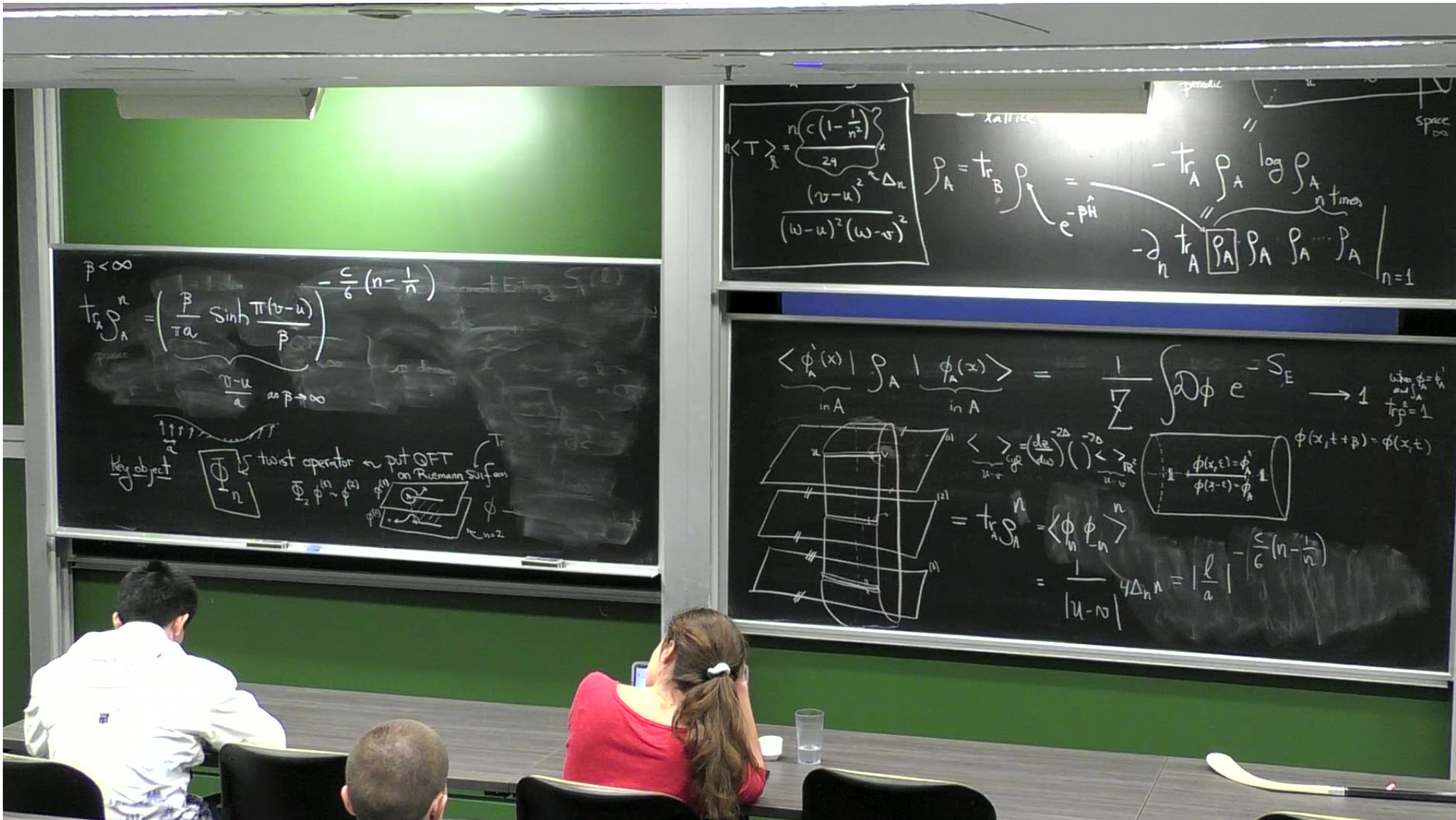


Key object Φ_n twist operator \sim put QFT on Riemann surface
 $\Phi_2 \phi^{(1)} \sim \phi^{(2)}$
 $\phi \rightarrow \phi^{(n)}$ which sheet we are
 $n=2$
 OFT $\sum_{\sigma} 2 \log \dots$
 $\text{Tr} \rho^n = \dots$
 Area Law
 in CFT

$\Phi_n(z)$
 $\Phi_n(z)$
 $|u-v|^{2\Delta_n + 2\bar{\Delta}_n} \sim \Delta_n = ?$
 conformal excess $2\pi(n-1)$ at pt z
 conf Ward id \Rightarrow good
 $\langle T(z) \Phi_n(w) \Phi_n(v) \rangle = \left[\frac{\Delta_n}{(z-u)^2} + \dots + \frac{\partial_v}{(z-v)} \right] \langle \Phi_n(u) \Phi_n(v) \rangle$
 1) LHS
 2) RHS \rightarrow QFT

$\rho_A = \text{tr}_B \rho = e^{-\rho \hat{H}}$
 $-\text{tr}_A \rho_A \log \rho_A$
 $-\partial_n \text{tr}_A \rho_A \dots \rho_A$
 n times
 $n=1$

$\langle \phi'_A(x) | \rho_A | \phi_A(x) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\phi e^{-S_E} \rightarrow 1$
 $\phi(x, t+\beta) = \phi(x, t)$
 $\phi(x, \epsilon) = \phi'_A$
 $\phi(\beta-\epsilon) = \phi_A$
 $\text{tr}_A \rho_A^3 = f_3(n, u)$
 find f 's by studying how they change \leftarrow insert T



$\beta < \infty$

$$\text{tr}_A \rho_A^n = \left(\frac{\beta}{\tau a} \sinh \frac{\pi(v-u)}{\beta} \right)^{-\frac{c}{6} (n - \frac{1}{n})}$$

$\frac{v-u}{a}$ as $\beta \rightarrow \infty$

Key object: Φ_n twist operator

put OFT on Riemann Surface

$\Phi_2 \phi^{(1)} \sim \phi^{(2)}$

ϕ

$n=2$

$\langle T \rangle = \frac{n \left\{ c \left(1 - \frac{1}{n^2} \right) \right\}}{24} \frac{\Delta_n}{(v-u)^2 (w-u)^2}$

matrix

$\rho_A = \text{tr}_B \rho = e^{-\beta H}$

$-\text{tr}_A \rho_A \log \rho_A$ n times

$-\partial_n \text{tr}_A \rho_A$

space ∞

$\langle \phi'_A(x) | \rho_A | \phi_A(x) \rangle = \frac{1}{\Sigma} \int \mathcal{D}\phi e^{-S_E} \rightarrow 1$ when $\phi'_A = \phi_A$ and $\text{tr} \rho_A = 1$

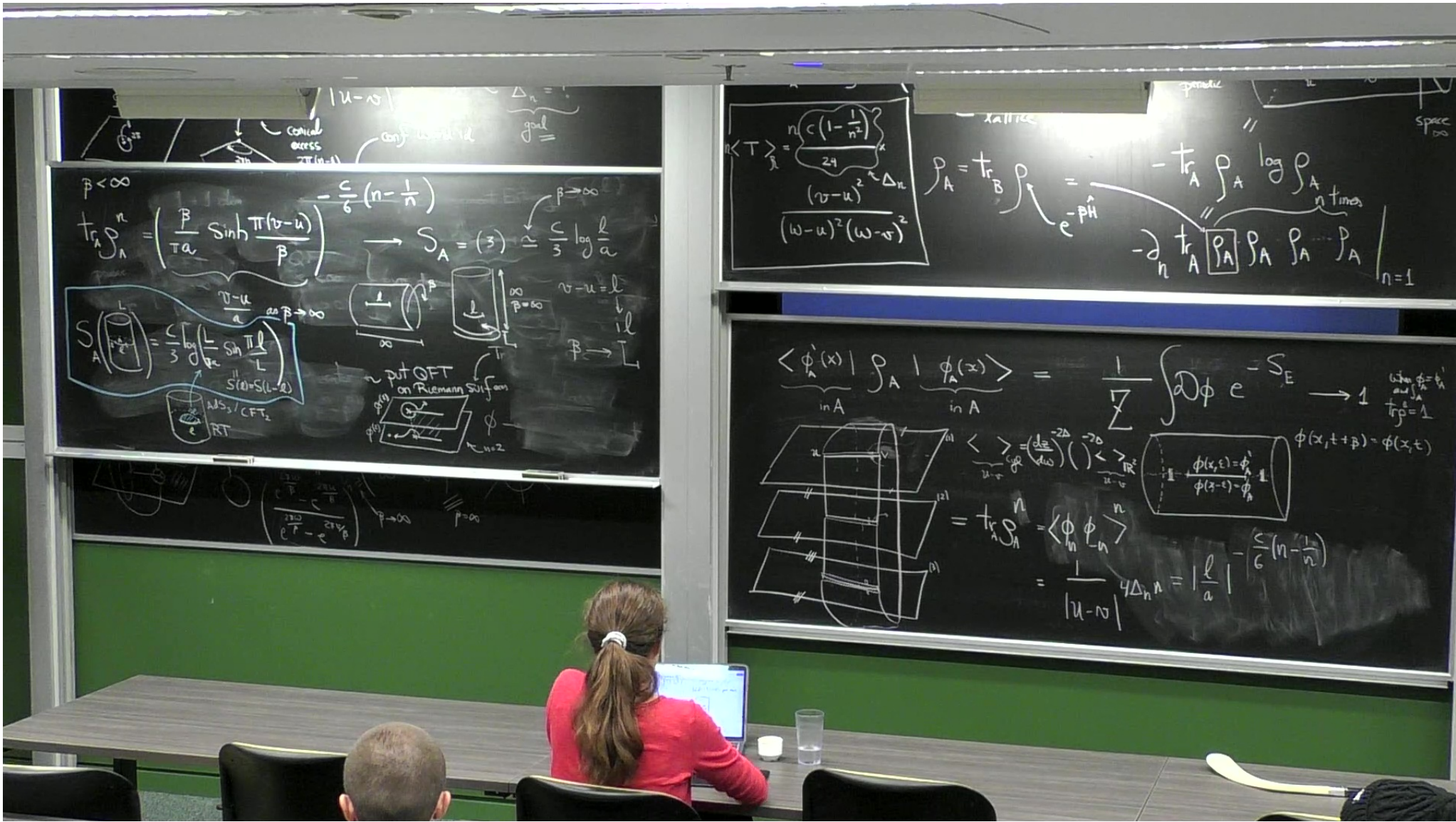
$\phi(x, t + \beta) = \phi(x, t)$

$\langle \phi'_A(x) | \rho_A | \phi_A(x) \rangle = \left(\frac{d\phi}{d\omega} \right) \left(\frac{d\phi}{d\omega} \right)$

$\langle \phi'_A(x) | \rho_A | \phi_A(x) \rangle = \text{tr}_A \rho_A^n = \langle \phi_n \phi_{-n} \rangle$

$= \frac{1}{|u-v|} 4\Delta_n^n = \left| \frac{l}{a} \right|^{-\frac{c}{6} (n - \frac{1}{n})}$

Diagram: A stack of three rectangular sheets representing a Riemann surface. The top sheet is labeled with u and v . A vertical line is drawn through the sheets, representing a twist operator. A cylinder is drawn around the sheets, with $\phi(x, \epsilon) = \phi'_A$ and $\phi(x, -\epsilon) = \phi_A$ written on it.



$\beta < \infty$
 $\text{tr}_A \rho_A^n = \left(\frac{\beta}{T a} \sinh \frac{\pi(v-u)}{\beta} \right)^n \rightarrow S_A = -\frac{c}{6} \left(n - \frac{1}{n} \right) \xrightarrow{\beta \rightarrow \infty} \frac{c}{3} \log \frac{l}{a}$

$S_A = \frac{c}{3} \log \frac{L}{a} = \frac{c}{3} \log \frac{L}{a} \frac{\sinh \frac{\pi l}{L}}{\sinh \frac{\pi l}{L}}$
 $\frac{v-u}{a} \xrightarrow{\beta \rightarrow \infty} \frac{2\pi l}{L}$

Diagrams showing a cylinder of length L and radius a , and a rectangular system with height l and width a .

$\langle T \rangle = \frac{n \left(1 - \frac{1}{n^2} \right)^n}{24 (v-u)^2 \Delta_n}$

$\rho_A = \text{tr}_B \rho = e^{-\beta H}$

$-\text{tr}_A \rho_A \log \rho_A$ (n times)
 $-\partial_n \text{tr}_A \rho_A^n$

Diagrams showing a lattice and a space ∞ .

$\langle \phi'_n(x) | \rho_A | \phi_n(x) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\phi e^{-S_E} \rightarrow 1$

$\langle \phi'_n(x) | \rho_A | \phi_n(x) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\phi e^{-S_E} \phi(x, t+\beta) = \phi(x, t)$

$\langle \phi'_n(x) | \rho_A | \phi_n(x) \rangle = \text{tr}_A \rho_A^n = \langle \phi_n | \rho_A^n | \phi_n \rangle$

$= \frac{1}{|u-v|} 4\Delta_n^n = \left| \frac{l}{a} \right|^{-\frac{c}{6} \left(n - \frac{1}{n} \right)}$

Diagrams showing a 3D lattice structure and a cylinder with wave functions $\phi(x, t)$ and $\phi(x, t+\beta)$.

$$\langle T \rangle = \frac{n c (1 - \frac{1}{n^2})}{24} \frac{\Delta_n}{(v-u)^2 (w-v)^2}$$

$$\rho_A = \text{tr}_B \rho = e^{-\text{tr}_A \rho_A \log \rho_A} = \frac{1}{Z_n} \text{tr}_A \rho_A \rho_A \rho_A \dots \rho_A \Big|_{n=1}$$

$$\langle \phi_A^i(x) | \rho_A | \phi_A^i(x) \rangle = \frac{1}{Z_n} \int d\phi e^{-S_E} \rightarrow 1$$

$\phi(x, t+p) = \phi(x, t)$
 $\phi(x, \epsilon) = \phi_A$
 $\phi(x, -\epsilon) = \phi_A$
 $\langle \phi_n | \phi_n \rangle = \frac{1}{|u-v|} 4\Delta_n = \frac{1}{a} \left(\frac{c}{6} (n - \frac{1}{n}) \right)$

$m > 0$

response due to changing only scale of ren. th.

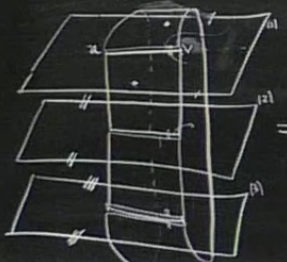
$S_n = \frac{A}{\text{Area}} \times S$
 $S = \partial_n \sum_p |_{n=1}$
 $1] \quad m \frac{\partial}{\partial m} \left(\log Z_n - n \log Z_1 \right) = \infty \xrightarrow{m \rightarrow 1} Z_n \xrightarrow{\partial/\partial n} S$



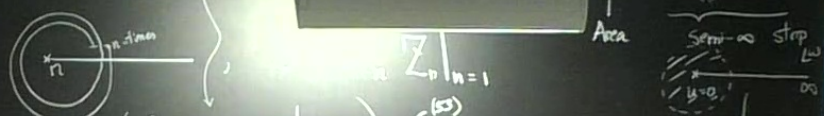
$$\langle T \rangle = \frac{n c (1 - \frac{1}{n^2})^2}{24 (v-u)^2 \Delta_n} \times \frac{(\omega-u)^2 (\omega-v)^2}{\dots}$$

$$\rho_A = \text{tr}_B \rho = e^{-\beta \hat{H}} = -\text{tr}_A \rho_A \log \rho_A \quad \text{[n times]}$$

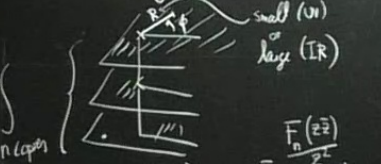
$$-\partial_n \text{tr}_A \rho_A \rho_A \rho_A \rho_A \big|_{n=1}$$

$$\langle \phi_A^i(x) | \rho_A | \phi_A^i(x) \rangle = \frac{1}{Z} \int d\phi e^{-S_E} \rightarrow 1$$


$$\langle \phi_n | \phi_n \rangle = \frac{1}{|u-v|} 4\Delta_n^n = \left| \frac{a}{6} \left(n - \frac{1}{n} \right) \right|$$

$$Z_n = \left(\text{Area} \right) \sum_{n=1}^{\infty} \dots$$


$$m \frac{\partial}{\partial m} \left(\log Z_n - n \log Z_1 \right) \xrightarrow{\text{int}} Z_n \xrightarrow{\partial/\partial n} S$$

$$\dots = \int dR \left(\langle \ominus \rangle_n - \langle \ominus \rangle_1 \right) \neq 0$$


$$\frac{\partial}{\partial R} \left(F + \frac{1}{4} G \right) \left(T_{zz} \right)$$

$$\text{total } \partial/\partial R \leftarrow \text{cons laws of } T, \bar{T}, \Theta$$

$$\bar{\partial} T + \frac{1}{4} \partial \Theta = 0 \quad 4 T_{z\bar{z}}$$

$$G = z\bar{z} \left(F + \frac{1}{4} G \right)$$

$$\langle T \rangle = \frac{n c (1 - \frac{1}{n^2})^2}{24 (v-u)^2 \Delta_n} \frac{1}{(w-u)^2 (w-v)^2}$$

$$\rho_A = \text{tr}_B \rho = e^{-\hat{P}H} = -\text{tr}_A \rho_A \log \rho_A \quad n \text{ times}$$

$$-\partial_n \text{tr}_A \rho_A \rho_A \rho_A \rho_A \big|_{n=1}$$

$$\langle \phi'_A(x) | \rho_A | \phi'_A(x) \rangle = \frac{1}{\sum \int d\phi e^{-S_E}} \rightarrow 1$$

with $\phi'_A = \phi_A$
 $\text{tr} \rho = 1$

$$\langle \phi(x, t+\beta) | \phi(x, t) \rangle = \text{tr}_A \rho_A = \langle \phi_n | \phi_n \rangle$$

$$= \frac{1}{|u-v|} 4\Delta_n = \frac{1}{a} \left(1 - \frac{c}{6} (n - \frac{1}{n}) \right)$$

$m > 0$

response due to changing only scale of ren. th.

$$Z = (ma)^{\frac{c}{6}(n-1/n)}$$

A end-points

$$S_A = A \times S$$

Area

$$S = \partial_n \sum_{n=1}^{\infty} \frac{\pi n c}{6} \left(1 - \frac{1}{n^2} \right)$$

Semi-infinite strip

Plan $m \frac{\partial}{\partial m} (\log Z_n - n \log Z_1) \xrightarrow{m \rightarrow \infty} Z_n \xrightarrow{\partial/\partial n} S$