Title: Lecture - Beautiful Papers

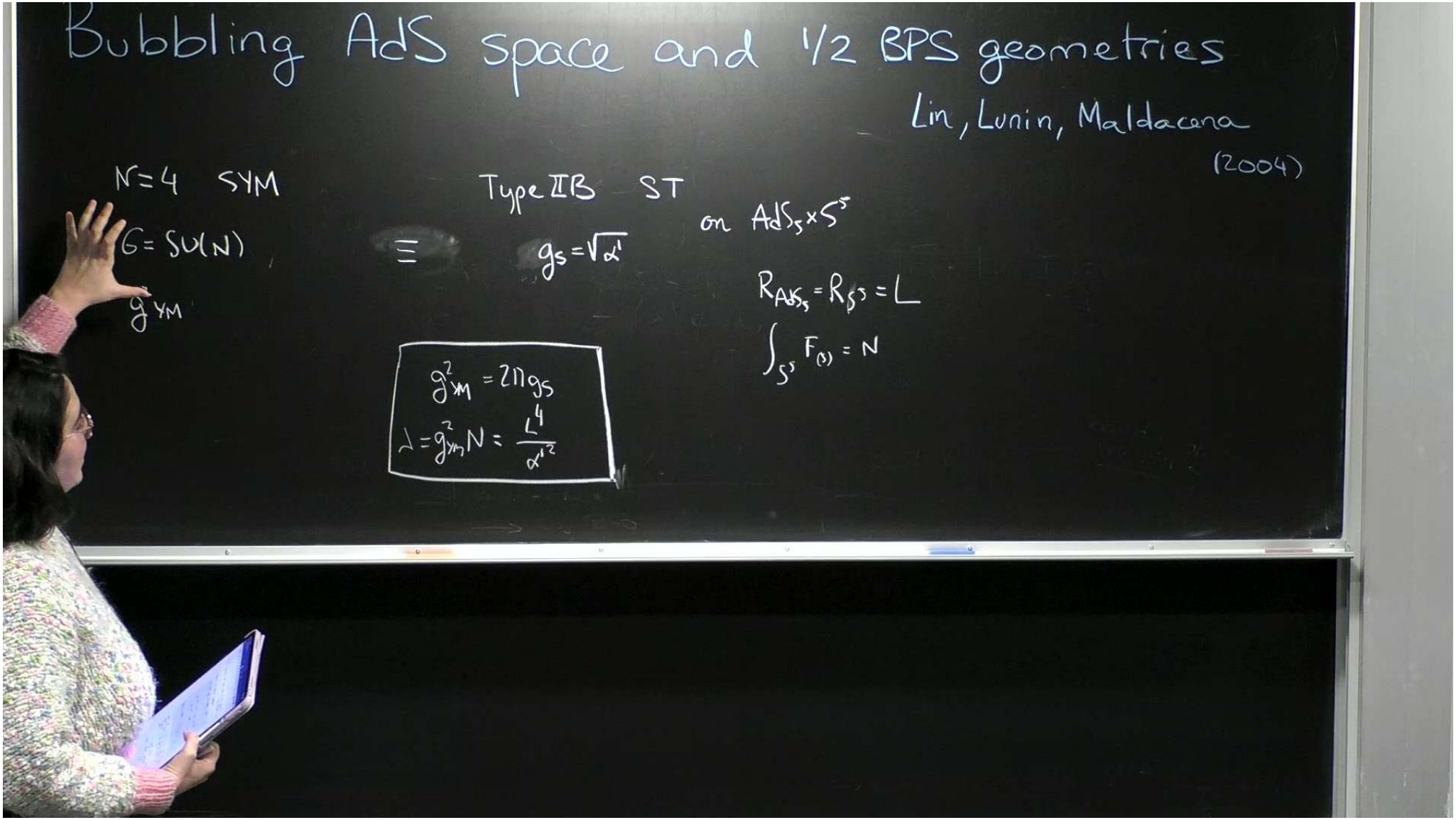
Speakers: Pedro Vieira

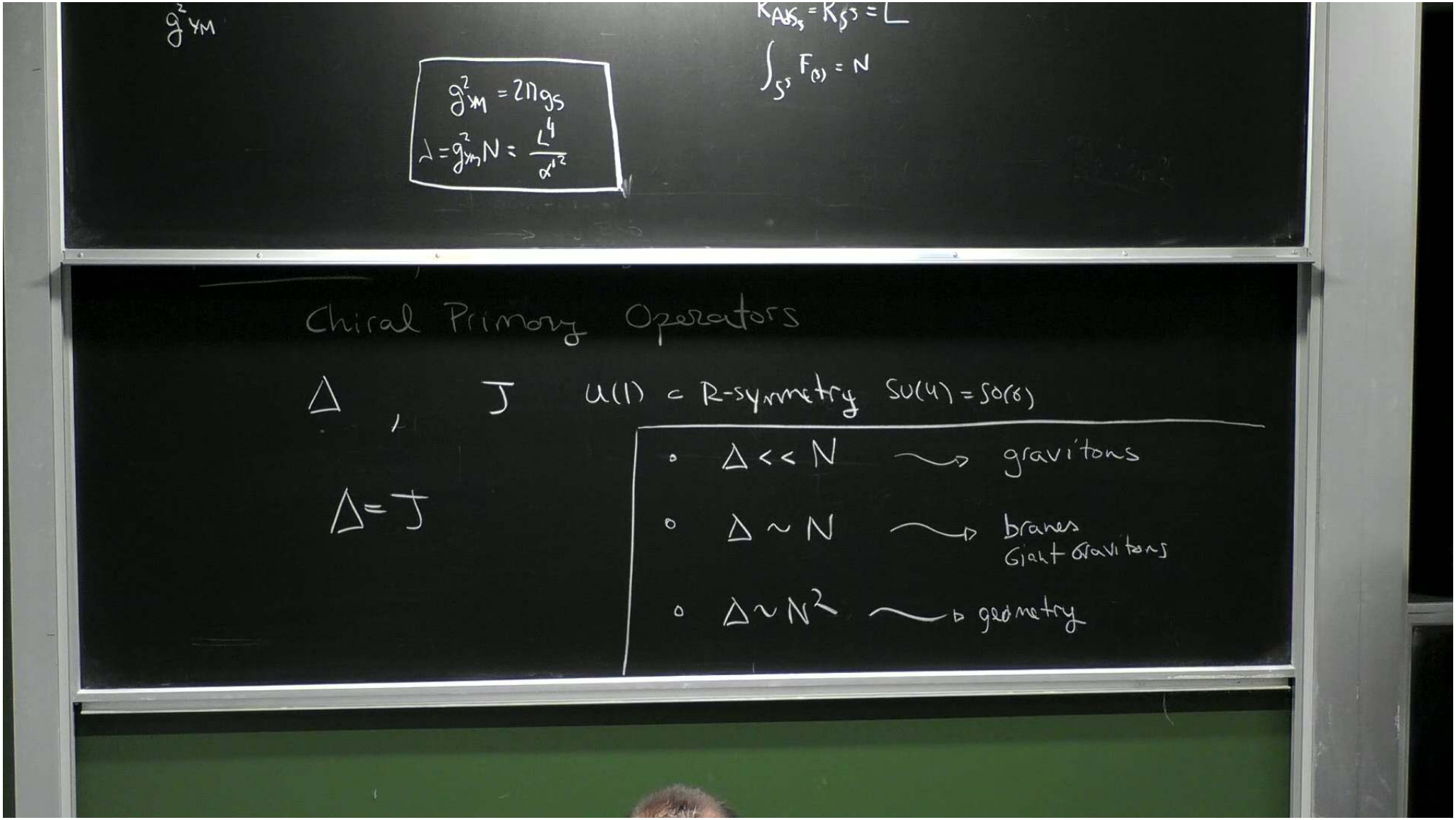
Collection/Series: Beautiful Papers - October 7, 2024 - January 31, 2025

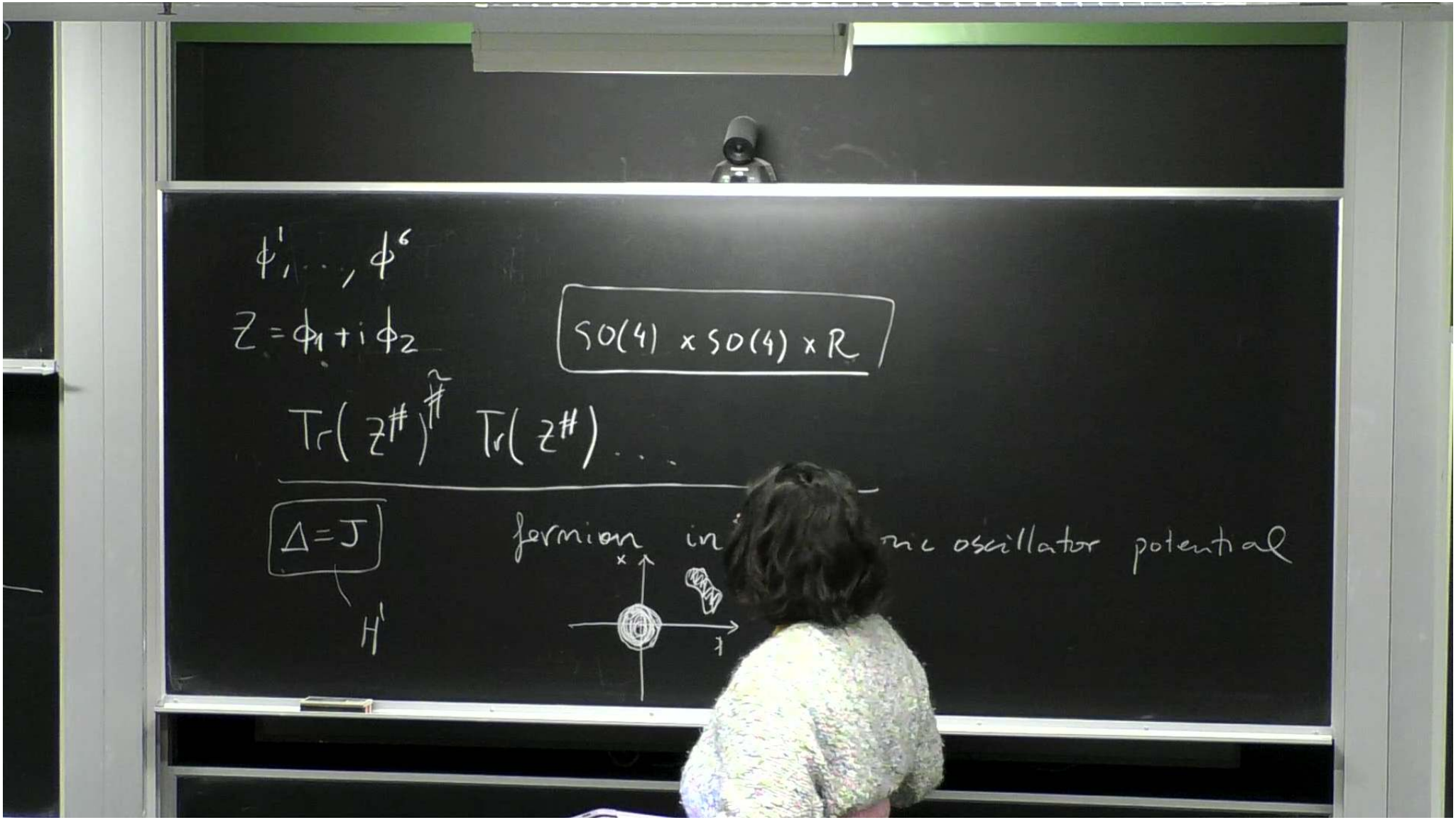
Subject: Other

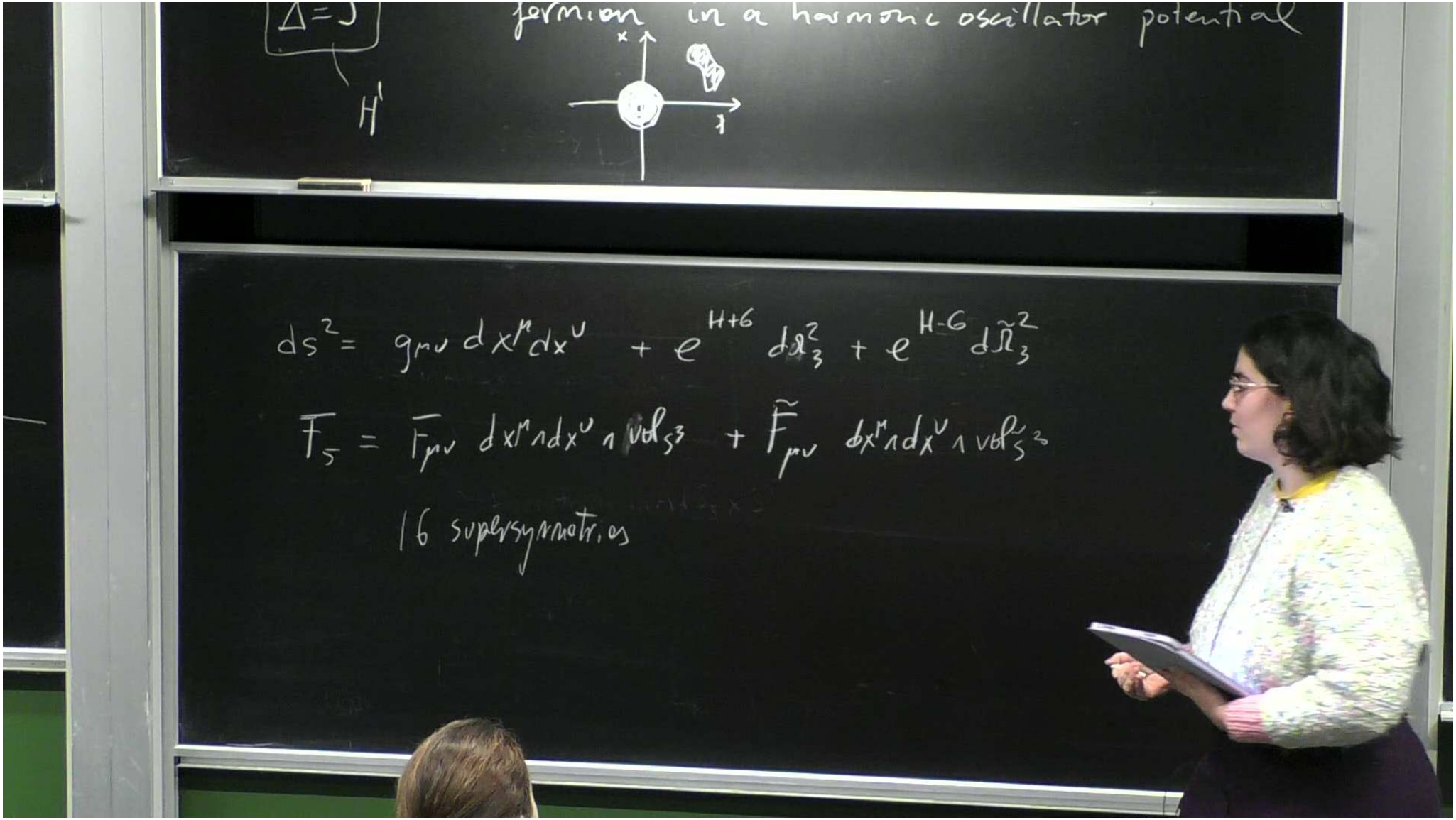
Date: January 20, 2025 - 9:15 AM

URL: https://pirsa.org/25010050


Bubbling AdS space and 1/2 BPS geometries
Lin, Lunin, Maldacena
(2004)
N=4 SYM
G=SU(N)
g_{YM}
≡
Type IIB ST
$g_s = \sqrt{\alpha'}$
on $AdS_5 \times S^5$
$R_{AdS_5} = R_{S^5} = L$
$\int_{S^5} F_{(5)} = N$
$g_{YM}^2 = 2\pi g_s$
$\lambda = g_{YM}^2 N = \frac{L^4}{\alpha'^2}$


g^2_{YM}
$R_{AdS_5} = R_{S^5} = L$
$g^2_{YM} = 2\pi g_s$
$\lambda = g^2_{YM} N = \frac{L^4}{\alpha'^2}$
$\int_{S^5} F_{(5)} = N$
Chiral Primary Operators
Δ , J $U(1) \subset$ R-symmetry $SU(4) = SO(6)$
$\Delta = J$
$\Delta << N$ ⇝ gravitons
$\Delta \sim N$ ⇝ branes Giant gravitons
$\Delta \sim N^2$ ⇝ geometry

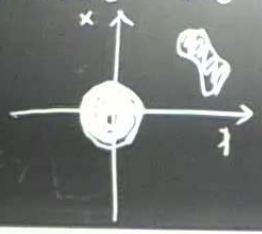

$\phi^1, \ldots, \phi^6$
$Z = \phi_1 + i\phi_2$
$SO(4) \times SO(4) \times R$
$Tr(Z^\#)^{\tilde{\#}}\ Tr(Z^\#) \ldots$
$\Delta = J$
H'
fermion in
oscillator potential
x
y


Δ=J
H'
fermion in a harmonic oscillator potential
x
1
$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{H+6} d\Omega_3^2 + e^{H-6} d\tilde{\Omega}_3^2$
$F_5 = \bar{F}_{\mu\nu} dx^\mu \wedge dx^\nu \wedge vol_{S^3} + \tilde{F}_{\mu\nu} dx^\mu \wedge dx^\nu \wedge vol_{\tilde{S}^3}$
16 supersymmetries

$\Delta = 3$

H^1

fermion in a harmonic oscillator potential



$$F_5 = \underbrace{F_{\mu\nu}}\, dx^\mu \wedge dx^\nu \wedge vol_{S^3} + \underbrace{F_{\mu\nu}}\, dx^\mu \wedge dx^\nu \wedge vol_{S^2}$$

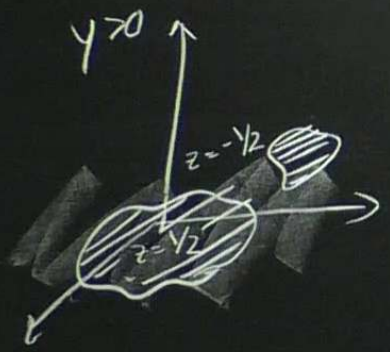
16 supersymmetries

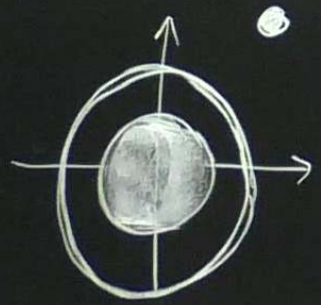
$$ds_5^2 = -\frac{2y}{\sqrt{4z^2-1}}(dt+V)^2 + \frac{\sqrt{4z^2-1}}{2y} ds_X^2 + y\sqrt{\frac{1+2z}{1-2z}}\, d\underbrace{\Omega_3^2} + y\sqrt{\frac{1-2z}{1+2z}}\, d\underbrace{\tilde{\Omega}_3^2}$$

$ds^2_X = dy^2 + dx_i^2$

$z : X \to \mathbb{R}$

$dV = \frac{1}{y} dz$





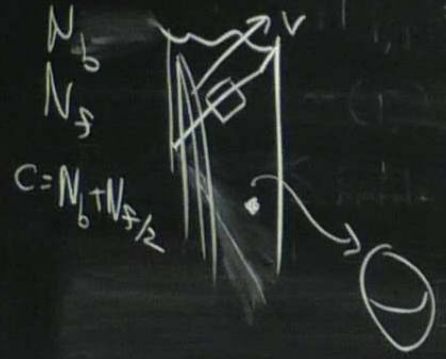
$\int_{S^5} F_{(5)} = \#$ = area

$\lambda = g_{YM}^2 N = \frac{L^2}{\alpha'^2}$

1905.08255

Slowly evaporating BH

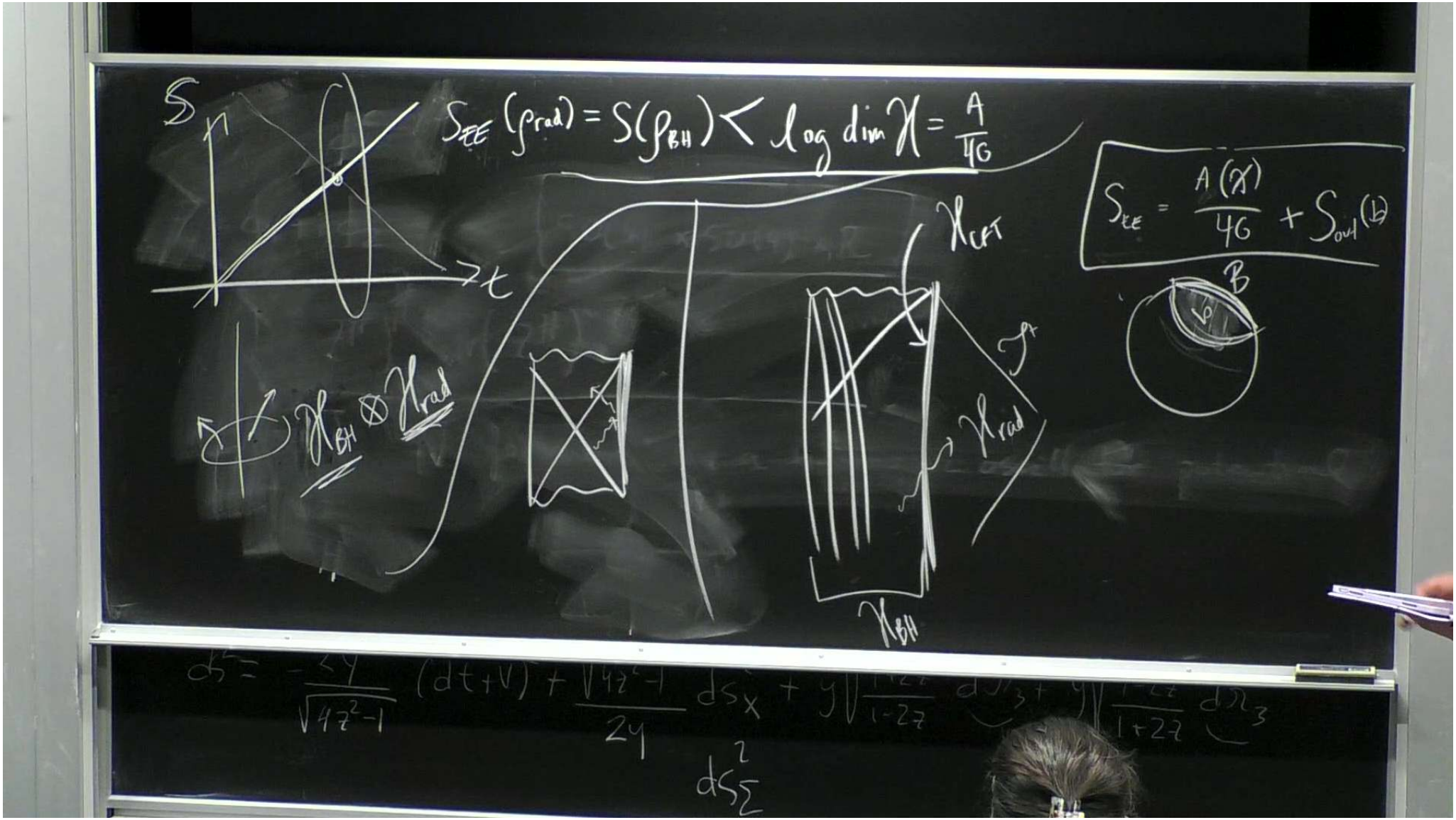
$$ds^2 = -f(r)dv^2 + 2dvdr + r^2 d\Omega^2$$

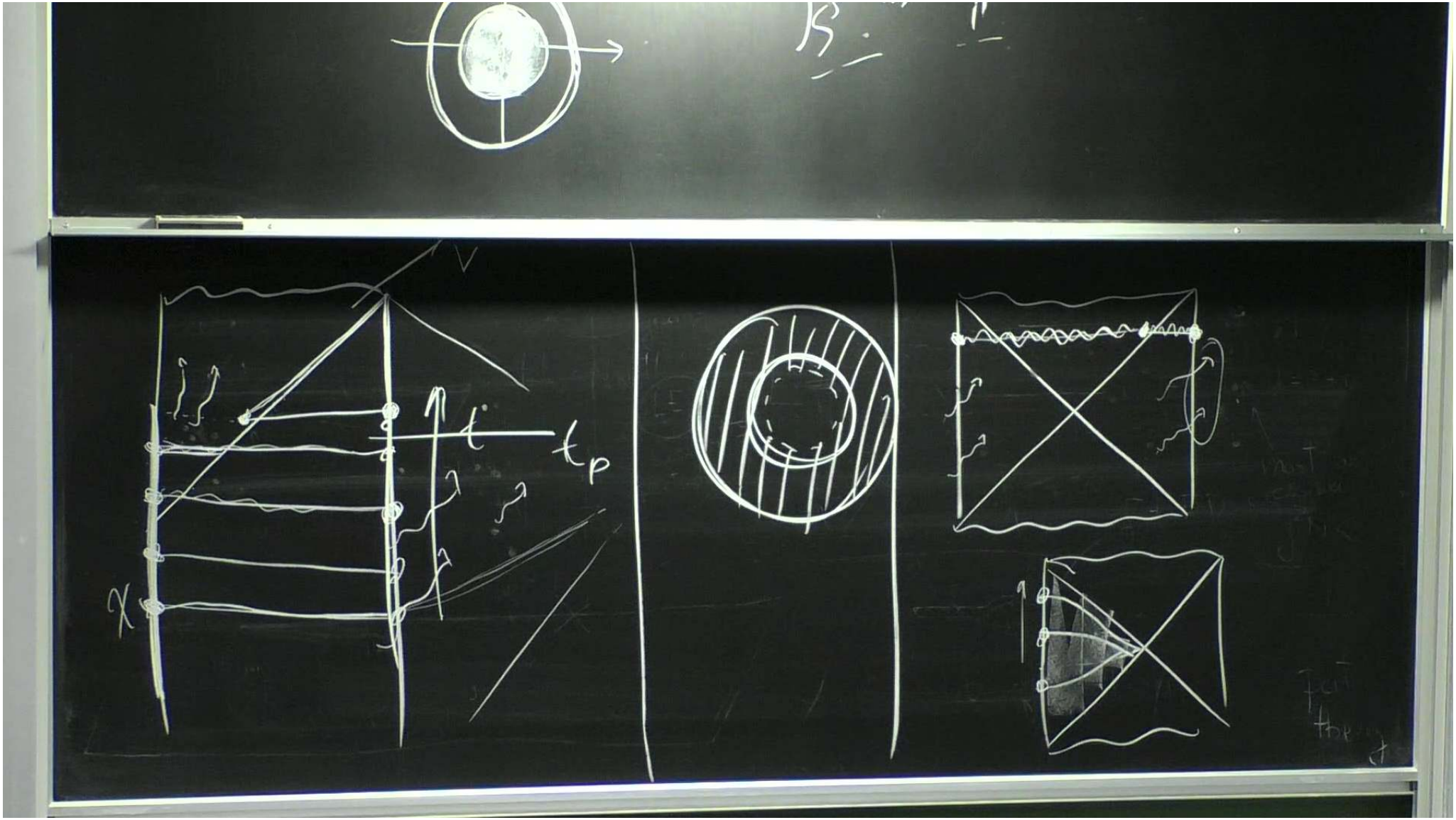


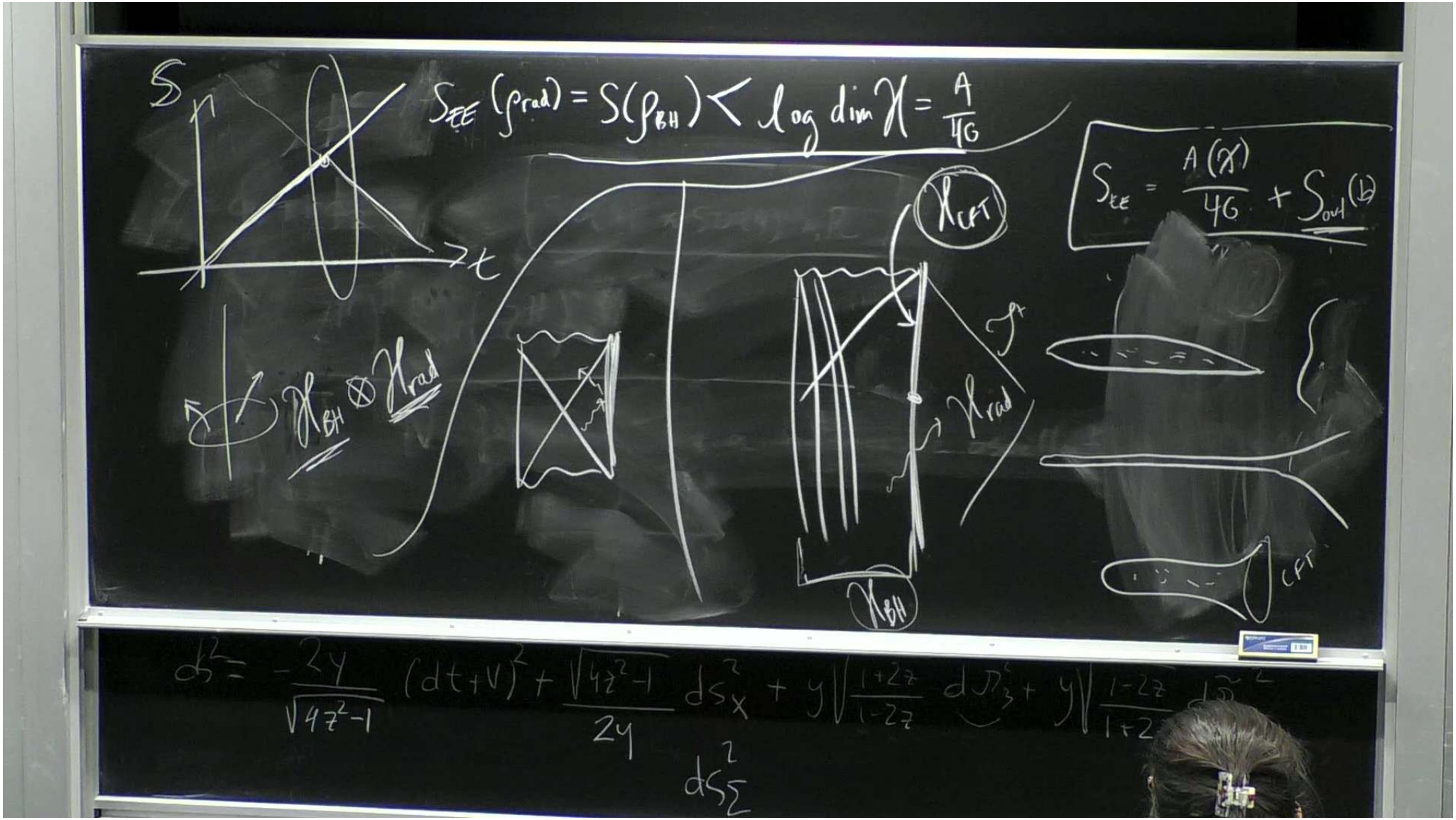
$$f(r) = 1 + \frac{r^2}{\ell^2} + \frac{2GM}{r^2}$$ AdS-Schw.

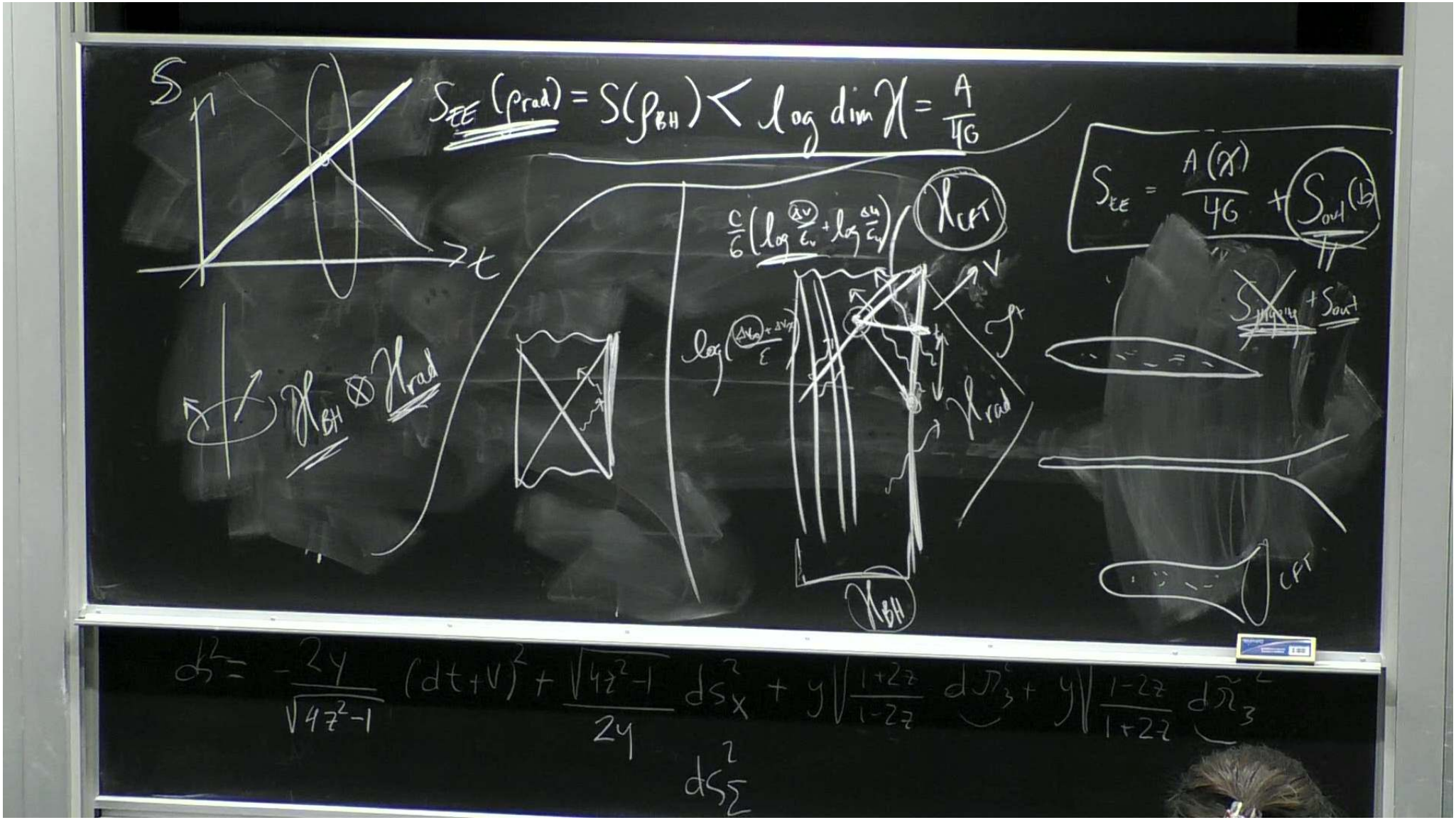
$M = M(v)$, $r_s = r_s(v)$, $\beta = \beta(v)$

$$\dot{M} = -\frac{C\pi}{12\beta(v)^2}$$


$S_{EE}(\rho_{rad}) = S(\rho_{BH}) < \log \dim \mathcal{H} = \frac{A}{4G}$
$S_{EE} = \frac{A(\gamma)}{4G} + S_{out}(b)$
$\mathcal{H}_{CFT}$
$\mathcal{H}_{BH} \otimes \mathcal{H}_{rad}$
$\mathcal{H}_{rad}$
$\mathcal{H}_{BH}$
B
ρ
σ^+
$ds^2 = \frac{-4\gamma}{\sqrt{4z^2-1}}(dt+V) + \frac{\sqrt{4z^2-1}}{2\gamma} ds_x^2 + \gamma\sqrt{\frac{1+2z}{1-2z}} d\Omega_3^2 + \gamma\sqrt{\frac{1-2z}{1+2z}} d\Omega_3^2$


V
t
t_p
x


S
t
$S_{EE}(\rho_{rad}) = S(\rho_{BH}) < \log \dim \mathcal{H} = \frac{A}{4G}$
$S_{EE} = \frac{A(\gamma)}{4G} + S_{out}(b)$
$\mathcal{H}_{CFT}$
$\mathcal{H}_{BH} \otimes \mathcal{H}_{rad}$
$\mathcal{H}_{rad}$
$\mathcal{H}_{BH}$
CFT
$ds^2 = -\frac{2y}{\sqrt{4z^2-1}}(dt+V)^2 + \frac{\sqrt{4z^2-1}}{2y} ds^2_x + y\sqrt{\frac{1+2z}{1-2z}} dz^2 + y\sqrt{\frac{1-2z}{1+2z}} d\Omega_3^2$
ds^2_Σ


$S_{EE}(\rho_{rad}) = S(\rho_{BH}) < \log \dim \mathcal{H} = \frac{A}{4G}$
$S_{EE} = \frac{A(\chi)}{4G} + S_{out}(b)$
$\mathcal{H}_{BH} \otimes \mathcal{H}_{rad}$
$\mathcal{H}_{CFT}$
$\mathcal{H}_{BH}$
$\mathcal{H}_{rad}$
S_{out}
CFT
$ds^2 = \frac{-2y}{\sqrt{4z^2-1}}(dt+V)^2 + \frac{\sqrt{4z^2-1}}{2y} ds^2_{\Sigma}$
$ds^2_{\Sigma} = ds^2_x + y\sqrt{\frac{1+2z}{1-2z}} d\vec{x}_3^2 + y\sqrt{\frac{1-2z}{1+2z}} d\vec{x}_3^2$

[1] Null Polygonal Wilson Loops

⊕

Minimal Surfaces

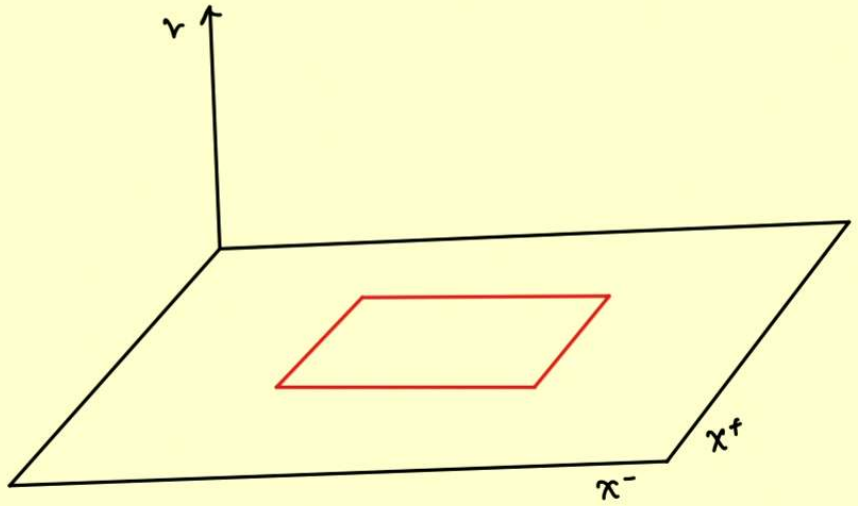
⊕

Stokes phenomenon

Based on: Alday, Maldacena. [0904.0663].

Wilson loops $\cong$ minimal Surface Area.

$$ds^2 = \frac{1}{r^2}(dx_+ dx_- + dr^2)$$



Holographically :

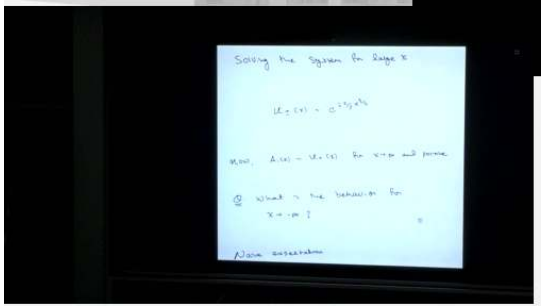
Solving the system for large x:

$$u_{\pm}(x) \sim e^{\mp 2/3\, x^{3/2}}$$

Now, $A_i(x) \sim u_+(x)$ for $x \to \infty$ and positive

Q: what is the behavior for $x \to -\infty$?

Naive expectation:



The classical world sheet

$$\psi_\mu(z,\bar{z}) = \psi_{a\dot{a}}(z,\bar{z})$$

$\mu = -1, \dots, 2$ Lorentz index

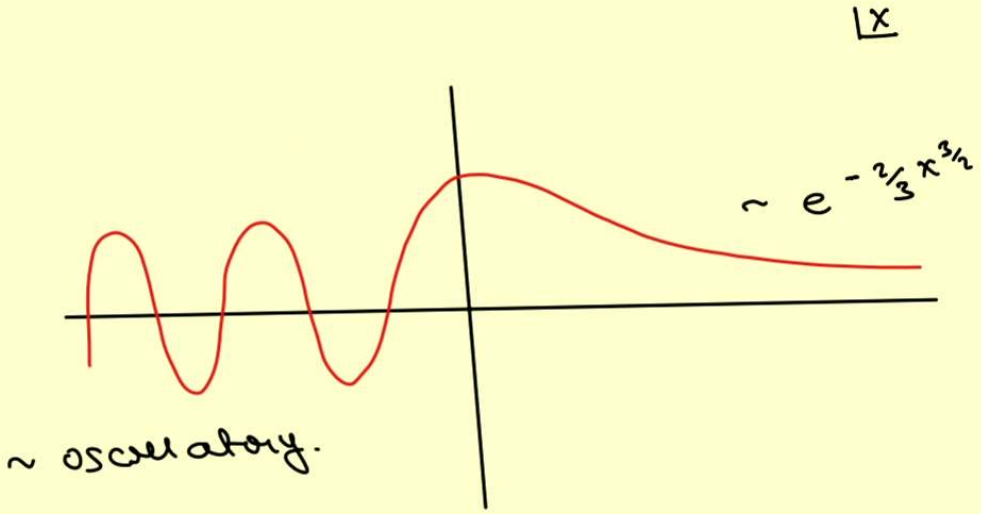
$a/\dot{a} = 1,2$ spinor indices.

3.2 The minimal Area.

$$A \propto \int d^2z \, e^{2\alpha}$$

where $\alpha(z,\bar{z})$ solves the Sinh-Gordon Eqn.

The actual behavior.



This is **Stokes phenomenon.**

Any function : $A : \mathbb{C} \to$

Solves : $(\partial_x^2 - x)\, u(x) = 0$

OR the linear system.

$$\left(\partial_x - \begin{pmatrix} 0 & 1 \\ x & 0 \end{pmatrix}\right) \psi = 0$$

with $\psi = \begin{pmatrix} u \\ v \end{pmatrix}$ and the connection

$$A(x) = \begin{pmatrix} 0 & 1 \\ x & 0 \end{pmatrix} \in \mathfrak{sl}_2(\mathbb{C}).$$

12

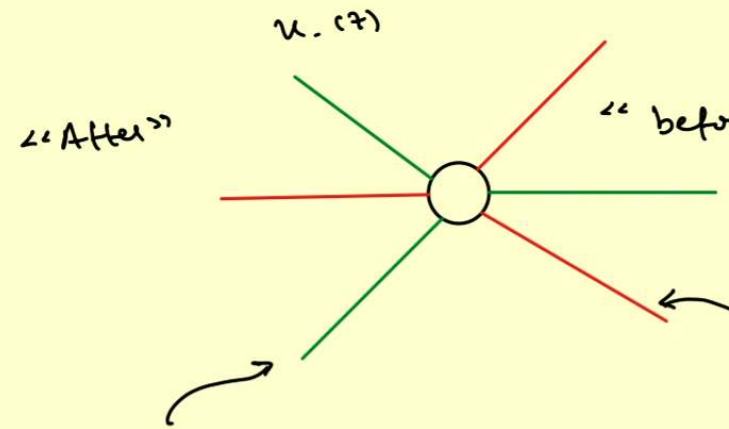
$$\mathrm{Exp}\left[\sqrt{\lambda}\ \mathrm{Area}_{\mathrm{cl.}}\left\{\ \right\}\right]$$

3.1 The setup.

$AdS_3 \hookrightarrow R^{2,2}$
embedding

given by the surface

$$\vec{y}\cdot\vec{y} = -y_{-1}^2 - y_0^2 + y_1^2 + y_2^2 = -1.$$



$u_+(z) = e^{-\frac{2}{3}z^{3/2}}$.

« Anti-Stokes lines »

$\mathrm{Re}\left(\int \sqrt{p}\right) = 0.$

« Stokes lines » given by

$$\mathrm{Im}\left(\int^{z} \sqrt{p(t)}\, dt\right)$$

where. For Any $p(t) = t$. Then, Stokes ph.

$$u_+(z) \longrightarrow u_+(z) + \boxed{\gamma}\, u_-(z)$$

« Before » « After ».

We want to compute:

$$\langle W\{\square\}\rangle$$

$$\square$$

$$\text{Exp}\left[\sqrt{\lambda}\ \text{Area}_{\text{cl.}}\left\{\ \right\}\right]$$

3.1 The setup.

$AdS_3 \hookrightarrow \mathbb{R}^{2,2}$
embedding

12

$$\text{Exp}\left[\sqrt{\lambda}\ \text{Area}_{\text{cl.}}\left\{\ \right\}\right]$$

3.1 The setup.

$AdS_3 \hookrightarrow \mathbb{R}^{2,2}$
embedding

given by the surface

$$\vec{\varphi}\cdot\vec{\varphi} = -\varphi_{-1}^2 - \varphi_0^2 + \varphi_1^2 + \varphi_2^2 = -1.$$

The classical world sheet

$$\chi_\mu(z,\bar{z}) = \chi_{a\dot{a}}(z,\bar{z})$$

$\mu = -1, \dots, 2$ Lorentz index

$a/\dot{a} = 1,2$ Spinor indices.

3.2 The minimal Area.

$$A \propto \int d^2z \, e^{2\alpha}$$

where $\alpha(z,\bar{z})$ solves the Sinh-Gordon Eqn.

3.3 The Auxiliary Linear Problem.

Q: How do we construct the world-sheet?

A: $$\mathcal{K}_{a\dot{a}}(z,\bar{z}) = \psi^{L}_{\alpha,a}\, M^{\alpha\dot{\beta}}\, \psi^{R}_{\dot{\beta},\dot{a}}$$

where the pair $(\psi^{L}_{\alpha}, \psi^{R}_{\dot{\alpha}})$ solve

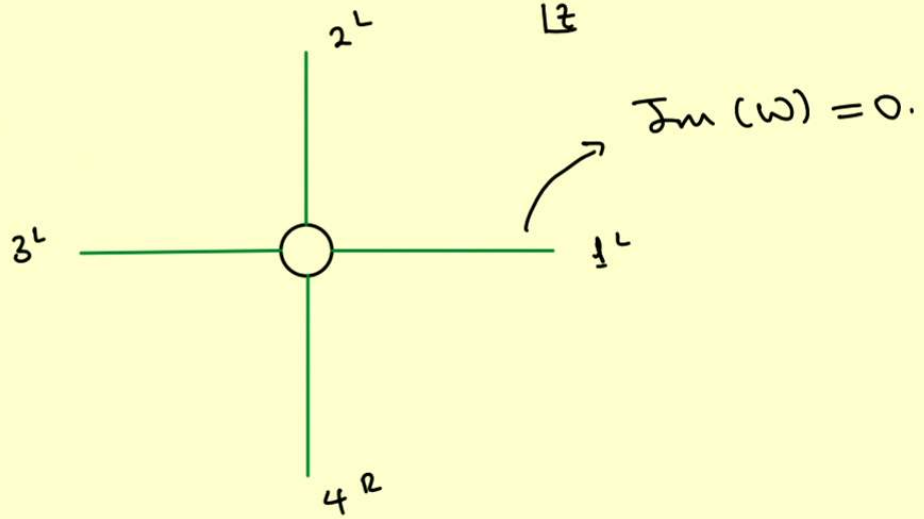
$$\left(\partial + (B^{L}_{z})_{\alpha}{}^{\beta}\right)\psi^{L}_{\beta} = 0$$

$$\left(\partial + (B^{R}_{z})_{\dot{\alpha}}{}^{\dot{\beta}}\right)\psi^{R}_{\dot{\beta}} = 0$$

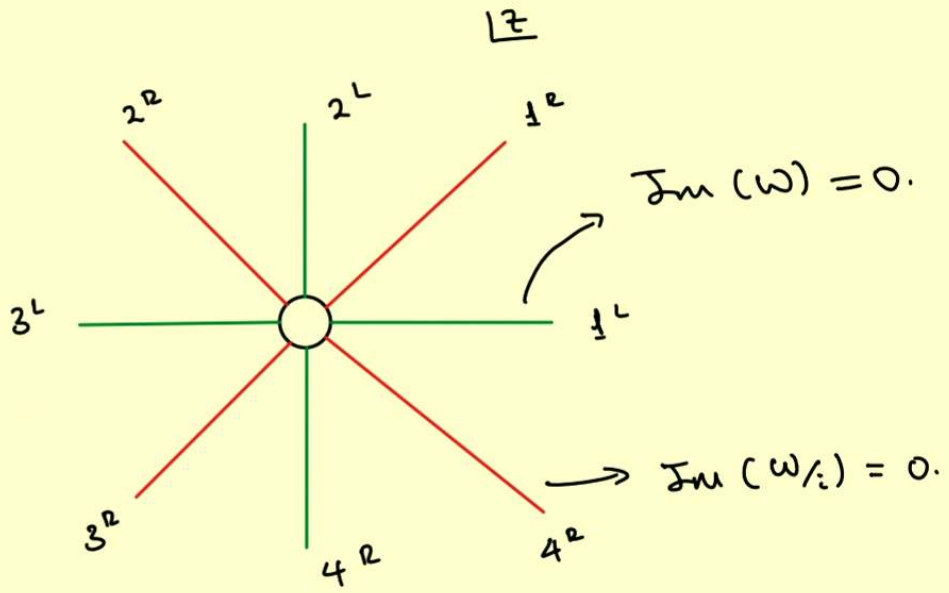
(and $\partial \to \bar{\partial}$, $B_z \to B_{\bar{z}}$)

with $\omega = \int^{z} \sqrt{p(t)}\, dt$.

Stokes graph



Stokes graph



$$\mathcal{X}_i^+ = \frac{\mathcal{Y}_2^L \wedge S_i^L}{\mathcal{Y}_1^L \wedge S_i^L}, \qquad \mathcal{X}_i^- = \frac{\mathcal{Y}_2^R \wedge S_i^R}{\mathcal{Y}_1^R \wedge S_i^R}.$$