Title: Lecture - Beautiful Papers

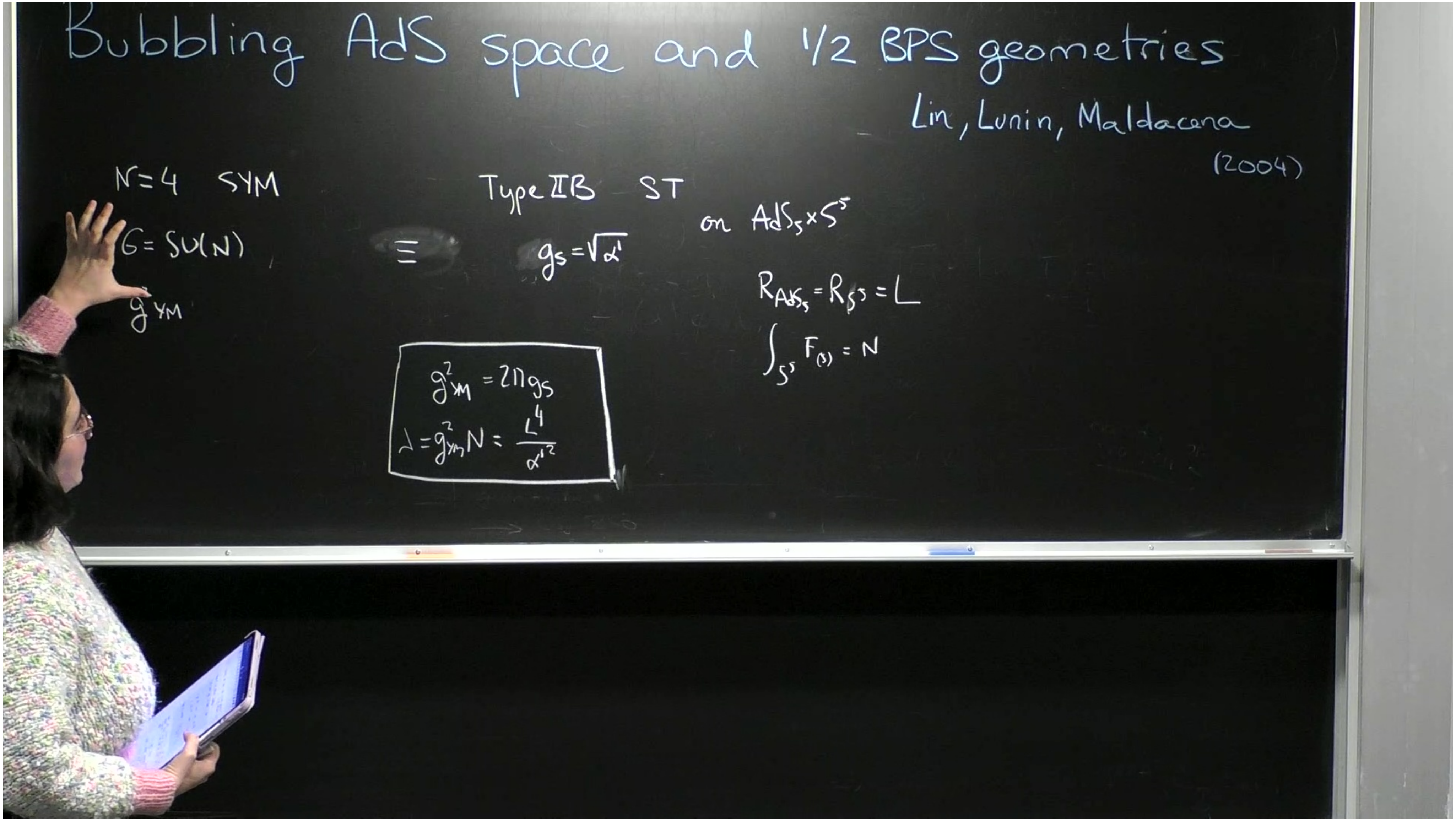
Speakers: Pedro Vieira

Collection/Series: Beautiful Papers - October 7, 2024 - January 31, 2025

Subject: Other

Date: January 20, 2025 - 9:15 AM

URL: https://pirsa.org/25010050



Bubbling AdS space and 1/2 BPS geometries
Lin, Lunin, Maldacena
(2004)
$\mathcal{N}=4$ SYM
$G = SU(N)$
g_{YM}
$\equiv$
Type IIB ST on $AdS_5 \times S^5$
$g_s = \sqrt{\alpha'}$
$R_{AdS_5} = R_{S^5} = L$
$\int_{S^5} F_{(5)} = N$
$g_{YM}^2 = 2\pi g_s$
$\lambda = g_{YM}^2 N = \frac{L^4}{\alpha'^2}$

g^2_{YM}

$R_{AdS_5} = R_{S^5} = L$

$\int_{S^5} F_{(5)} = N$

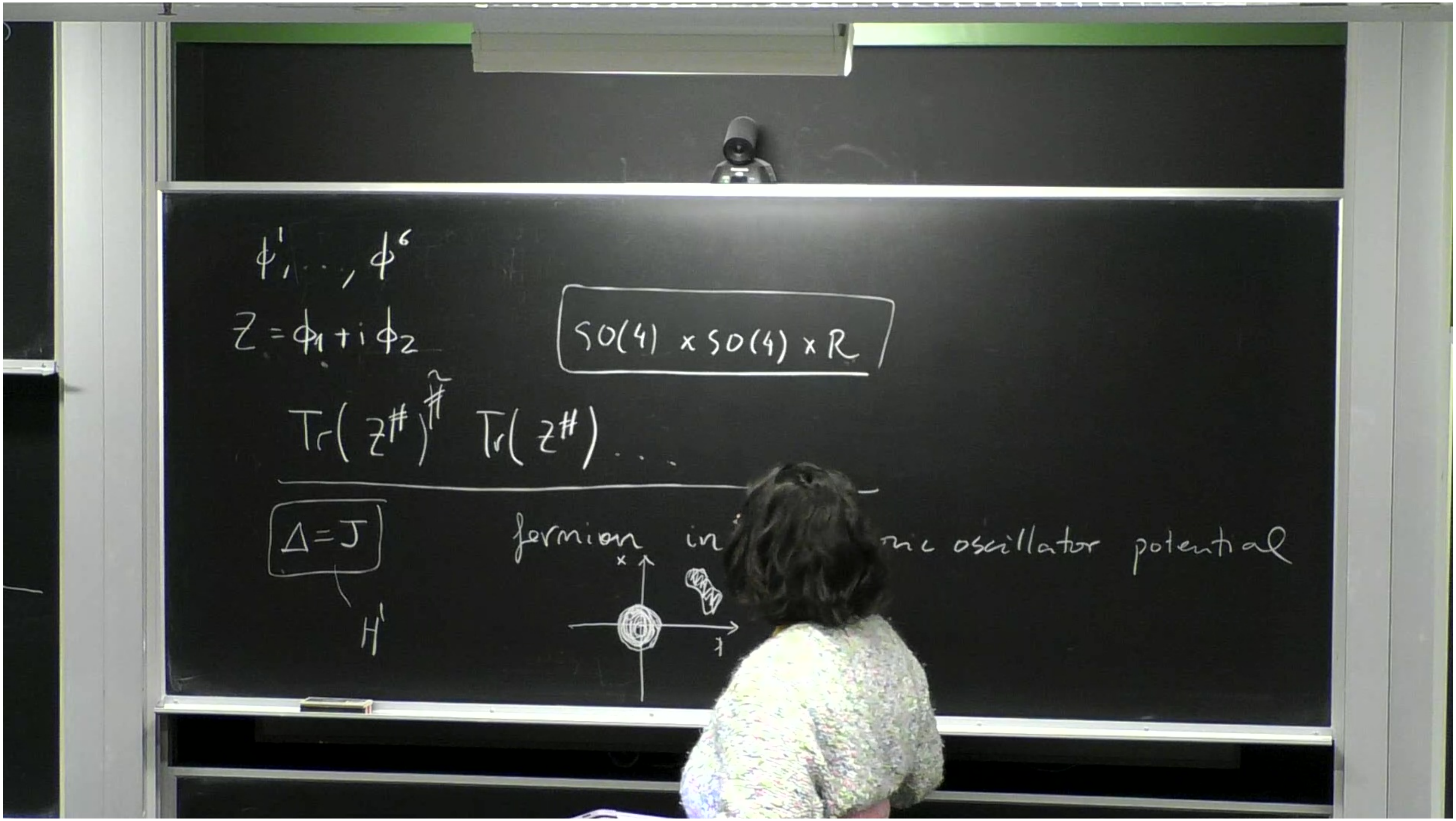
$$\boxed{\begin{aligned} g^2_{YM} &= 2\pi g_s \\ \lambda = g^2_{YM} N &= \frac{L^4}{\alpha'^2} \end{aligned}}$$

Chiral Primary Operators

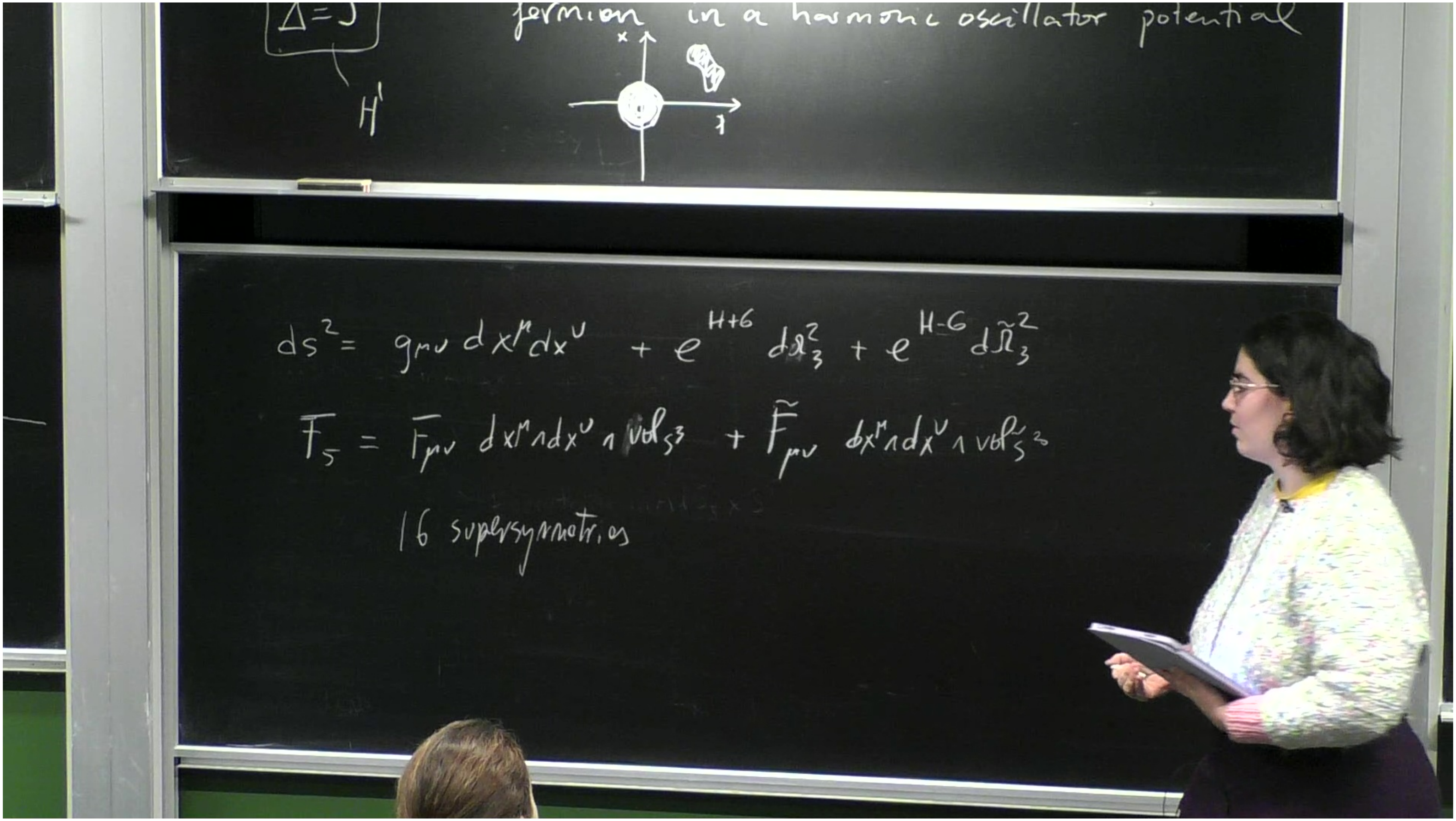
Δ , J $U(1) \subset$ R-symmetry $SU(4) = SO(6)$

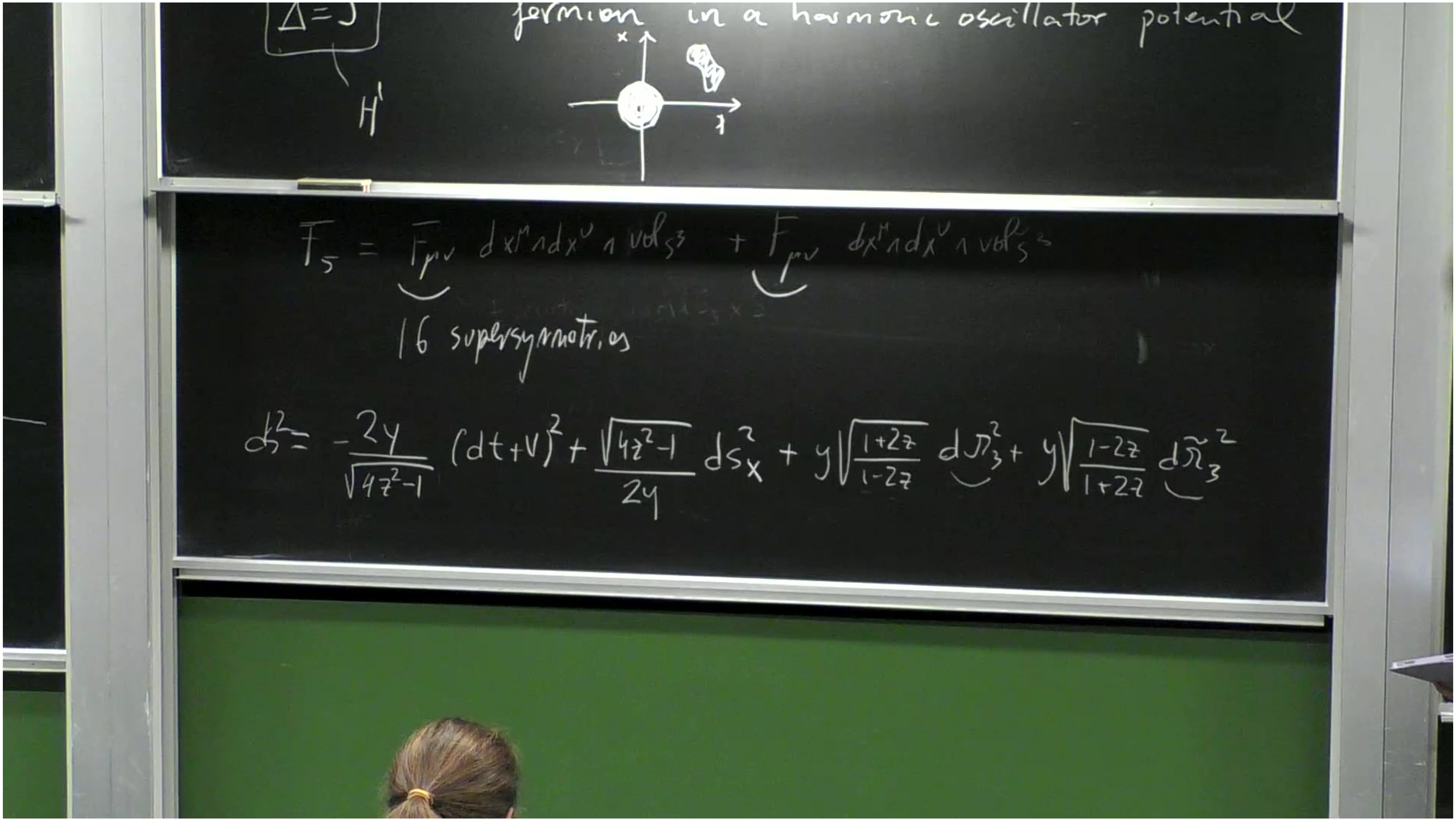
$\Delta = J$

- $\Delta << N \rightsquigarrow$ gravitons
- $\Delta \sim N \rightsquigarrow$ branes Giant Gravitons
- $\Delta \sim N^2 \rightsquigarrow$ geometry

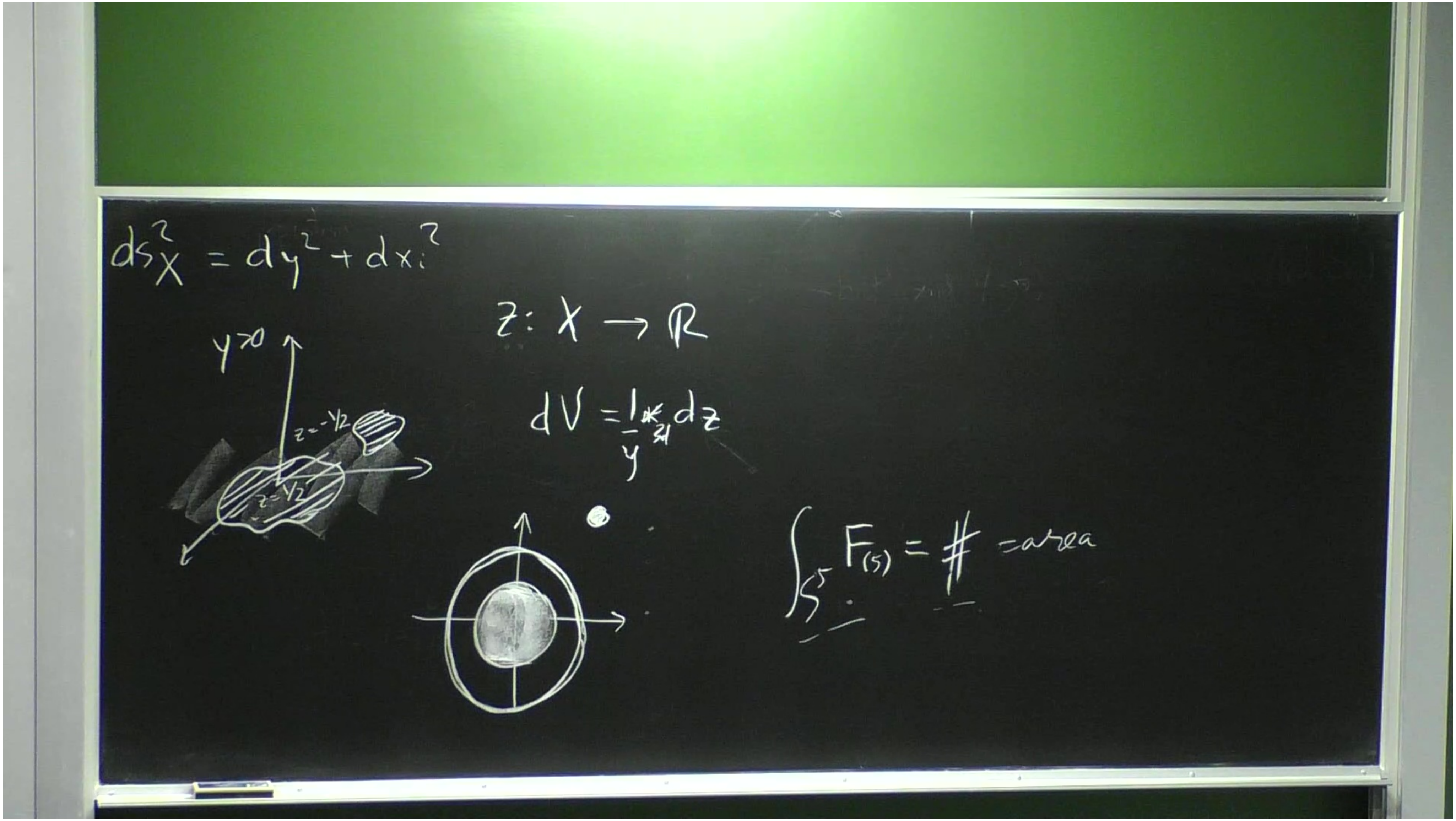


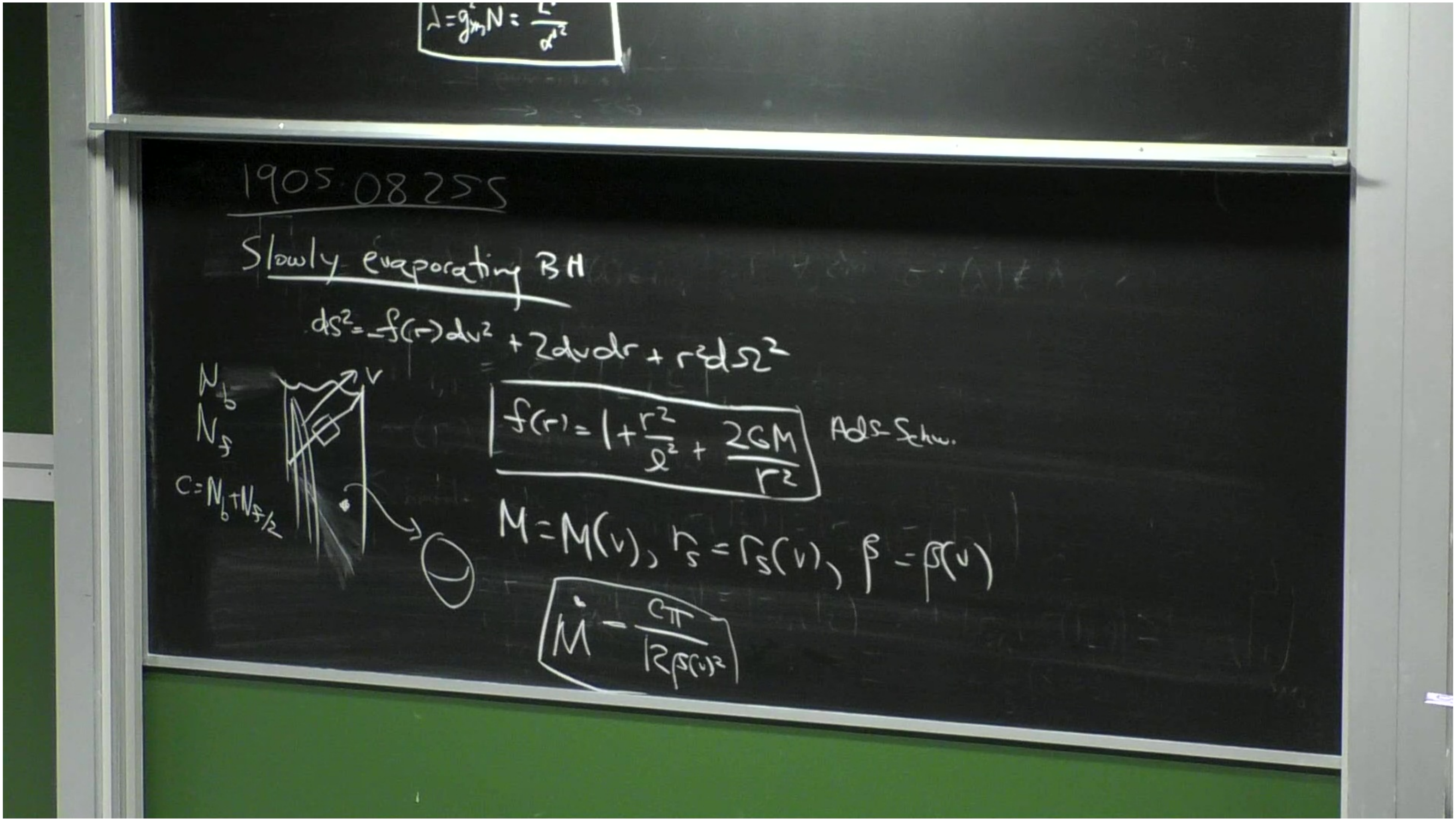
$\phi^1, \dots, \phi^6$
$Z = \phi_1 + i\phi_2$
$SO(4) \times SO(4) \times R$
$Tr(Z^{\#})^{\tilde{\#}}\ Tr(Z^{\#}) \dots$
$\Delta = J$
fermion in
oscillator potential


fermion in a harmonic oscillator potential
$\Delta = J$
H'
$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{H+G} d\Omega_3^2 + e^{H-G} d\tilde{\Omega}_3^2$
$F_5 = F_{\mu\nu} dx^\mu \wedge dx^\nu \wedge vol_{S^3} + \tilde{F}_{\mu\nu} dx^\mu \wedge dx^\nu \wedge vol_{\tilde{S}^3}$
16 supersymmetries

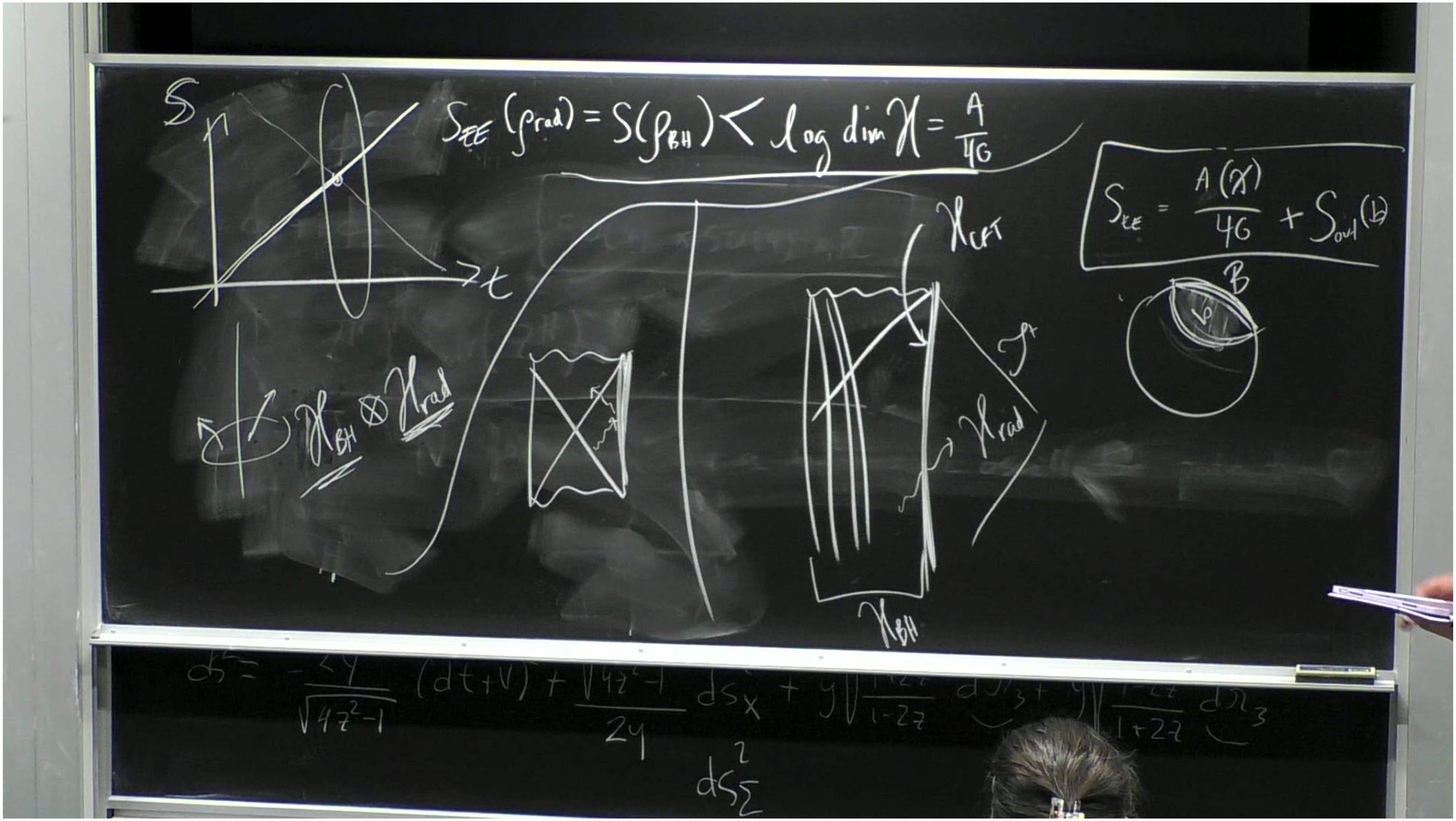


$\Delta = J$
H'
fermion in a harmonic oscillator potential
$F_5 = F_{\mu\nu}\, dx^\mu \wedge dx^\nu \wedge \mathrm{vol}_{S^3} + F_{\mu\nu}\, dx^\mu \wedge dx^\nu \wedge \mathrm{vol}_{S^3}$
16 supersymmetries
$ds^2 = -\frac{2y}{\sqrt{4z^2-1}}(dt+V)^2 + \frac{\sqrt{4z^2-1}}{2y} ds_X^2 + y\sqrt{\frac{1+2z}{1-2z}}\, d\Omega_3^2 + y\sqrt{\frac{1-2z}{1+2z}}\, d\tilde{\Omega}_3^2$

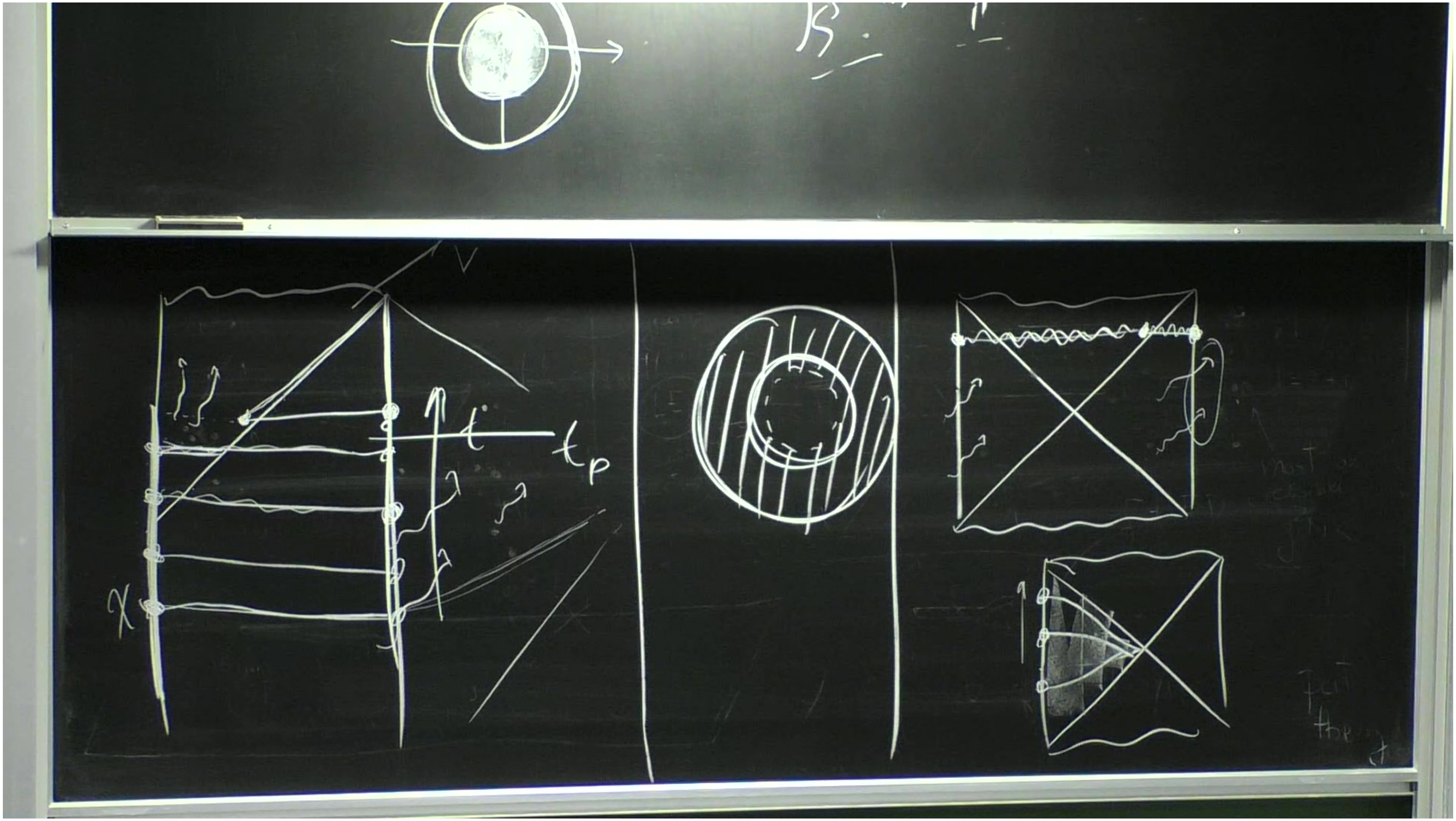

$ds_X^2 = dy^2 + dx_i^2$
$z: X \to \mathbb{R}$
$dV = \frac{1}{y} *_{3d}\, dz$
$y>0$
$z=-1/2$
$z=1/2$
$\int_{S^5} F_{(5)} = \# = \text{area}$



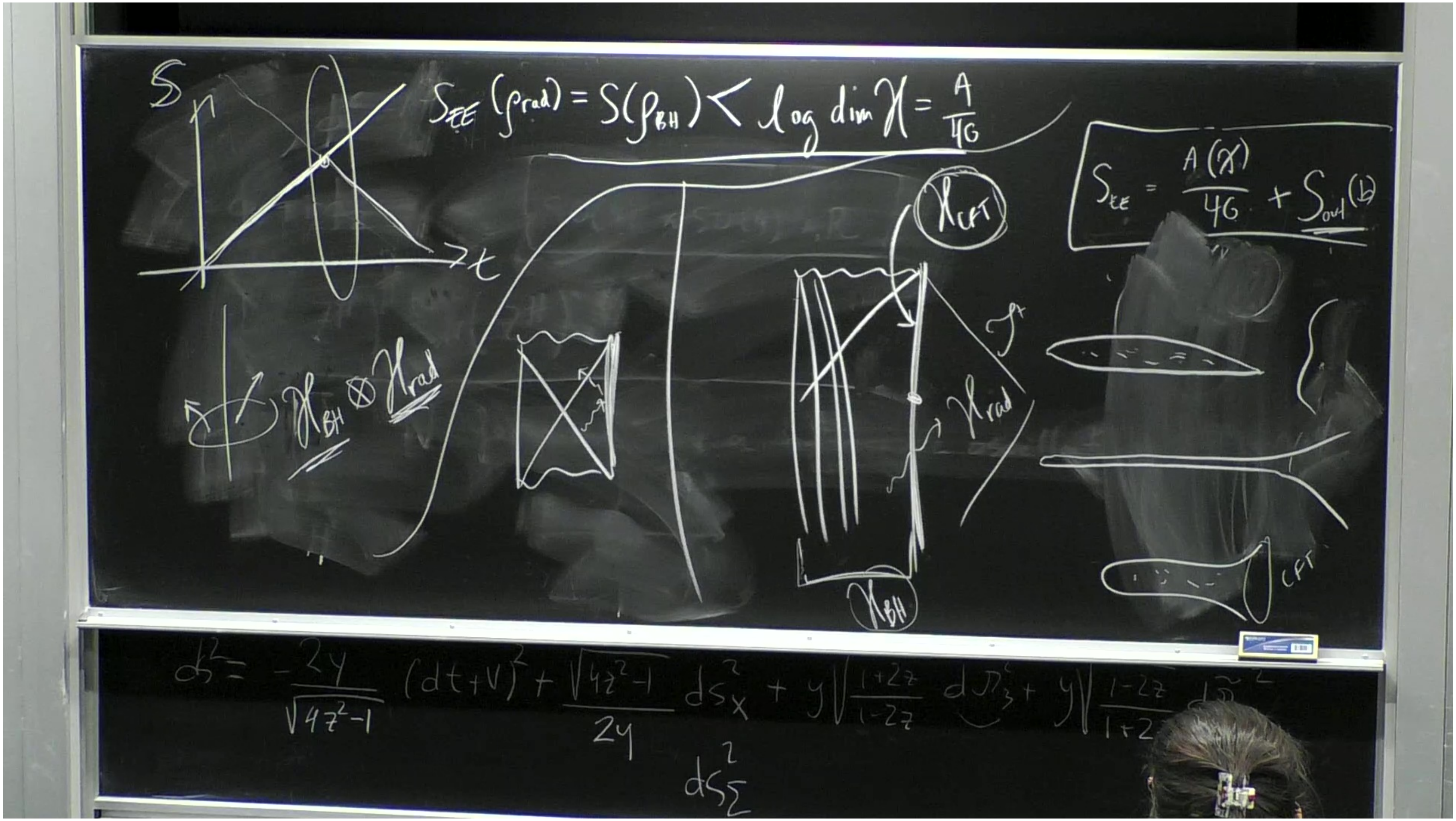
λ = g²YM N = L⁴/α'²
1905.08255
Slowly evaporating BH
ds² = -f(r)dv² + 2dvdr + r²dΩ²
Nb
Nf
c = Nb + Nf/2
v
f(r) = 1 + r²/ℓ² + 2GM/r²
AdS Schw.
M = M(v), rs = rs(v), β = β(v)
Ṁ - cπ/12β(v)²



$S_{EE}(\rho_{rad}) = S(\rho_{BH}) < \log \dim \mathcal{H} = \frac{A}{4G}$
$S_{EE} = \frac{A(\chi)}{4G} + S_{out}(b)$
S
t
$\mathcal{H}_{BH} \otimes \mathcal{H}_{rad}$
$\mathcal{H}_{CFT}$
$\mathcal{H}_{rad}$
$\mathcal{H}_{BH}$
B
ds_Σ^2



t
tp
x


S
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$S_{EE}(\rho_{rad}) = S(\rho_{BH}) < \log \dim \mathcal{H} = \frac{A}{4G}$
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$\mathcal{H}_{BH} \otimes \mathcal{H}_{rad}$
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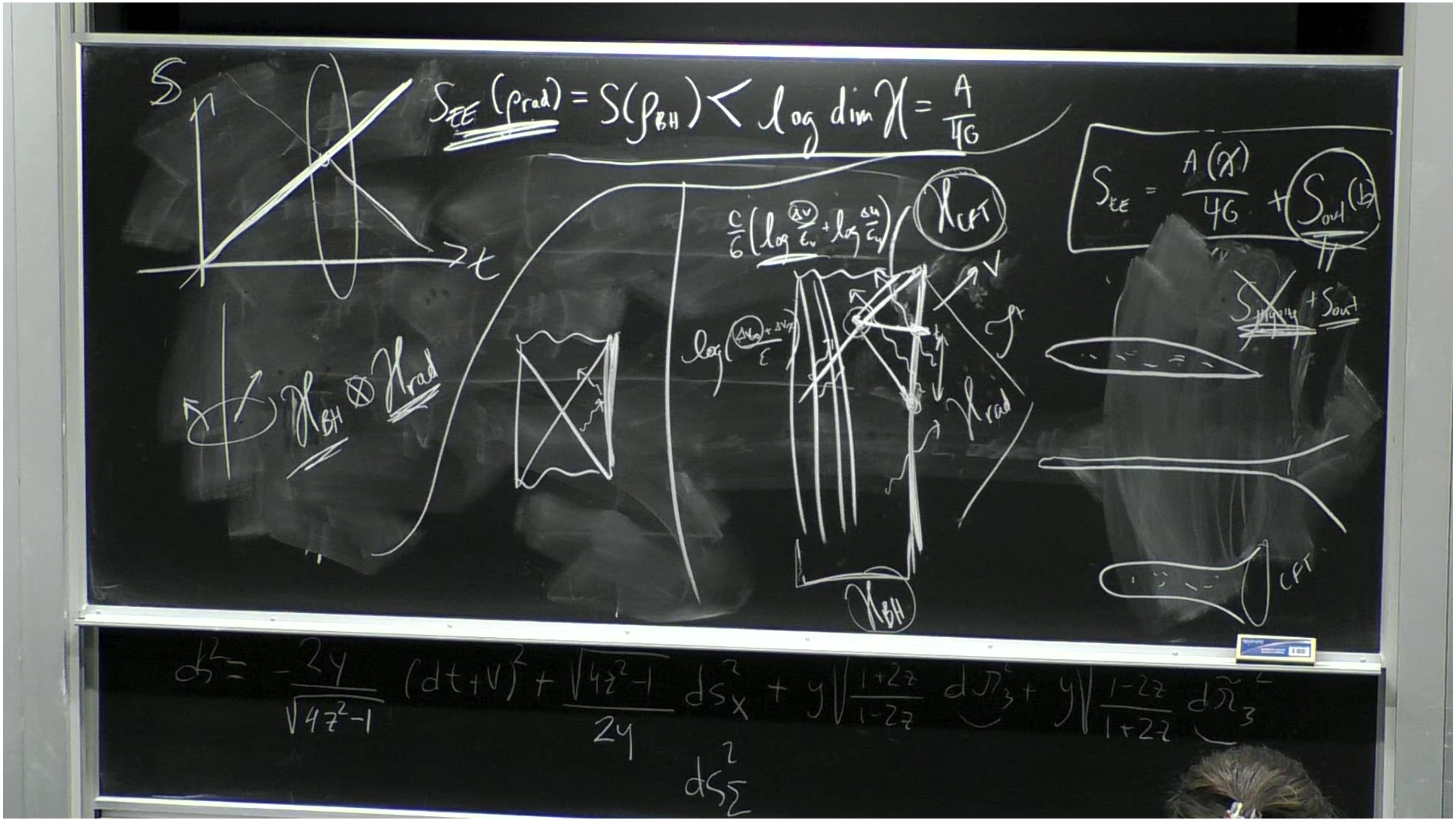
CFT
$ds^2 = -\frac{2y}{\sqrt{4z^2-1}}(dt+V)^2 + \frac{\sqrt{4z^2-1}}{2y} ds_X^2 + y\sqrt{\frac{1+2z}{1-2z}}\, d\Omega_3^2 + y\sqrt{\frac{1-2z}{1+2z}}$
ds_Σ^2


S_EE (ρ_rad) = S(ρ_BH) < log dim H = A/4G
S_EE = A(X)/4G + S_out(b)
H_BH ⊗ H_rad
H_CFT
H_rad
H_BH
c/6 (log Δv/ε_v + log Δu/ε_u)
CFT
ds² = −2y/√(4z²−1) (dt+v)² + √(4z²−1)/2y ds_x² + y√((1+2z)/(1−2z)) dΩ₃² + y√((1−2z)/(1+2z)) dΩ̃₃²
ds_Σ²

$\boxed{1}$ Null Polygonal Wilson Loops

$\oplus$

Minimal Surfaces

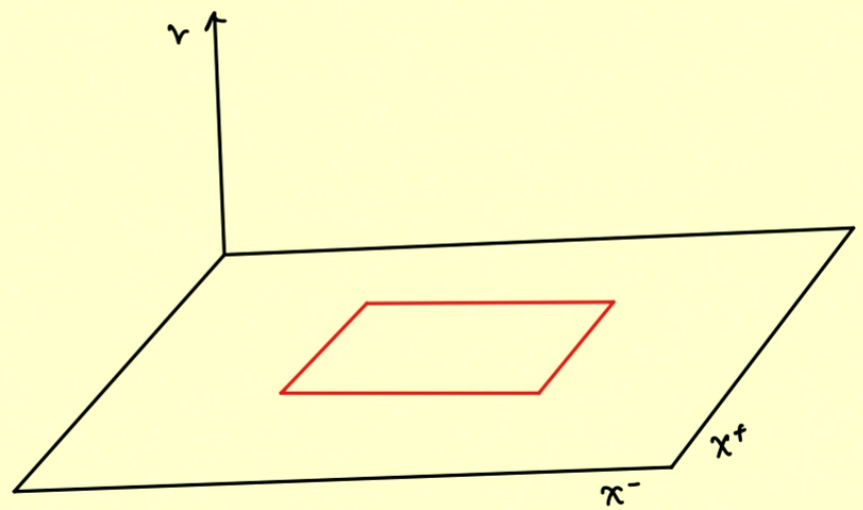
$\oplus$

Stokes phenomenon

Based on: Alday, Maldacena [0904.0663].

Wilson loops $\cong$ minimal Surface Area.

$$ds^2 = \frac{1}{r^2}\left(dx_+ dx_- + dr^2\right)$$



Holographically :

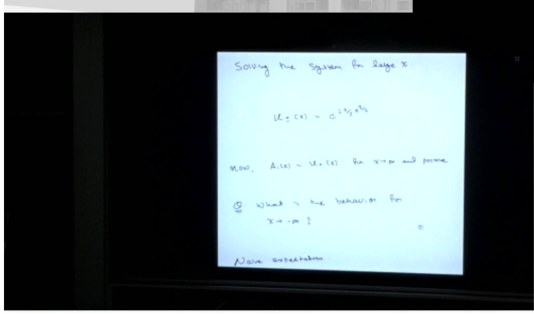
Solving the system for large x:

$$u_{\pm}(x) \sim e^{\mp \frac{2}{3} x^{3/2}}$$

Now, $Ai(x) \sim u_{+}(x)$ for $x \to \infty$ and positive

Q: What is the behavior for $x \to -\infty$?

Naive expectation:



The classical world sheet

$$Y_\mu(z,\bar{z}) = Y_{a\dot{a}}(z,\bar{z})$$

$\mu = -1, \ldots, 2$ Lorentz index

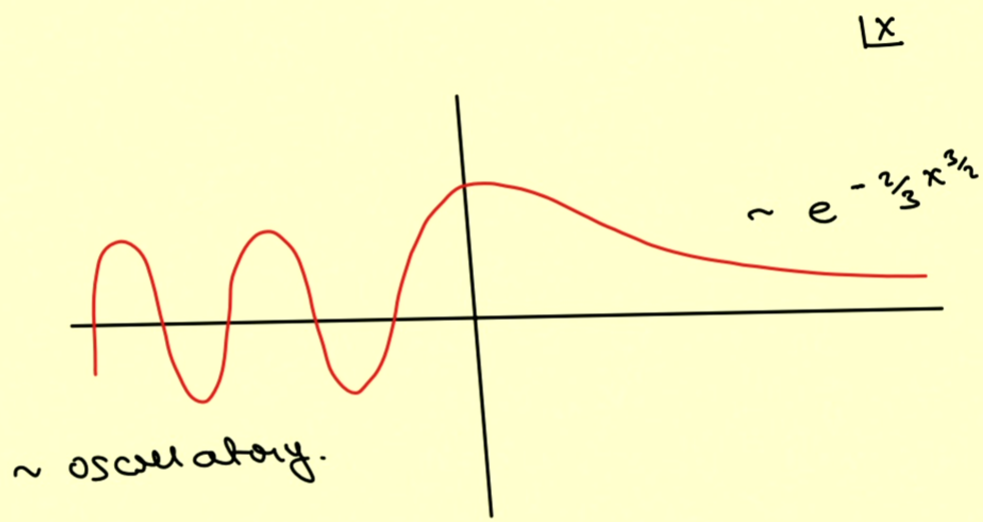
$a/\dot{a} = 1, 2$ spinor indices.

$\boxed{3.2}$ The minimal Area.

$$\boxed{A \propto \int d^2z \, e^{2\alpha}}$$

Where $\alpha(z,\bar{z})$ solves the Sinh-Gordon Eqn.

The actual behavior.



This is $\boxed{\text{Stokes Phenomenon.}}$

Airy function : $A_i(x)$

Solves : $(\partial_x^2 - x)\, u(x) = 0$

OR the linear system.

$$\left(\partial_x - \begin{pmatrix} 0 & 1 \\ x & 0 \end{pmatrix} \right) \psi = 0$$

with $\psi = \begin{pmatrix} u \\ v \end{pmatrix}$ and the connection

$$A(x) = \begin{pmatrix} 0 & 1 \\ x & 0 \end{pmatrix} \in sl_2(\mathbb{C}).$$

12

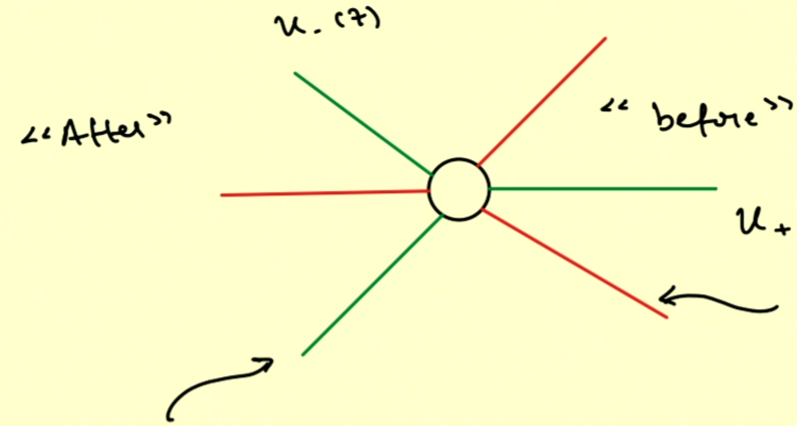
$\mathrm{Exp}\left[\sqrt{\lambda}\ \mathrm{Area}_{cl.}\left\{\right.\right.$  $\left.\left.\right\}\right]$

$\boxed{3.1}$ The setup.

$$AdS_3 \underset{\text{embedding}}{\hookrightarrow} \mathbb{R}^{2,2}$$

given by the surface

$$\vec{Y}\cdot\vec{Y} = -Y_{-1}^2 - Y_0^2 + Y_1^2 + Y_2^2 = -1.$$



$$u_+(z) = e^{-\frac{2}{3} z^{3/2}}.$$

"Anti-Stokes lines"

$\mathrm{Re}\left(\int \sqrt{p} \right) = 0.$

"Stokes lines" given by

$$\mathrm{Im}\left(\int^{z} \sqrt{p(t)}\, dt \right)$$

where. For Airy $p(t) = t$. Then, Stokes ph.

$$\underset{\text{"Before"}}{u_+(z)} \longrightarrow \underset{\text{"After".}}{u_+(z) + \boxed{\gamma}\, u_-(z)}$$

We want to compute:

$\langle W \{$  $\} \rangle$

$\simeq$

$\mathrm{Exp}\left[\sqrt{\lambda}\ \mathrm{Area}_{\mathrm{cl.}} \{$  $\} \right]$

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$\boxed{3.1}$ The setup.

$$AdS_3 \xhookrightarrow[\text{embedding}]{} \mathbb{R}^{2,2}$$

given by the surface

$$\vec{\varphi}\cdot\vec{\varphi} = -\varphi_{-1}^2-\varphi_0^2+\varphi_1^2+\varphi_2^2 = -1.$$

The classical world sheet

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Where $\alpha(z,\bar z)$ solves the Sinh-Gordon Eqn.

$\boxed{3.3}$ The Auxiliary Linear Problem.

Q: How do we construct the world-sheet ?

A: $\quad X_{a\dot{a}}(z,\bar{z}) = \psi^{L}{}_{\alpha, a}\, M^{\alpha\dot{\beta}}\, \psi^{R}_{\dot{\beta}, \dot{a}}$

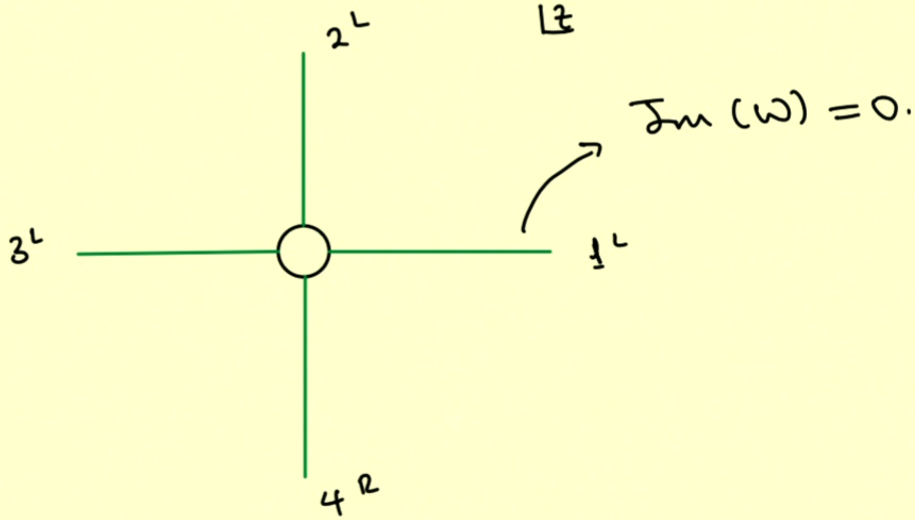
where the pair $(\psi^{L}{}_{\alpha}, \psi^{R}_{\dot{\alpha}})$ solve

$$\boxed{\begin{aligned} &(\partial + (B^{L}{}_{z})_{\alpha}{}^{\beta})\, \psi^{L}{}_{\beta} = 0 \\ &(\partial + (B^{R}{}_{z})_{\dot{\alpha}}{}^{\dot{\beta}})\, \psi^{R}_{\dot{\beta}} = 0 \end{aligned}}$$

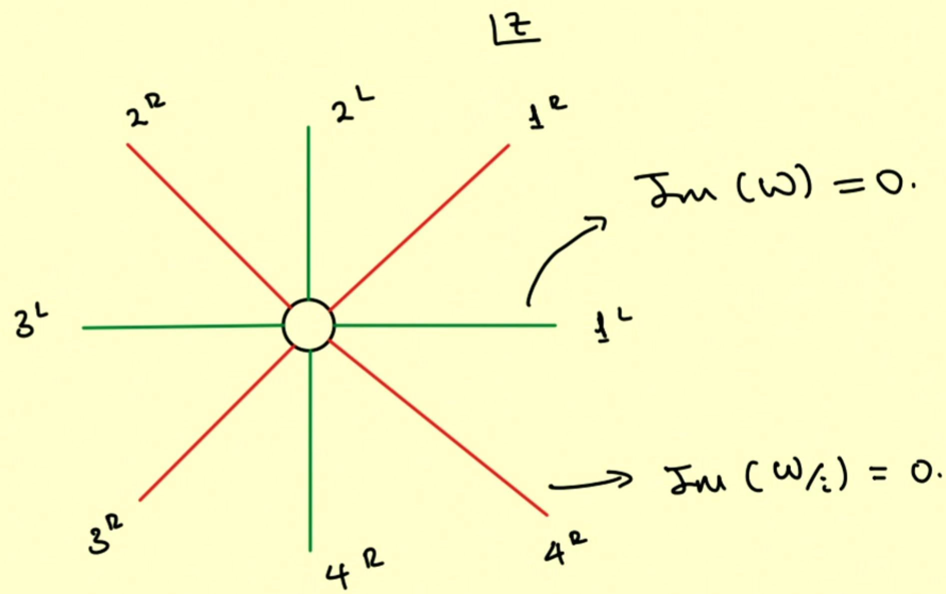
(and $\partial \to \bar{\partial}$, $B_z \to B_{\bar{z}}$)

with $\omega = \int^{z} \sqrt{p(t)}\, dt.$

Stokes graph



Stokes graph



$$\chi_i^+ = \frac{\psi^L_2 \wedge s_i^L}{\psi^L_1 \wedge s_i^L}, \qquad \chi_i^- = \frac{\psi^R_2 \wedge s_i^R}{\psi^R_1 \wedge s_i^R}.$$