

Title: Lecture - Quantum Foundations, PHYS 639

Speakers: Lucien Hardy

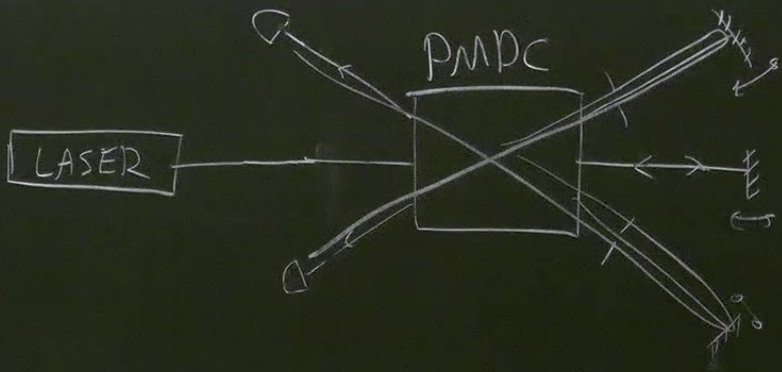
Collection/Series: Quantum Foundations (Elective), PHYS 639, January 6 - February 5, 2025

Subject: Quantum Foundations

Date: January 20, 2025 - 11:30 AM

URL: <https://pirsa.org/25010043>

The Tirol cross



$$|\alpha\rangle_s |0\rangle_a |0\rangle_b + c |\alpha\rangle |1\rangle |1\rangle +$$

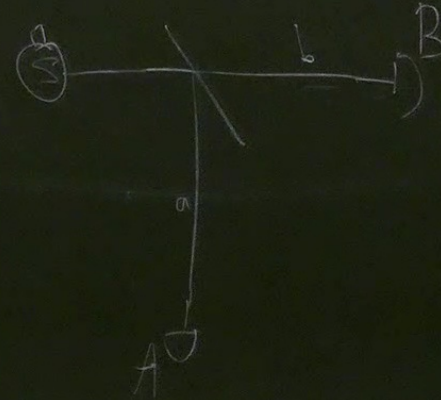
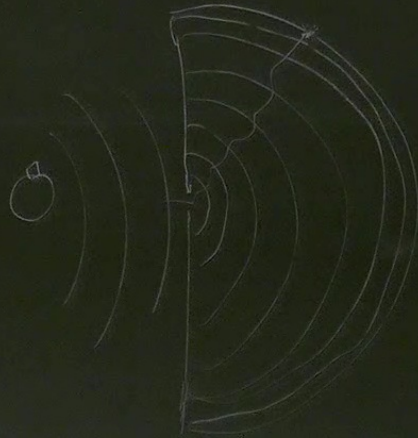
$$|\alpha\rangle_s |0\rangle_a |0\rangle_b + c |\alpha\rangle |1\rangle |1\rangle + c e^{i\theta} |\alpha\rangle |1\rangle |1\rangle$$

$$|c+c|^2 = 4|c|^2$$

EPR 1935

Einstein 1927

completeness
locality



Defn.

Completeness "every element of physical reality (epr) must have a counterpart in the physical theory"

Ψ -completeness All elements of physical reality follow from the quantum state

$$[A=\alpha]_a \text{ is an epr iff } \langle \Psi | \hat{P}_{[A=\alpha]_a} | \Psi \rangle = 1$$

Soft. cond. for the existence of an epr "If, without in any way disturbing

Suff. cond. for the existence of an epr

certainty \downarrow probs = 1

$$\left[\lim_{N \rightarrow \infty} \frac{N-100}{N} = 1 \right] \text{ QB/SM}$$

"If, without in any way disturbing probability equal to unity) exists an element of physical

If $\text{probs}((A=\alpha)_a | (B = \beta)) = 1$

and if measurement B does then \exists an epr $[A=\alpha]_a$ for

Suff. cond. for the existence of an epr

certainty \downarrow probs = 1

$$\left[\lim_{N \rightarrow \infty} \frac{N-100}{N} = 1 \right] \text{ QB/SM}$$

"If, without in any way disturbing a system (probability equal to unity) the system exhibits an element of physical reality

If $\text{probs}((A=\alpha)_a | (B=\beta)_b) = 1$
and if measurement B does not disturb A
then \exists an epr $[A=\alpha]_a$ for a given system

"If, without in any way disturbing a system, we can predict with certainty (i.e. probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this quantity."

$$\text{If } \text{prob}((A = \alpha)_a | (B = \beta)_b) = 1$$

and if measurement B does not disturb a
then \exists an epr $[A = \alpha]_a$ for a even when we don't measure B

locality A system is undisturbed by choices made at spacelike separation from it.

We can prove

$(\text{QT predictions}) \wedge (\psi\text{-completeness}) \wedge \text{locality} \Rightarrow \text{contradiction}$

$$\text{prob}\left(\left(A=1\right)_a \mid \left(B=0\right)_b\right) = 1$$

If we perform B and see $B=0$

then \exists epr $[A=1]_a$

If we perform B and see $B=1$

then \exists epr $[A=0]_a$

locally \Rightarrow have an epr

$$\text{prob}(A=1 | B=0) = 1$$

If we perform B and see B=0

then \exists epr $[A=1]_a$

If we perform B and see B=1

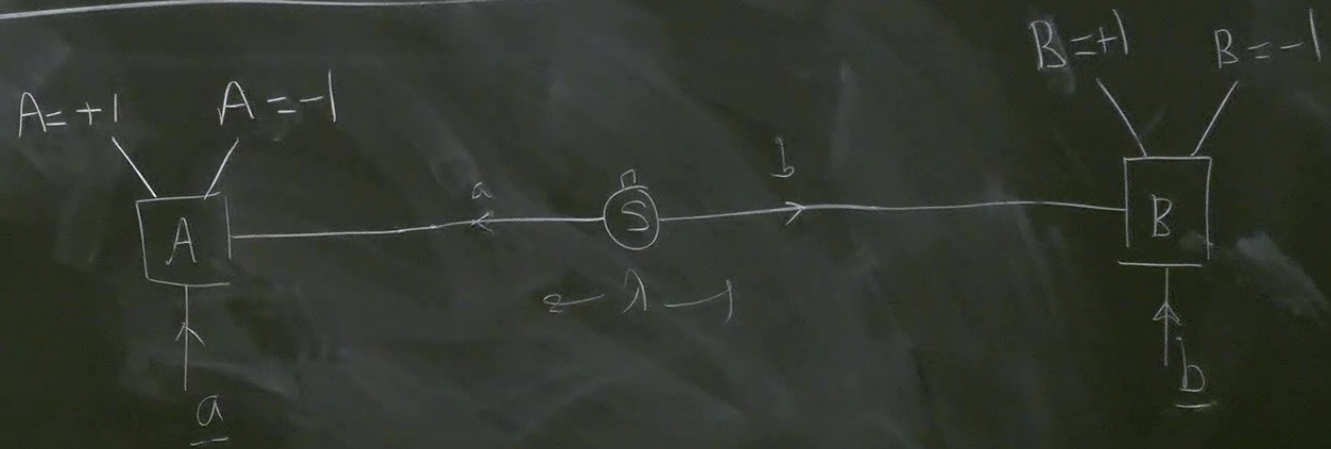
then \exists epr $[A=0]_a$

locally \Rightarrow have an epr $[A=1]_a$
 even if we don't perform meas B.

But if $|\psi\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle)$

$$\text{then } \langle \psi | \hat{P}_{A=1} | \psi \rangle = \frac{1}{2}$$

J.S. Bell 1964



Introduce λ (hidden variables)

$$A(\underline{a}, \lambda) = \pm 1$$

$$B(\underline{b}, \lambda) = \pm 1$$

ψ -completeness

All elements of physical system
quantum state

$$[A=\alpha]_a \text{ is an epr iff } \langle \psi | \hat{P}_{[A=\alpha]_a} | \psi \rangle = 1$$

$$E(\underline{a}, \underline{b}) = \int A(\underline{a}, \lambda) B(\underline{b}, \lambda) \rho(\lambda) d\lambda$$

$\rho(\lambda)$ is a distribution over λ

$$\rho(\lambda) \geq 0$$

$$\int \rho(\lambda) d\lambda = 1$$

CHSH 1969

$$X, X', Y, Y' = \pm 1$$

$$X'Y' + X'Y + XY' - XY = \pm 2$$

$$X'(Y' + Y) + X(Y' - Y) = \pm 2$$

$+2, 0, -2$ $0, +2, 0$