Title: Lecture - Quantum Foundations, PHYS 639

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Subject: Quantum Foundations

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The Quantum Zeno effect 1977 Sudarshan Misra.

$|0\rangle_{a_1}$ at time $t=0$

$$|\psi(\delta t)\rangle_{a_1} = e^{-iH\delta t/\hbar}|0\rangle_{a_1}$$

$$\simeq |0\rangle_{a_1} - \frac{iH\delta t}{\hbar}|0\rangle_{a_1}$$

Measure onto

$$\hat{P}_0 = |0\rangle_{a_1}\langle 0|$$

fect 1977 Sudarshan Misra.

0

$/\hbar \; |0\rangle_{a_1}$

$- \frac{iH\delta t}{\hbar}|0\rangle_{a_1}$

Measure onto

$$\hat{P}_0 = |0\rangle_{a_1}\langle 0| \qquad \hat{P}_{\bar{0}} = \mathbb{1} - |0\rangle\langle 0| = \sum_{n=1}^{N-1} |n\rangle\langle n|$$

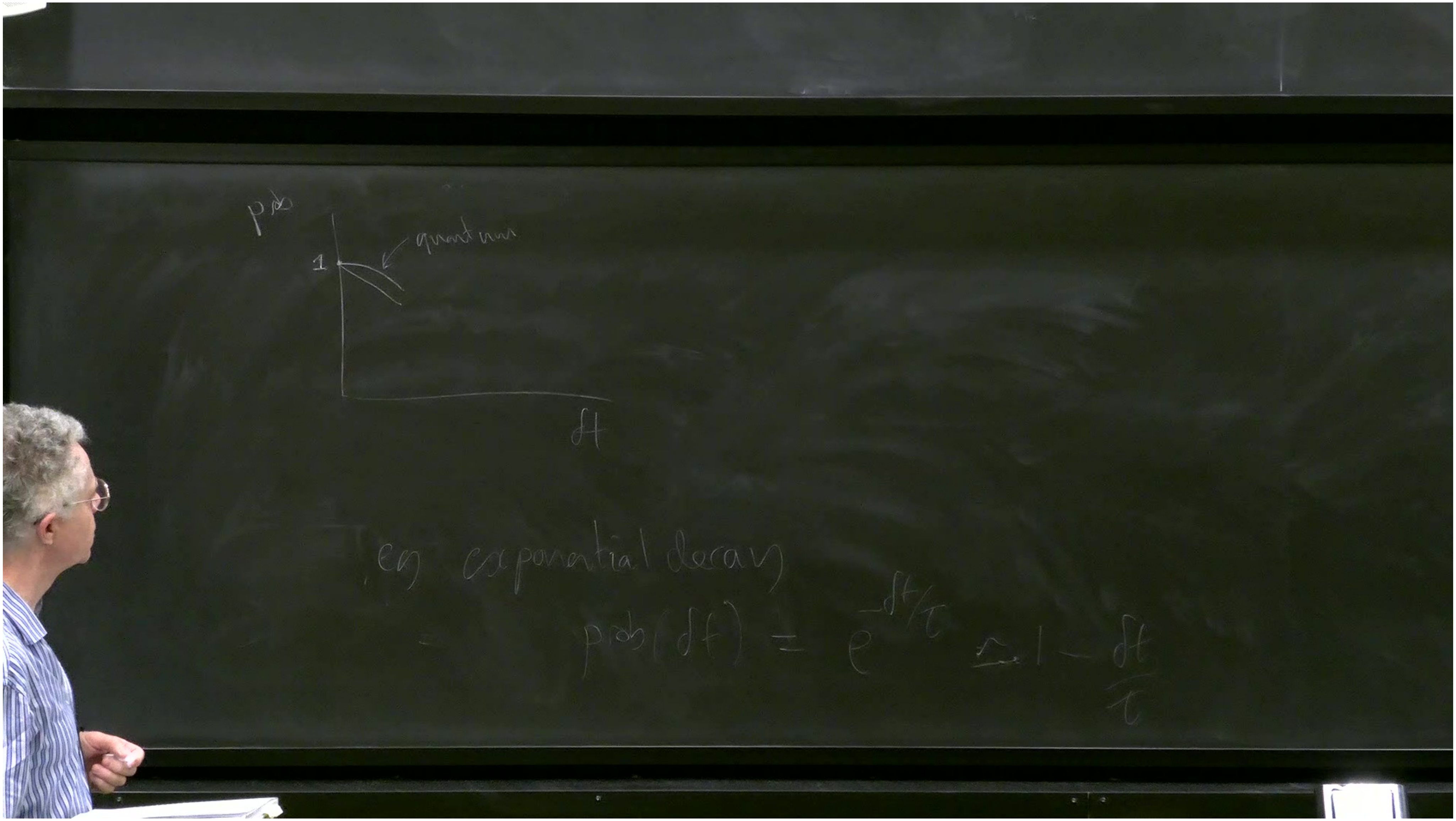
$$\text{prob}(P_0, \delta t) \simeq 1 - k^2(\delta t)^2$$

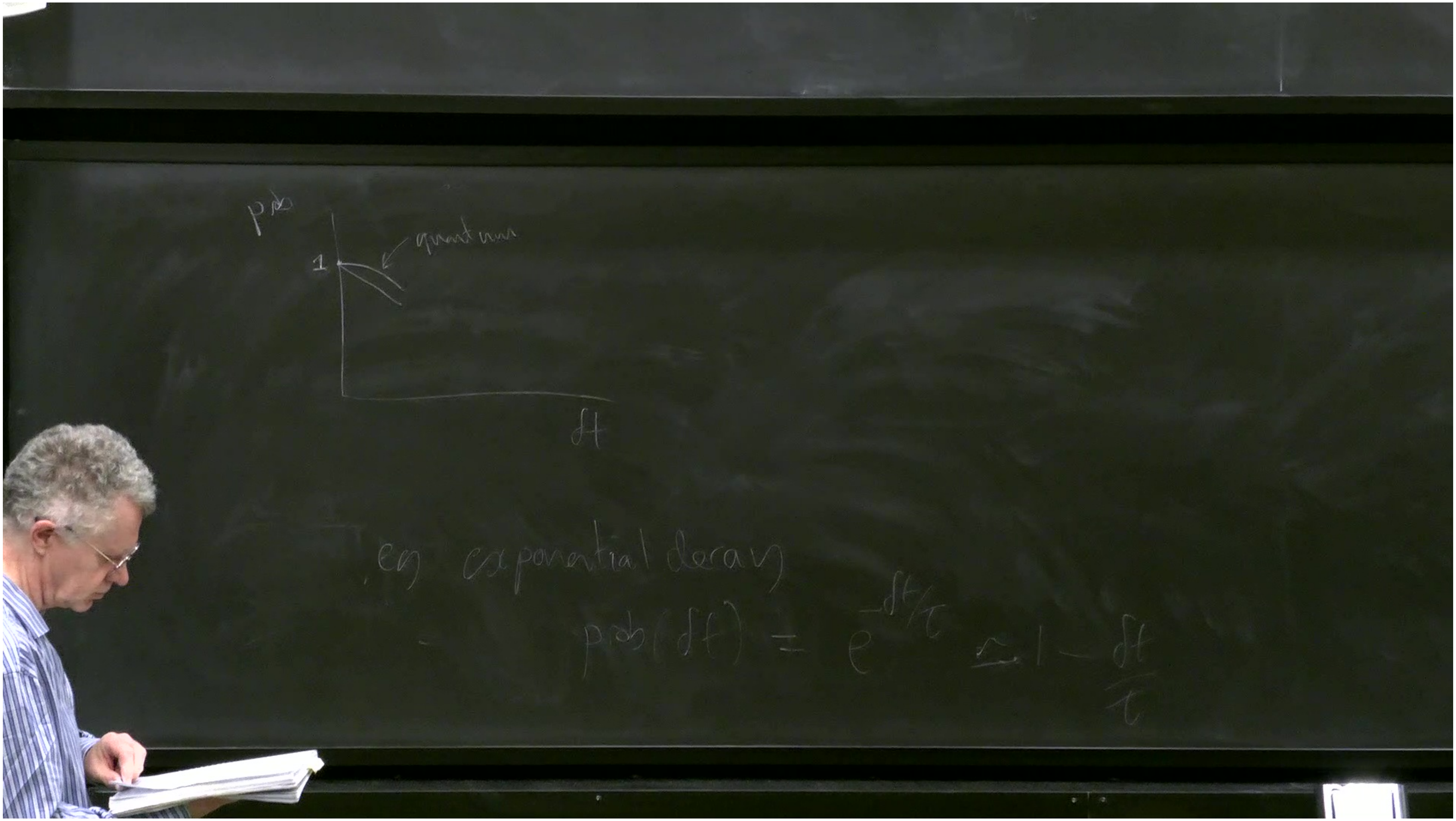
$$= \langle \psi(\delta t)| \sum_{n=1}^{N-1} |n\rangle\langle n| \,|\psi(\delta t)\rangle$$

$$\hbar \qquad d_1 \qquad = \langle \psi(\delta t) | \sum_{n=1} |n\rangle\langle n| \psi(\delta t) \rangle$$

$$\langle 0 | \left(\cancel{\mathbb{1}} + \frac{i\hat{H}\delta t}{\hbar} \right) \sum_{n=1}^{N-1} |n\rangle\langle n| \left(\cancel{\mathbb{1}} - \frac{i\hat{H}\delta t}{\hbar} \right) | 0 \rangle$$

$$= \underbrace{\frac{1}{\hbar^2} \sum_{n=1}^{N-1} \langle 0|\hat{H}|n\rangle\langle n|\hat{H}|0\rangle}_{K^2} \delta t^2$$


prob
1
quantum
δt
eg exponential decay
prob(δt) = e^{-δt/τ} ≃ 1 − δt/τ


prob
quantum
1
δt
eg exponential decay
prob(δt) = e^{-δt/τ} ≃ 1 − δt/τ

$$\text{prob}(\delta t) = e^{-\delta t/\tau} \simeq 1 - \frac{\delta t}{\tau}$$

Look at system every δt, for time T - we look $\frac{T}{\delta t}$ times

$$\text{prob}(\underbrace{P_0, \ldots, P_0}_{T/\delta t}) = \left(1 - k^2 \delta t^2\right)^{T/\delta t} = 1 - \left(\frac{T}{\delta t}\right) k^2 (\delta t)^2 + \ldots$$

$$= 1 - k^2 T \delta t$$

as $\delta t \to 0$ prob $\to 1$

$\frac{T}{\delta t}$ times

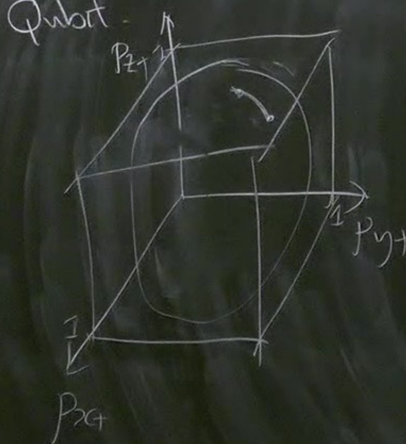
$\left(\frac{T}{\delta t}\right) k^2 (\delta t)^2 + \cdots$

δt

$\to 0$ prob $\to 1$

Have linear evolution in $\mathcal{H}$

Qubit.



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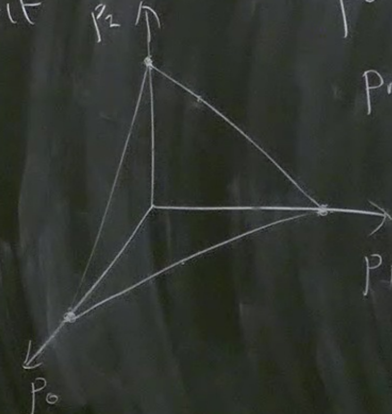


$p_0 + p_1 + p_2 = 1$

$p_n \geq 0$

$$\text{prob}(\delta t) = e^{\cdots} \simeq 1 - \frac{\delta t}{\tau}$$

Look at system every δt, for time T – we look $\frac{T}{\delta t}$ times

$$\text{prob}(\underbrace{P_0, \ldots, P_0}_{T/\delta t}) = \left(1 - k^2 \delta t^2\right)^{T/\delta t} = 1 - \left(\frac{T}{\delta t}\right) k^2 (\delta t)^2 + \cdots$$

$$= 1 - k^2 T \delta t$$

as $\delta t \to 0$ prob $\to 1$

Quantum Optical Interferometry

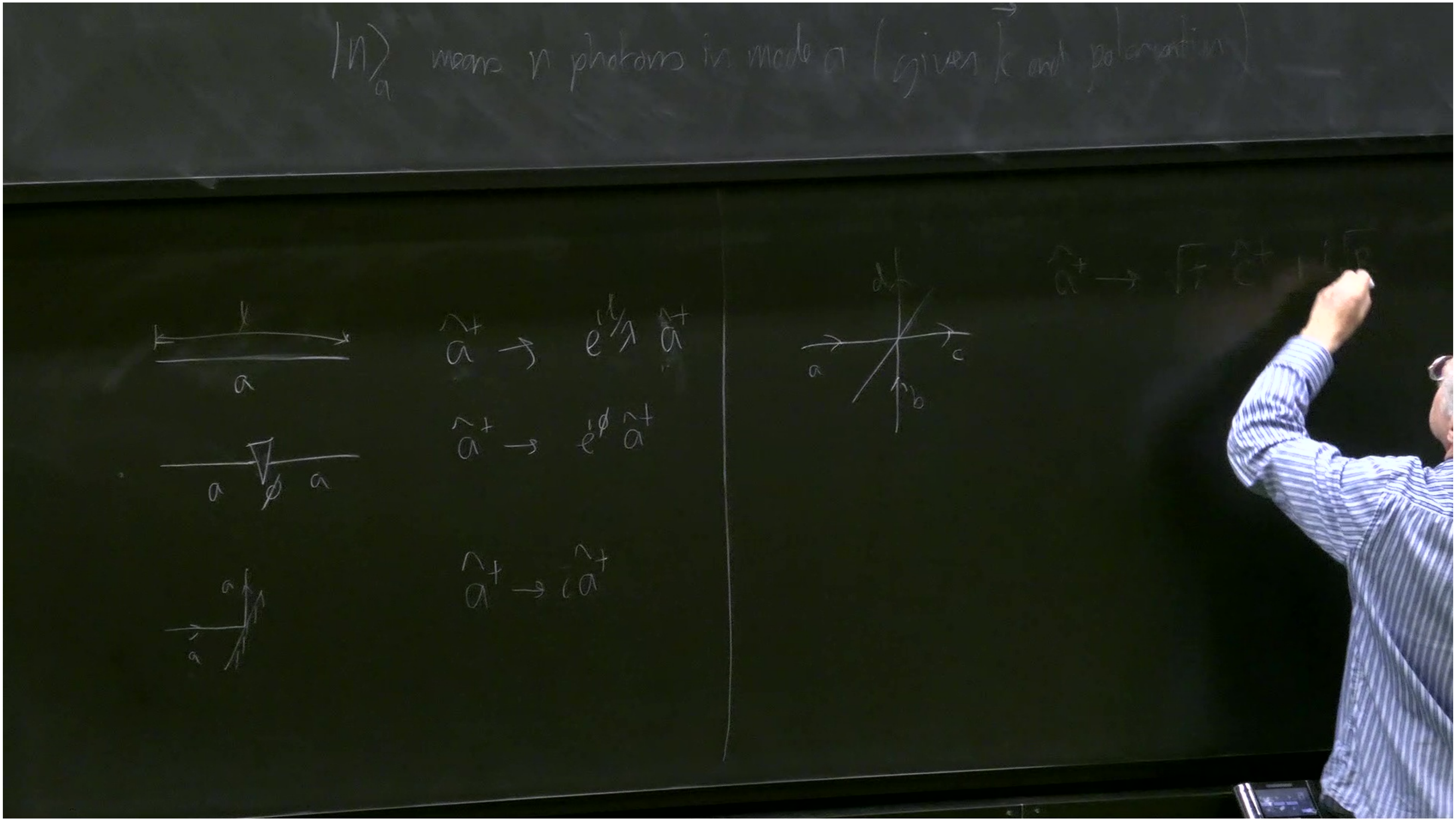
single mode

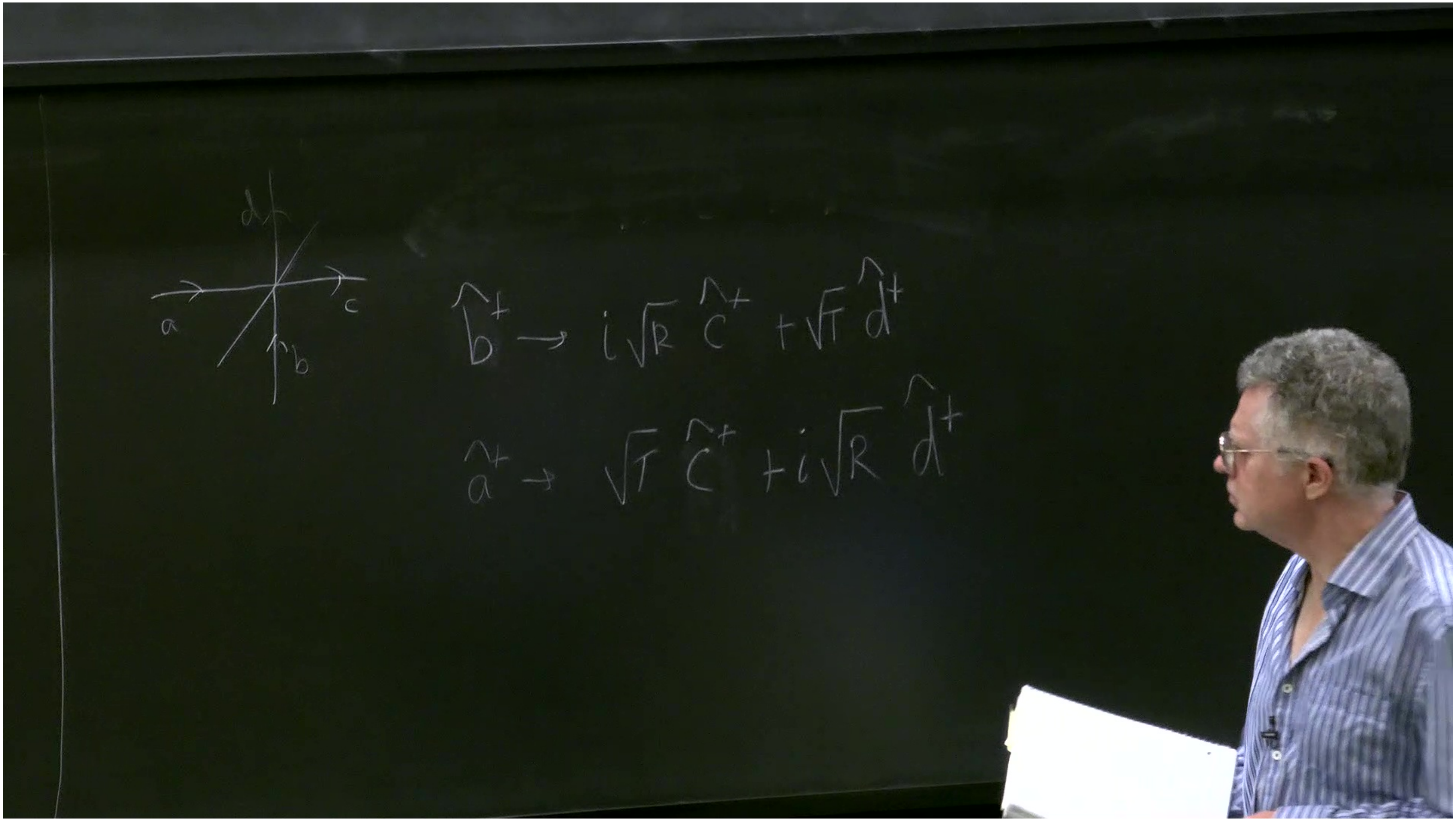
The "trick" is to use creation, $\hat{a}^\dagger$, and annihilation, $\hat{a}$, operators

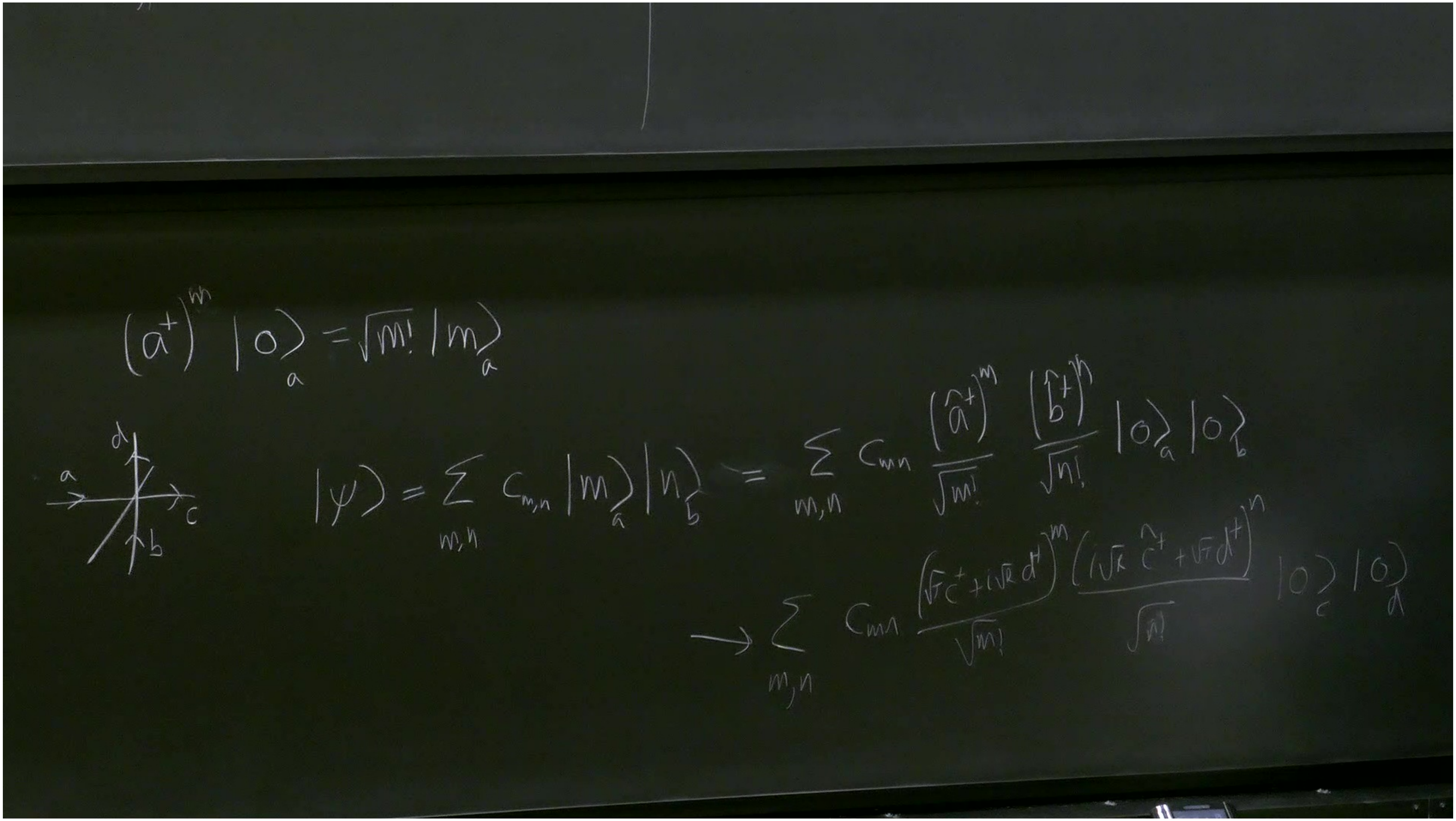
$$\hat{a}|0\rangle_a = 0 \qquad \hat{a}|n\rangle_a = \sqrt{n}\,|n-1\rangle_a\,, \qquad \hat{a}^\dagger|n\rangle_a = \sqrt{n+1}\,|n+1\rangle_a$$

$$[\hat{a}, \hat{a}^\dagger] = \hat{\mathbb{1}}$$

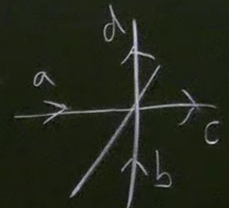
$|n\rangle_a$ means n photons in mode a (given $\vec{k}$ and polarization)


|n⟩_a means n photons in mode a (given k and polarization)
â† → e^{iL/λ} â†
â† → e^{iφ} â†
â† → i â†
â† → √T ĉ† + i√R


d
a
c
b
$\hat{b}^\dagger \to i\sqrt{R}\,\hat{c}^\dagger + \sqrt{T}\,\hat{d}^\dagger$
$\hat{a}^\dagger \to \sqrt{T}\,\hat{c}^\dagger + i\sqrt{R}\,\hat{d}^\dagger$


$(a^{+})^{m}|0\rangle_a = \sqrt{m!}\,|m\rangle_a$
d
a
c
b
$|\psi\rangle = \sum_{m,n} c_{m,n}|m\rangle_a|n\rangle_b = \sum_{m,n} c_{mn}\frac{(\hat{a}^{+})^{m}}{\sqrt{m!}}\frac{(\hat{b}^{+})^{n}}{\sqrt{n!}}|0\rangle_a|0\rangle_b$
$\longrightarrow \sum_{m,n} c_{mn}\frac{(\sqrt{T}c^{+}+i\sqrt{R}d^{+})^{m}}{\sqrt{m!}}\frac{(i\sqrt{R}\,\hat{c}^{+}+\sqrt{T}d^{+})^{n}}{\sqrt{n!}}|0\rangle_c|0\rangle_d$

$$(a^{\dagger})^{m}\,|0\rangle_a = \sqrt{m!}\,|m\rangle_a$$



$$|\psi\rangle = \sum_{m,n} c_{m,n}\,|m\rangle_a |n\rangle_b = \sum_{m,n} c_{mn}\,\frac{(\hat{a}^{\dagger})^{m}}{\sqrt{m!}}\,\frac{(\hat{b}^{\dagger})^{n}}{\sqrt{n!}}\,|0\rangle_a |0\rangle_b$$

$$\rightarrow \sum_{m,n} c_{mn}\,\frac{\left(\sqrt{T}\,\hat{c}^{\dagger} + i\sqrt{R}\,d^{\dagger}\right)^{m}}{\sqrt{m!}}\,\frac{\left(i\sqrt{R}\,\hat{c}^{\dagger} + \sqrt{T}\,d^{\dagger}\right)^{n}}{\sqrt{n!}}\,|0\rangle_c |0\rangle_d$$

$$|\psi\rangle = \sum_{m,n} c_{mn}\,|m\rangle|n\rangle$$

$$= \sum k_n\,|n'\rangle|n'\rangle$$

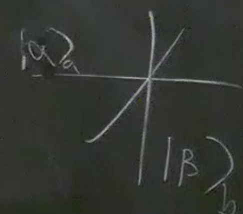
e.g. exponential decay

$$\text{prob}(dt) = e^{-dt/\tau} \simeq 1 - dt$$

Detectors.

$$\xrightarrow{a} D^A \qquad \hat{a}^\dagger \hat{a}\, |n\rangle_a = n\,|n\rangle_a$$

$$|\alpha\rangle_a = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\, |n\rangle_a$$



Sources

$$\boxed{\text{LASER}} \xrightarrow{a} \qquad |\alpha\rangle_a$$

$$\hat{a}\,|\alpha\rangle_a = \alpha\,|\alpha\rangle_a$$

$$|\alpha\rangle_a |\beta\rangle_b \longrightarrow |\sqrt{T}\alpha + i\sqrt{R}\,\beta\rangle_c\, |i\sqrt{R}\alpha +$$

n

$n \geq 1$

K^2

$\beta/2 \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle_a$

PMDC.

$\to |\sqrt{T}\alpha + i\sqrt{R}\,\beta\rangle_c \, |i\sqrt{R}\alpha + \sqrt{T}\,\beta\rangle_d$