

**Title:** Lecture - Quantum Foundations, PHYS 639

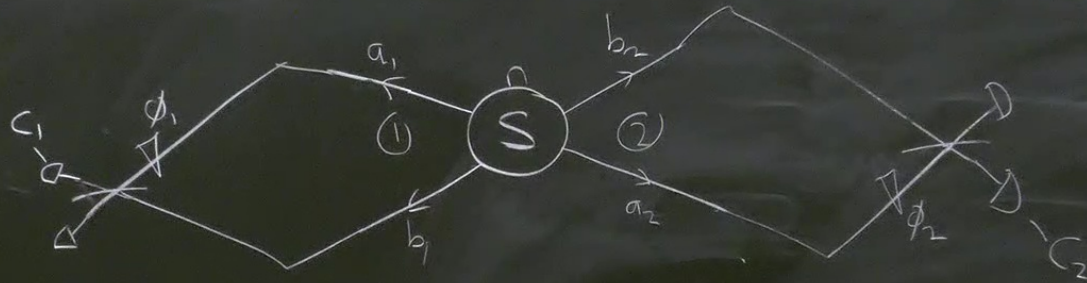
**Speakers:** Lucien Hardy

**Collection/Series:** Quantum Foundations (Elective), PHYS 639, January 6 - February 5, 2025

**Subject:** Quantum Foundations

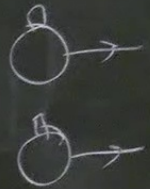
**Date:** January 07, 2025 - 10:15 AM

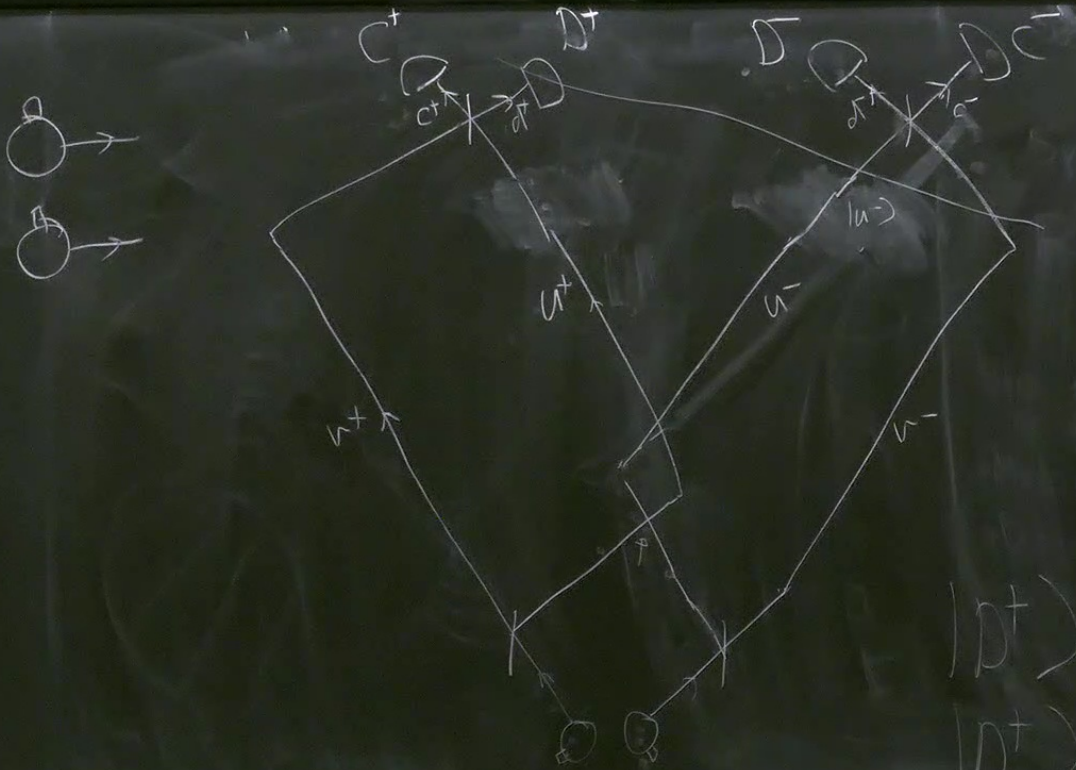
**URL:** <https://pirsa.org/25010038>



$$|\psi\rangle = \frac{1}{\sqrt{2}} (|a_1\rangle |a_2\rangle + |b_1\rangle |b_2\rangle)$$

prob(C, D) ~ sinusoidal fn of  $\phi_1 + \phi_2$





$D^+ \& D^-$  happens sometimes

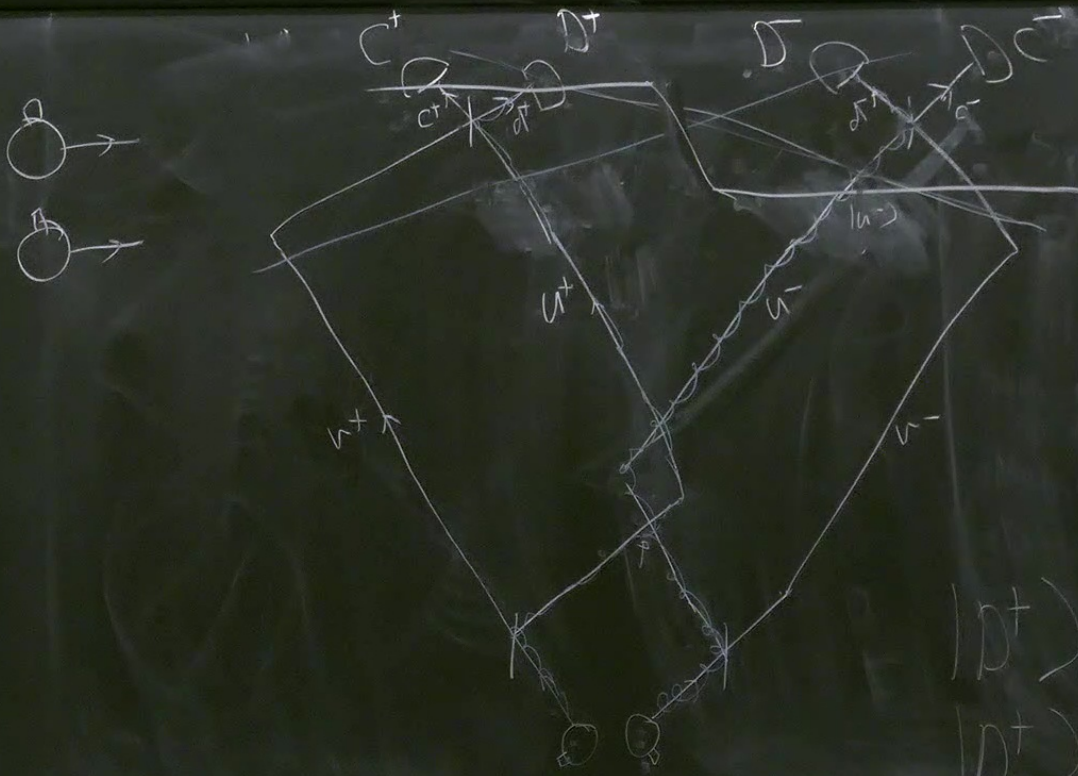
$$D^+ \Rightarrow U^-$$

$$U^+ \Leftarrow D^-$$

$U^+ \& U^-$  never happens

$$|D^+\rangle |u^-\rangle$$

$$|D^+\rangle |s^-\rangle \times$$



$D^+$  &  $D^-$  happens sometimes

$D^+ \Rightarrow U^-$

$U^+ \Leftarrow D^-$

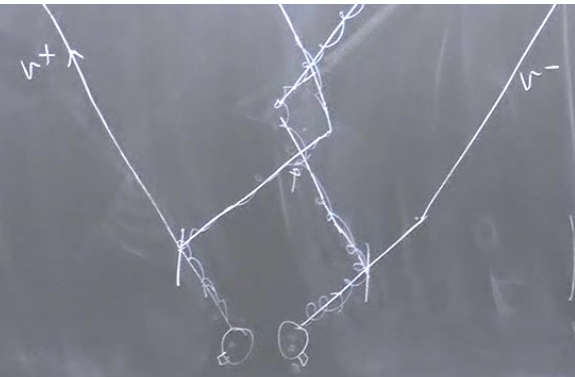
$U^+$  &  $U^-$  never happens

$|D^+ \rangle |U^- \rangle$

$|D^+ \rangle |U^- \rangle \times$

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|a_1\rangle|a_2\rangle + |b_1\rangle|b_2\rangle)$$

prob( $C_1, C_2$ )  $\sim$  sinusoidal fn of  $\phi_1 + \phi_2$



① Associated with each system,  $a_i$ , is a Hilbert space  $\mathcal{H}_{a_i}$  of dimension  $N_{a_i}$  where  $a_i$  is the system type and  $i$  is the instance.

② For a composite system ( $a_1, a_2, b_3, c_4$  for example) the Hilbert space is the tensor product of the components Hilbert spaces

$$\mathcal{H}_{a_1} \otimes \mathcal{H}_{a_2} \otimes \mathcal{H}_{b_3} \otimes \mathcal{H}_{c_4}$$

$$(H_{a_1} \otimes H_{a_2} \otimes H_{a_3} \otimes H_{a_4})$$

③ The state of a system at any given time is an (initially normalized) element of the associated Hilbert space  $|\psi\rangle_{a_i} \in H_{a_i}$

④ Evolution. Two cases.

Ⓐ When there is no measurement the state evolves unitarily

$$|\psi\rangle \rightarrow \hat{U}|\psi\rangle \quad \hat{U} \text{ is unitary.}$$

Ⓑ When there is a measurement the state is projected with the projector associated with the measurement outcome we see

$$|\psi\rangle \rightarrow \hat{P}|\psi\rangle$$

$$H_3 \otimes H_4$$

my question is an (initially normalized) element of

$$|\psi\rangle_{a_1} \in H_{a_1}$$

but the state evolves unitarily

$\hat{U}$  is unitary.

the state is projected with the projector

and outcome we see

$$|\psi\rangle \rightarrow \hat{P}|\psi\rangle \quad \hat{P} \text{ is a projector.}$$

an (initially normalized) element of

$$\hat{P}_3 \left( \hat{P}_2 \left( \hat{P}_1 |\psi\rangle \right) \right)$$

with the projector

$$|\psi\rangle \rightarrow \hat{P} |\psi\rangle$$

$\hat{P}$  is a projector.



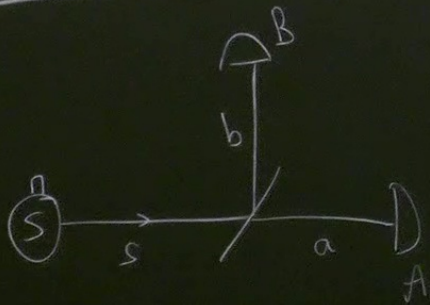
⑤ The probability of a given measurement outcome is given by the square magnitude after projection

$$\text{prob} = \langle \psi | \hat{P} | \psi \rangle = \langle \psi | \hat{P} | \psi \rangle$$

The Born rule.

$$\frac{N_{\text{outcomes}}}{N}$$

# The measurement problem

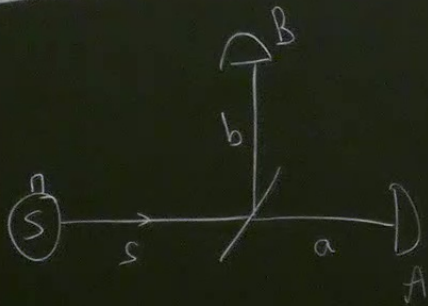


$$|s\rangle |A_0\rangle |B_0\rangle \rightarrow (\sqrt{T} |a\rangle + i\sqrt{R} |b\rangle) |A_0\rangle |B_0\rangle$$

$$\rightarrow \sqrt{T} |A\rangle |B_0\rangle + i\sqrt{R} |A_0\rangle |B\rangle$$

$$\xrightarrow[\text{clicks}]{A} \sqrt{T} |A\rangle |B_0\rangle \quad \text{prob} = T$$

# The measurement problem

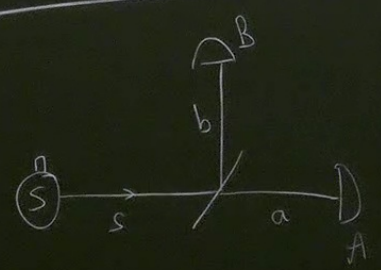


$$|s\rangle |A_0\rangle |B_0\rangle |me\rangle \rightarrow \left( \sqrt{T} |a\rangle + i\sqrt{R} |b\rangle \right) |A_0\rangle |B_0\rangle |me\rangle$$

$$\rightarrow \sqrt{T} |A\rangle |B_0\rangle \left| \begin{array}{c} I \\ \text{see} \\ A \\ \text{click} \end{array} \right\rangle + i\sqrt{R} |A_0\rangle |B\rangle \left| \begin{array}{c} I \\ \text{see} \\ B \\ \text{click} \end{array} \right\rangle$$

$$\xrightarrow[A \text{ clicks}]{} \sqrt{T} |A\rangle |B_0\rangle \quad \text{prob} = T$$

measurement problem



$$|s\rangle |A_0\rangle |B_0\rangle |me\rangle \rightarrow (\sqrt{T} |a\rangle + \sqrt{R} |b\rangle) |A_0\rangle |B_0\rangle |me\rangle$$

$$\rightarrow \sqrt{T} |A\rangle |B_0\rangle \left| \begin{smallmatrix} I \\ \text{see} \\ A \\ \text{click} \end{smallmatrix} \right\rangle + \sqrt{R} |A_0\rangle |B\rangle \left| \begin{smallmatrix} I \\ \text{see} \\ B \\ \text{click} \end{smallmatrix} \right\rangle$$

— many worlds  
 — additional h.v. deBB

$$\xrightarrow[\text{click}]{A} \sqrt{T} |A\rangle |B_0\rangle \quad \text{prob} = T$$