Title: Lecture - Gravitational Physics, PHYS 636

Speakers: Ruth Gregory

Collection/Series: Gravitational Physics (Elective), PHYS 636, January 6 - February 5, 2025

Subject: Cosmology, Strong Gravity

Date: January 29, 2025 - 9:00 AM

URL: https://pirsa.org/25010035

LECTURE 11: Extended Thermodynamics

Consider AdS black holes:

$$ds^2 = f\,dt^2 - \frac{dr^2}{f} - r^2 d\Omega_{k}^2$$

$$f = 1 - \frac{2m}{r} + \frac{r^2}{L^2} \left(+\frac{Q^2}{r^2}\right)$$

LATER

$G \to 1$ For convenience.

Recall $T = \frac{1}{4\pi r_+}\left[1 + \frac{3r_+^2}{L^2}\right]$

∂

∂

T

$(GM$

mics

$$\frac{\partial T}{\partial r_+} = \frac{1}{4\pi r_+^2}\left[-1 + \frac{3r_+^2}{L^2}\right]$$

T stationary at $r_+ = L/\sqrt{3}$

$\left(GM = \frac{2}{3\sqrt{3}}L\right)$ The heat capacity tells us how much heat is needed to raise the temp. of a body y

body

$$C = \frac{\delta Q}{\delta T} = T\,\frac{\delta S}{\delta T}$$

Here $S = \pi r_+^2$

$$C = \frac{2\pi r_+ \cdot 4\pi r_+^2}{\left[\frac{3r_+^2}{L^2} - 1\right]} = 2\pi r_+^2\,\frac{\left[\frac{3r_+^2}{L^2} + 1\right]}{\left[\frac{3r_+^2}{L^2} - 1\right]}$$

See that for $L \to \infty$ (or $r_+ \to 0$) $C < 0$

but for $r_+ \gg L$, $C > 0$

In vacuum, $c < 0$, so the b.h radiates, loses mass, gets hotter, ...

a runaway process occurs:

Black Hole Evaporation.

Estimate energy loss:

$$\frac{dM}{dt} \sim \sigma T^4 \cdot 4\pi r_+^2$$

$$\sigma = \left(\pi^2/60\right)$$

b.h

For SCH, $r_+ = 2M$

$$\frac{dM}{dT} \sim \frac{1}{256 \times 60\pi M^2}$$

$$\Rightarrow \frac{M^3}{3} \sim \frac{(t_0 - t)}{15360\pi}$$

$M \to 0$ in finite $\Delta t \sim 5120\pi M^3$

$5120\pi M^3$

A black hole of mass $\gtrsim 5 \times 10^{14}$ g has a lifetime longer than age of Universe.

How to form? Via v. large fluctuations in primordial power spectrum – currently "Ultra Slow Roll" inflation can give such large ptns. These are Primordial Black Holes (PBH)

PBH could be/contribute to Dark Matter

$$\frac{d}{dt} \qquad L_+(\pi^2/60)$$

In AdS, $C > 0$ for $r_+ > L/\sqrt{3}$

thus large black holes are thermodynamically stable – but is this all?

Look at Gibbs free energy cf to radn

$$G = \underline{M} - TS = \frac{r_+}{2}\left(1 + \frac{r_+^2}{L^2}\right) - \frac{r_+}{4}\left(1 + \frac{3r_+^2}{L^2}\right) = \frac{r_+}{4}\left(1 - \frac{r_+^2}{L^2}\right)$$

PBH could be/contribute to Dark Matter

$G = 0$ for $r_+ = L$

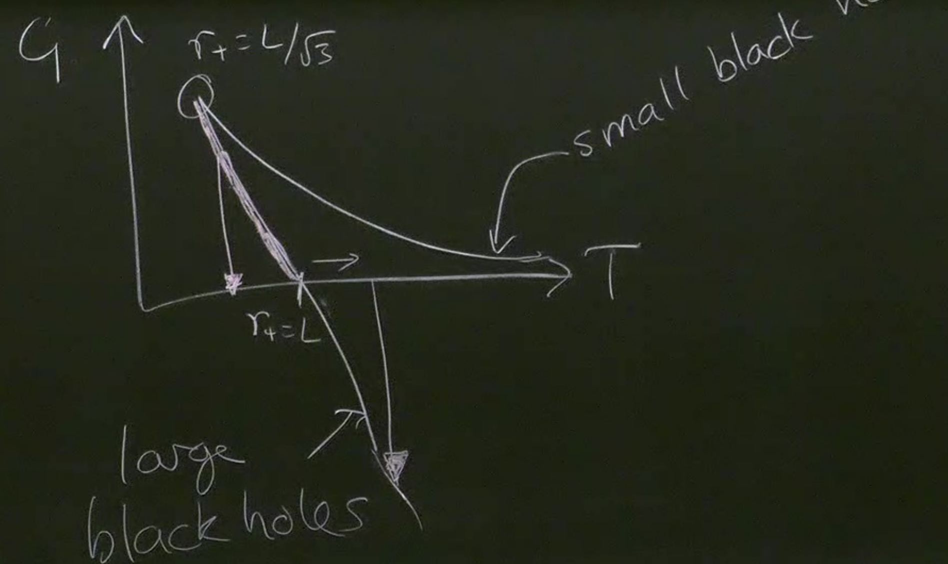
For radn:

$$G = U - pV - TS$$

$$U = \frac{\pi^2}{15} V T^4 \qquad S = \frac{4U}{3T}$$

$$p = \frac{1}{3}\rho = \frac{U}{3V} \Rightarrow G = 0$$

Hawking - Page



$dM = T\,dS - p\,dV$

In AdS, have Λ, hence

a pressure $\sim \dfrac{-\Lambda}{8\pi G}$

If think of Λ as vacuum energy

$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 - V\right]$$

If $\phi = \phi_0$ & $V'(\phi_0) = 0$ then $T_{\mu\nu} = V_0\, g_{\mu\nu}$.

$$f(r_+) = 1 - \frac{2m}{r_+} + \frac{Q^2}{r_+^2} + \frac{r_+^2}{L^2} = 0$$

Consider $f(r_+ + \delta r_+)$:

$$0 = f'(r_+)\,\delta r_+ - \frac{2\,\delta m}{r_+} + 2\frac{Q\,\delta Q}{r_+}$$
$$- 2\frac{\delta L}{L^3} r_+^2$$

$$A = \frac{Q}{r}\,dt \qquad \left(Q = \frac{1}{4\pi}\int \star F \right)$$

$$T = \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi r_+}\left[1 - \frac{Q^2}{r_+^2} + 3\frac{r_+^2}{L^2}\right] \quad (\text{Eucl}$$

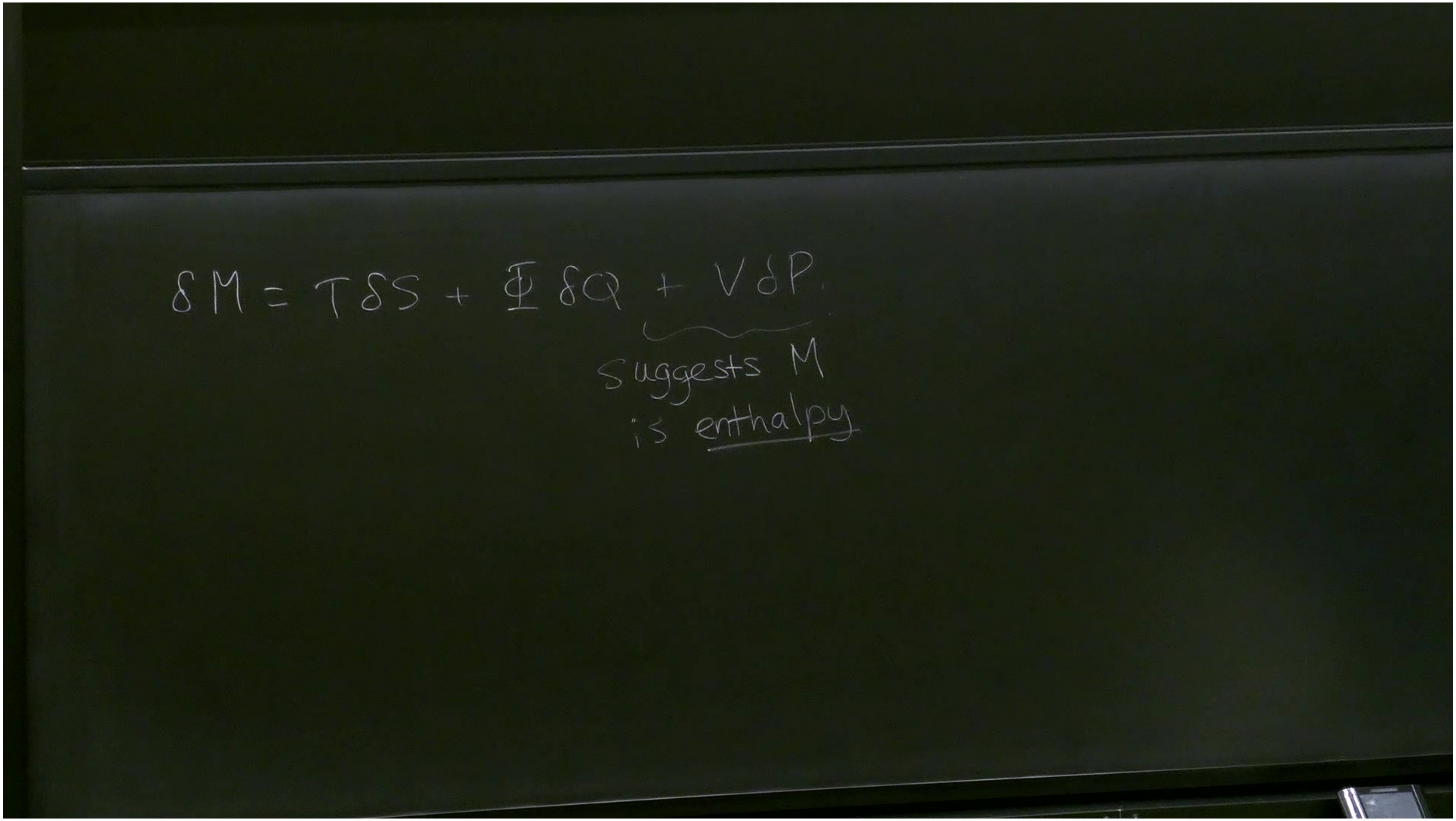
$$T = \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi r_+}\left[1 - \frac{Q^2}{r_+^2} + 3\frac{r_+^2}{L^2}\right]$$ (Euclidean trick)

$$S = \pi r_+^2$$

$$\Phi = \frac{Q}{r_+}$$

$$\delta m = \underbrace{\frac{f'(r_+)\,\delta(r_+^2)}{4}}_{T\delta S} + \Phi\,\delta Q + \underbrace{\frac{1}{2} r_+^3\,\delta\!\left(\frac{1}{L^2}\right)}_{\underbrace{\frac{4\pi}{3} r_+^3}_{V}\,\delta p}$$

We associate $\frac{4\pi}{3} r_+^3$ with a thermodynamic volume to get a 1^{st} Law.


$\delta M = T\delta S + \Phi\,\delta Q + V\delta P$
suggests M
is enthalpy

$$\delta M = T\delta S + \Phi\,\delta Q \underbrace{+ V\delta P}_{\text{suggests } M \text{ is enthalpy}}$$

Look again at G at fixed Q

$$T = \frac{1}{4\pi r_+}\left(1 - \frac{Q^2}{r_+^2} + \frac{3r_+^2}{L^2}\right)$$

$$\frac{\partial T}{\partial r_+} = \frac{1}{4\pi r_+^2}\left(-1 + \frac{3Q^2}{r_+^2} + \frac{3r_+^2}{L^2}\right)$$

$$\frac{\partial T}{\partial r_+} = 0 \quad \text{at} \quad r_+^2 = \frac{L^2}{6}\left[1 \pm \sqrt{1 - \frac{36Q^2}{L^2}}\right].$$

If $Q > L/6$ – no real root

(As we increase Q, f develops a repeated root – $Q = M$ if $\Lambda = 0$ – this is the <u>extremal limit</u> with $T = 0$).

$\pm\sqrt{1-\frac{36Q^2}{L^2}}\Big]$.

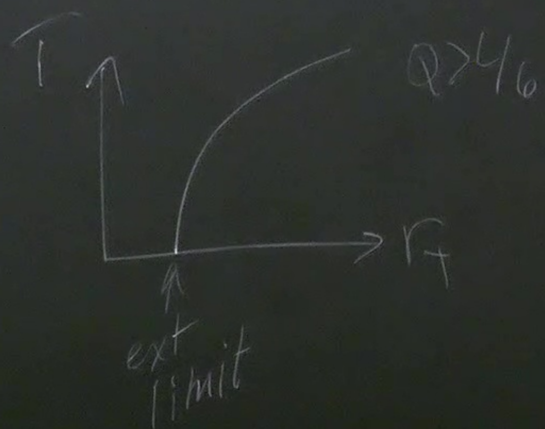
root

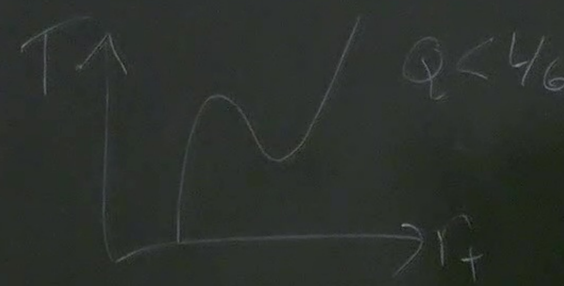
elops a

=0 - this

ith T=0).

If $Q < L/6$, get 2 turning pts in T





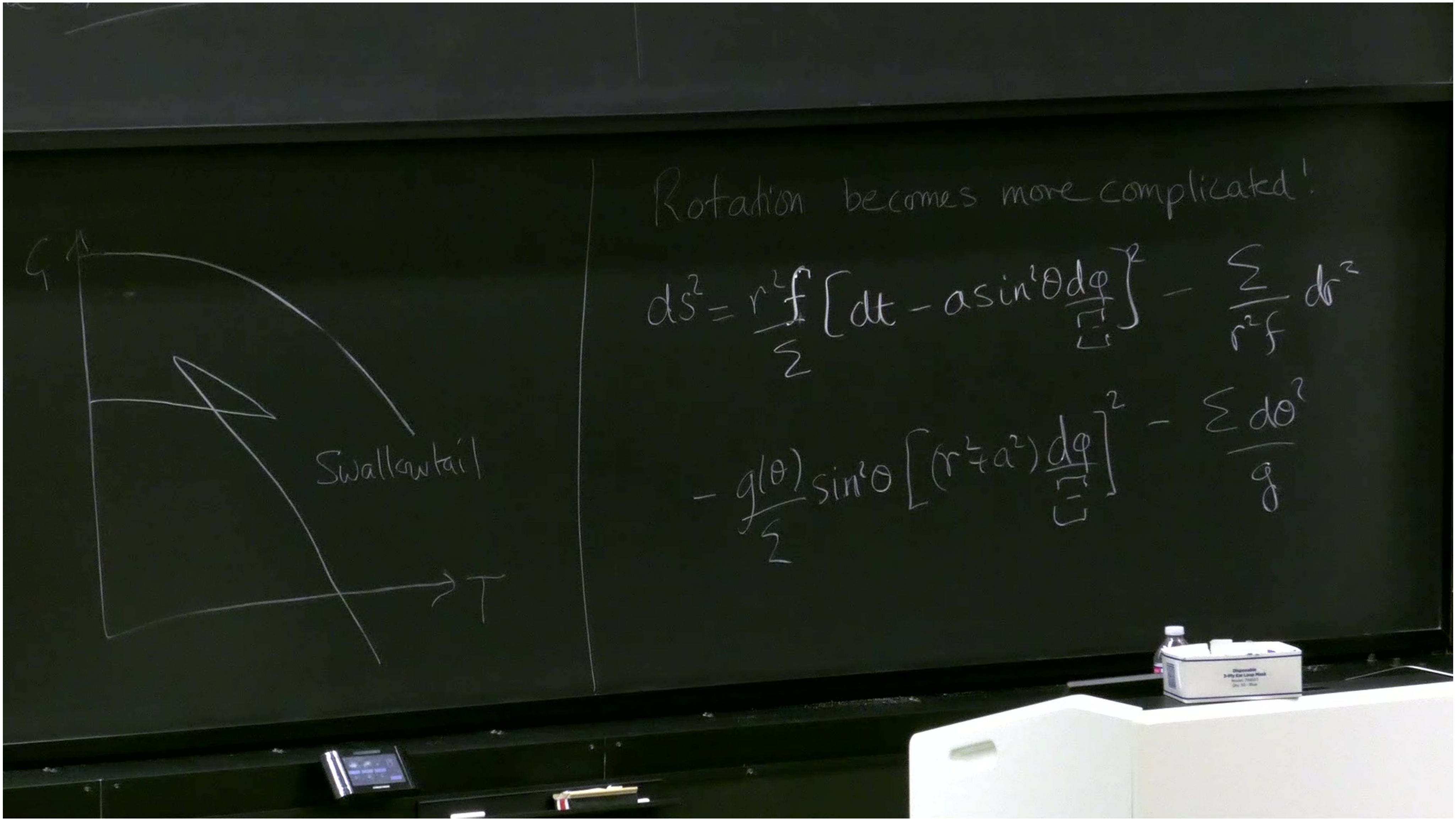
$$\frac{\partial I}{\partial r_+} = \frac{1}{4\pi r_+^2}\left(-1 + \frac{3Q^2}{r_+^2} + \frac{3r_+^2}{L^2}\right)$$

Free energy $M - TS$

$$G = \frac{r_+}{2}\left[1 + \frac{Q^2}{r_+^2} + \frac{r_+^2}{L^2}\right] - \frac{r_+}{4}\left[1 - \frac{Q^2}{r_+^2} + \frac{3r_+^2}{L^2}\right]$$

$$= \frac{r_+}{4}\left[1 + \frac{3Q^2}{r_+^2} - \frac{r_+^2}{L^2}\right]$$

$$\frac{\partial G}{\partial r_+} = \frac{1}{4}\left[1 - \frac{3Q^2}{r_+^2} - \frac{3r_+^2}{L^2}\right]$$


G
Swallowtail
T
Rotation becomes more complicated!
$ds^2 = \frac{r^2 f}{\Sigma}\left[dt - a\sin^2\theta \frac{d\varphi}{\Xi}\right]^2 - \frac{\Sigma}{r^2 f}dr^2$
$- \frac{g(\theta)}{\Sigma}\sin^2\theta\left[(r^2+a^2)\frac{d\varphi}{\Xi}\right]^2 - \frac{\Sigma\, d\theta^2}{g}$

$$f = 1 - \frac{2m}{r} + \frac{a^2 + Q^2}{r^2} + \frac{\boxed{r^2 + a^2}}{L^2}$$

$$\Sigma = r^2 + a^2 \cos^2\theta$$

$$g = 1 - \frac{a^2}{L^2}\cos^2\theta$$

Ξ is chosen to make metric regular

on axis

$+ \frac{\boxed{r^2+a^2}}{L^2}$

ic regular

$$ds^2_{\theta,\varphi} \underset{\theta\to 0}{\sim} d\theta^2 + \theta^2\left(1-\frac{a^2}{L^2}\right)^2 \frac{d\varphi^2}{\Xi^2}$$

$$\Rightarrow \quad \Xi = 1 - a^2/L^2$$

As $r \to \infty$

$$\frac{g_{t\varphi}}{g_{\varphi\varphi}} \sim \frac{-a r^2 \sin^2\theta / L^2 \Xi}{-r^2 \frac{\sin^2\theta}{\Xi^2}(1-a^2/L^2)}$$

$$\to -a/L^2$$

In B-L coords - the boundary is rotating!

$$M = \frac{m}{\Xi^2} \qquad J = aM \qquad \Omega = \frac{a\Xi}{r_+^2+a^2} + \frac{a}{\ell^2}$$

$$\Phi = \frac{Q r_+ \Xi}{r_+^2+a^2} \qquad V = \frac{4\pi}{3\Xi}\left[r_+(r_+^2+a^2) + \frac{m a^2}{\Xi} \right]$$